Beam Scheduling and Relay Assignment in Wireless Relay Networks with Smart Antennas

Brendan Mumey, Jian Tang, Ivan Judson, Richard S. Wolff

***Abstract*—Relay Stations (RSs) can be deployed in a wireless network to extend its coverage and improve its capacity. Smart (directional) antennas can enhance the functionalities of RSs by forming the beam only towards intended receiving Subscriber Stations (SSs). In this paper, we study a joint problem of selecting a beam width and direction for the smart antenna at each RS and determining the RS assignment for SSs in each scheduling period. The objective is to maximize a utility function that can lead to a stable and high-throughput system. We define this as the Beam Scheduling and Relay Assignment Problem (BS-RAP). We show that BS-RAP is NP-hard, present a Mixed Integer Linear Programming (MILP) formulation to provide optimal solutions and present two polynomial-time greedy algorithms, one of which is shown to have a constant factor approximation ratio.**

***Index Terms*– Wireless relay networks, smart antenna, beam scheduling, relay assignment, approximation algorithm**

I. Introduction

A wireless relay network consists of a Base Station (BS), multiple Relay Stations (RSs) and a large number of Subscriber Stations (SSs).The BS serves as a gateway connecting the network to external networks such as the Internet. If an SS is out of the transmission range of the BS, it can communicate with the BS via one or multiple RSs in a multihop manner. Such a network architecture has been adopted by emerging wireless networking standards such as IEEE 802.16j. The IEEE 802.16j [2] was proposed to extend the scope of IEEE 802.16e [1] to support multihop relay. Compared to a single-hop wireless network in which each SS directly communicates with the BS, a relay network can significantly extend the coverage range, improve network capacity and reduce dead spots [2]. Therefore, such relay networks are considered as a promising solution to provide low-cost, high-speed and long-range wireless communications for various applications such as broadband Internet access and emergency communications.

Compared to a conventional omni-directional antenna, a smart (directional) antenna offers a longer transmission range and lower power consumption by forming one or multiple beams only toward intended receivers without wasting energy in other directions. Therefore, smart antennas can enhance the functionalities of RSs and help a wireless relay network better achieve its goal. We focus on a smart adaptive antenna with an adjustable beamwidth and beam orientation.

Wireless relay networks have attracted extensive attention from the research community recently and various resource allocation problems have been studied in recent works [9], [13], [21], [22]. However, most of them focused on relay networks with omni-directional antennas. In this paper, we exploit the benefits of using smart antennas in wireless relay networks by jointly considering two fundamental problems: Beam Scheduling (selecting a beamwidth and direction for the smart antenna at each RS in each scheduling period) and Relay Assignment (determining how the RSs should be assigned to serve SSs in each scheduling period). Our objective is to maximize a utility function that can lead to a stable and high-throughput system. To the best of our knowledge, we are the first to study such a joint beam scheduling and relay assignment problem in the context of wireless relay networks, and present theoretically well founded and practically useful algorithms to solve it. Specifically, we summarize our contributions in the following:

1) We define the Beam Scheduling and Relay Assignment Problem (BS-RAP), show it is NP-hard and present a Mixed Integer Linear Programming (MILP) formulation to provide optimal solutions, which can serve as a benchmark for performance evaluation.
2) We present two polynomial-time greedy algorithms for the BS-RAP and show that one of them has a constant approximation ratio (i.e., if the problem is a maximization problem, then the objective value of a solution given by the algorithm is guaranteed to be no smaller than the optimal value multiplied by a constant less than 1).

The rest of this paper is organized as follows. We discuss related work in Section II. We describe the system model and present the problem formulation in Sections III and IV, respectively. The proposed algorithms are presented in Section V. The simulation results are presented in Section VI and the paper is concluded in Section VII.

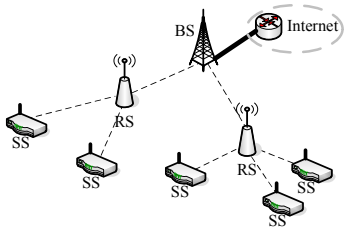

Fig. 1. A wireless relay network

II. Related Work

Resource allocation in wireless relay networks has received much research attention. In [21], Sundaresan *et al.* showed

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that the scheduling problem to exploit diversity gains alone in 2-hop WiMAX relay networks is NP-hard, and provided polynomial time approximation algorithms to solve it. They also proposed a heuristic algorithm to exploit both spatial reuse and diversity gains. In [9], a similar scheduling problem was studied for OFDMA-based WiMAX relay networks. The authors provided an easy-to-compute upper bound. They presented three heuristic algorithms which were shown to provide close-to-optimal solutions and outperform other existing algorithms in simulations. Other recent works on this topic include [13], [22].

Smart antennas have have also bee well studied. MAC protocols were proposed in [7], [11] for 802.11-based ad-hoc networks with switched beam antennas. The authors of these papers modified the original 802.11 MAC protocol to explore the benefits of directional antennas. In [20], Sundaresan *et al.* presented a constant factor approximation algorithm for Degree-Of-Freedom (DOF) assignment and a distributed algorithm for joint DOF assignment and scheduling in ad-hoc networks with Digital Adaptive Array (DAA) antennas. A unified representation of the physical layer capabilities of different types of smart antennas, and unified medium access algorithms were presented in [19]. Another important type of smart antenna are Multiple Input Multiple Output (MIMO) antennas which are able to support multiple concurrent streams over a single link. Resource allocation with MIMO links has been studied in [4], [14], [18].

We summarize the differences between our work and this related work as follows: (1) Most related work on wireless relay networks [9], [13], [21], [22] dealt with resource allocation problems with omni-directional antennas, which are mathematically different from the optimization problems studied here. (2) Relay assignment is a special problem for wireless relay networks, which, however, was not a concern for most previous work on directional antennas. (3) Some related work on directional antennas focused on switched beam (sectorized) antennas [7], [11], [12], [15], [17], that can only form main beams towards a few pre-defined directions. However, we consider a smart adaptive antenna with an adjustable beam orientation and beamwidth, which makes the corresponding optimization problems much harder. (4) We present fast and effective algorithms to determine antenna patterns and relay assignments in a real-time manner, which can be applied to networks with mobile nodes. However, the topology control algorithms [10], [12], [15], [17] may only be used in relatively static networks. (5) Generally, directional antenna related resource allocation problems are NP-hard. Most related work, including [7], [11], [4], [18], [19], presented heuristic algorithms that cannot provide any performance guarantees. Our work, however, presents a constant factor approximation algorithm for the joint beam scheduling and relay assignment problem and a polynomial-time optimal algorithm for the relay beam scheduling problem.

III. System Model

We consider a 2-hop wireless relay network with a BS, m RSs $\{R_1, \ldots, R_i, \ldots, R_m\}$ and n SSs $\{M_1, \ldots, M_j, \ldots, M_n\}$. Each SS can communicate with the BS through an RS. The BS has an omni-directional antenna and transmits at a fixed high power level such that it can reach every RSs with a high data rate. Each RS R_i is equipped with an adaptive directional antenna that can form a main beam in any direction with a beamwidth chosen from a set of angles $\Theta = \{\theta_1 < \ldots < \theta_W\}$. We do not make any assumption on antennas at SSs, i.e., an SS can have either an omni-directional antenna or a directional antenna. Both RSs and SSs transmit at fixed power levels. For uplink communications (i.e, from an SS to an RS or from an RS to the BS), the transmitting node can simply point its main beam towards the receiving node. Hence, we focus on determining antenna orientations of RSs for downlink communications from RSs to SSs.

Let $r_{ijk} \geq 0$ be the maximum data rate that can be supported by assigning M_j to R_i and adjusting the directional antenna at R_i to cover M_j with a beam of width θ_k, i.e. the capacity of the wireless link from R_i to M_j with beamwidth θ_k. Typically, r_{ijk} depends on the transmit power at R_i, the beamwidth, the distance between M_j and R_i, the operating frequency, and maybe other factors. The optimization schemes proposed in this work is independent of the propagation model. Practically, if a radio is capable of Adaptive Modulation and Coding (AMC), the maximum link data rate r_{ijk} is given by a discrete step increasing function of SNR at the receiver (instead of the continuous Shannon's function). The main beam of a directional antenna at RS R_i is modeled as a sector with an angle of $\theta_k \in \Theta$ and a radius of R_i^T.

IV. Problem Formulations

In this section we formulate the Beam Scheduling and Relay Assignment Problem (BS-RAP) and the Beam Scheduling Problem (BSP). We provide an MILP formulation for the BS-RAP and prove that this problem is NP-hard.

A. The Beam Scheduling and Relay Assignment Problem

For each RS R_i, we imagine rotating the main beam direction through 360 degrees (recall that the width of the beam is some angle $\theta_k \in \Theta$ degrees). As the direction changes, SSs will enter and leave the beam sector. For any fixed direction α and beamwidth θ_k there will be a set of SSs that are currently covered by the beam. We refer to this set of SSs as a *beam set* for α and θ_k. We note that there will be finite collection of distinct beams sets for all $\alpha \in [0, 360]$ degrees and $\theta_i \in \Theta$. We further assume that any beam set that is a proper subset of another is removed from the collection. Let these *beam sets* be $\mathcal{B}_{ik} = \{B_{ik1}, B_{ik2}, \ldots, B_{ikn_{ik}}\}$. Even thought the main beam of a directional antenna can be pointed in any direction, as explained above, we only need to consider a finite number of directions in terms of coverage for SSs.

We are interested in the problem of choosing the beam direction of the smart antenna at each RS as well as which

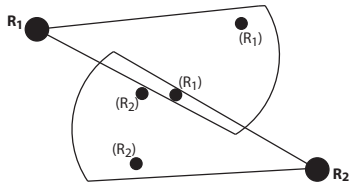

Fig. 2. An illustration of a BS-RAP instance. Each RS must select a beam width and direction to point. SSs have been assigned the indicated RSs. We assume $K_1 = K_2 = 2$, meaning at most 2 SSs can be assigned the same RS. Suppose that all queue lengths are identically 1 and that all transmission rates are also identically 1 for the beam configurations shown. In which case, the four SSs that have been assigned to RSs will have their queues reduced to 0 in one scheduling period (this is optimal). Observe that it is not always optimal to assign the closest RS to an SS.

RS should serve each SS in each scheduling period. Note that if a scheduling-based MAC protocol (such as WiMAX) is used, then a scheduling period consists of several consecutive frames. The beam direction problem is equivalent to the problem of selecting a beam set for each RS since once a beam set is chosen, the RS can point its main beam towards any direction whose corresponding sector can cover all the SSs belonging to that beam set. Once a beamwidth θ_{k_i} and beam set B_{i,k_i,l_i} is selected for each RS R_i, all SSs will know which RSs (if any) can cover it. Because each RS R_i has a limited number of channels to use simultaneously, we assume it can serve some maximum number K_i of SSs during the scheduling period. Deciding which RS (if any) should serve each SS is thus a joint problem to be solved along with selecting the relay beams. Let $r(j)$ be the RS that is assigned to SS M_j. We will use the convention that $r(j) = -1$ indicates that no RS is assigned to M_j. To summarize, in order for the RS assignment to be *valid*, we require that $|j : r(j) = i| \leq K_i$ for all $i = 1, \ldots, m$ and $r(j) = i > 0 \Rightarrow j \in B_{i,k_i,l_i}$.

The beam scheduling problem becomes the problem of creating a beam set schedule for each relay in order to best serve the SSs. We assume that at each scheduling period, a RS is able to select one of its beam sets to be active. Following [3], we assume that each SS M_j has a capacity demand represented as a queue length q_j. We formally define the optimization problem as follows.

Definition 1: For a given scheduling period where each SS M_j has a queue length q_j, the **Beam Scheduling and Relay Assignment Problem (BS-RAP)** is to select for each RS R_i, a beamwidth θ_{k_i} and beam set B_{i,k_i,l_i} and jointly determine a valid RS assignment, $\langle r(j) \rangle$, such that the utility function $\sum_{\{j:r(j)\neq -1\}} q_j \min(q_j, r_{r(j),j,k_i})$ is maximized.

Figure 2 provides an illustration of a simple instance of the BS-RAP problem in the case where each RS can serve at most two SSs ($K_1 = K_2 = 2$). It is known that if a scheduling algorithm can maximize the above utility function in each scheduling period, then it can keep the system stable, i.e., keep the length of each queue finite [3]. Such a stable scheduling algorithm is also considered to achieve 100% throughput [5].

V. Proposed Algorithms

We present two algorithms to solve the BS-RAP. Both are based on greedy strategies. The first algorithm has a constant factor approximation guarantee and the second is shown to be somewhat more effective in practice. Finally, we present an polynomial-time optimal algorithm for the BSP.

A. BS-RAP: A Basic Greedy Algorithm

The idea of the *BS-RAP-Greedy1* algorithm is to first get a tentative assignment of RSs to SSs and then use that assignment to guide selecting a beam set of reach RS to use. Steps 1 and 2 of the algorithm assigns RSs to SSs optimistically assuming that any SS M_j can communicate with any RS R_i using the best narrow-beam transmission rate available r_{ij1}. Thus, beam directions are ignored for the time being. This simplifies the problem to a form of the *generalized assignment problem (GAP)*; this problem considers that there are some number of activities (subscriber stations) and agents (relay stations) with various capabilities [6]. Assigning an agent to an activity provides some value (v^r_{ij1}) and requires some cost (always 1). The objective is to assign agents to activities in order to maximize total value, with agents constrained to given budgets (in this case K_i, the maximum number of SSs that RS R_i can be assigned to). Once this tentative RS assignment is found, each RS choses a beam set that maximizes its reward given this assignment (Step 3). Any remaining SSs that have not yet been assigned a RS are then assigned RSs if possible (Step 4). Pseudocode for BS-RAP-Greedy1 is given in Algorithm 1.

Theorem 1: The BS-RAP-Greedy1 algorithm runs in $O(mn \log n)$ time and provides a solution that is within a factor $\frac{1}{2\lceil 2\pi/\theta_1 \rceil}$ of the optimal value.

The proof is omitted due to space considerations and will appear in the full version of this paper.

B. BS-RAP: A Joint Greedy Algorithm

The second greedy algorithm is a variation on the first. The main difference is that beam set selection and relay assignment are done jointly. This is done in Step 2 of the algorithm, which loops through all the RSs and for each, chooses the best beam set to use given the previously made SS assignments to that RS. Once a beam set is chosen for a RS R_i then up to K_i SSs are assigned that RS and the loop continues. We refer to the algorithm as the BS-RAP-Greedy2, pseudocode is given in Algorithm 2.

VI. Numerical Results

In this section, we present simulation results to show the performance of the proposed algorithms. The ILOG CPLEX [8] optimization software was used to solve all the MILP and LP problems. In the simulation, n SSs were randomly deployed within a square $l \times l$ km region, with a single BS placed at the center of the square. Then, m RSs were deployed radially from the center of the region with uniform angular spacing and random radii between zero and the maximum distance. In the simulation, we calculated SNRs

Algorithm 1 BS-RAP-Greedy1

Step 1 Let $v_{ijk} = q_j \min(q_j, r_{ijk})$ for all i, j.
Let $r(j) = -1$ for all j.
We define (and keep updated)

$$v^r_{ijk} = \begin{cases} v_{ijk} & \text{if } r(j) = -1 \\ v_{ijk} - v_{r(j)jk} & \text{otherwise.} \end{cases}$$

Step 2 **for** $i = 1$ **to** m :
Let π^i sort $\{v^r_{ij1}\}_{j=1}^n$ into decreasing order.
for $p = 1$ **to** K_i :
if $v^r_{i\pi^i(p)1} > 0$
Set $r(\pi^i(p)) = i$.
endif
endfor
endfor

Step 3 **for** $i = 1$ **to** m :
Calculate the *reward* w_{ikl} of each beam set B_{ikl}, defined as follows:

$$w_{ikl} = \sum_{j \in B_{ikl}, r(j)=i} v_{ijk}$$

Compute $(k_i, l_i) = \text{argmax}_{k,l}\, w_{ikl}$.
Set $s_{ik_il_i} = 1$ and $s_{ikl} = 0$ for $k, l \neq k_i, l_i$.
For $j \notin B_{ik_il_i}$ s.t. $r(j) = i$, set $r(j) = -1$.
endfor

Step 4 **for** $j = 1$ **to** n :
Set $x_{ijk} = 0$ for all i, k.
if $r(j) = -1$
Let $\mathcal{R}_j = \{i : j \in B_{ik_il_i}, |\{j : r(j) = i\}| < K_i\}$
if $\mathcal{R}_j \neq \emptyset$
Set $r(j) = \text{argmax}_{i \in \mathcal{R}_j} r_{ijk_i}$
Set $x_{r(j)jk_{r(j)}} = 1$.
endif
else
Set $x_{r(j)jk_{r(j)}} = 1$.
endif
endfor

Algorithm 2 BS-RAP-Greedy2

Step 1 Follow Step 1 of BS-RAP-Greedy1.

Step 2 **for** $i = 1$ **to** m :
Calculate the *reward* w_{ikl} of each beam set B_{ikl}, defined as follows:
(a) Let π^{ikl} sort B_{ikl} by decreasing v^r_{ijk}.
(b) Let $W_{ikl} = \{p \leq K_i : v^r_{i\pi^{ikl}(p)k} > 0\}$.
(c) Let $w_{ikl} = \sum_{p \in W_{ikl}} v^r_{i\pi^{ikl}(p)k}$.
Compute $k_il_i = \text{argmax}_{kl}\, w_{ikl}$.
Set $s_{ik_il_i} = 1$ and $s_{ikl} = 0$ for $kl \neq k_il_i$.
for $p \in W_{ik_il_i}$:
Set $r(\pi^{ikl}(p)) = i$.
endfor
endfor

Step 3 Follow Step 4 of BS-RAP-Greedy1.

using the free space path loss model [16]. Parameters relevant to the propagation model and other related parameters were set according to the IEEE 802.16e [1] standard.

To simplify the scenarios, we just considered that each RS could use either the beamwidth θ or the beamwidth 2θ. In each simulation scenario, we changed the value of one parameter and fixed the values of the others. We first evaluated the performance of the proposed BS-RAP algorithms, i.e., the two greedy algorithms (labeled as "Greedy #1" and "Greedy #2"), in terms of the *utility* function (Definition 1). In all scenarios, we also computed optimal solutions by solving the MILP for the BS-RAP (labeled as "Optimal"). We summarize our BS-RAP simulation scenarios below and present the corresponding simulation results in Figs. 3–6. Each plotted value in these figures is an average over 10 runs per parameter combination, each with a different randomly generated network. We assume that each RS has angles θ and 2θ available and that each SS queue length is drawn from the uniform distribution on $[0, 2\mu]$, where μ is the mean queue length.

We make the following observations from the simulation results, noting that the BS-RAP-Greedy2 algorithm is close to optimal in almost all cases, while the BS-RAP-Greedy1 algorithm performs less well in all cases:

1) From Fig. 3–6, we see the BS-RAP-Greedy2 algorithm closely tracks the optimal solution; it is always within 15% of the optimal solution. The BS-RAP-Greedy1 algorithm degrades to less than 75% of optimal as the number of SSs increases in Scenario 1.

2) Fig. 4 illustrates that a near linear increase in performance results as the number of RSs is increased in Scenario 2. Even as the number of RSs increases to the maximum value, both greedy algorithms do a relatively good job at allocating these additional resources. Eventually, there must be diminishing returns as more RSs are added but this did not occur in the range considered ($m \in [2, 6]$).

3) Fig. 5 suggests that θ in the range 40-50 degrees is optimal for the simulation parameters examined. Recall that each RS could form beams of width θ or 2θ. Across the simulations performed, there were somewhat more beams of width 2θ chosen versus θ. Using a narrower beam can increase the potential throughput from an RS to an SS assigned to it, but a wider beam potentially allows the RS to serve more SSs during the scheduling period.

4) From Fig. 6 we see that the BS-RAP utility increases close to linearly with the average queue length of each SS. BS-RAP-Greedy2 tracks the optimal solution closely, whereas the relative performance of BS-RAP-Greedy1 falls off as the queue lengths are increased.

VII. Conclusions

In this paper, we have exploited how to leverage smart antennas for efficient communications in wireless relay networks. A corresponding optimization problem was formally defined as the BS-RAP and was proven to be NP-hard. We first presented an MILP formulation to provide optimal solutions

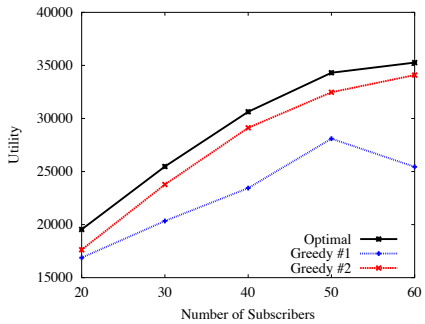

Fig. 3. Scenario 1: Performance VS. the number of SSs (n)

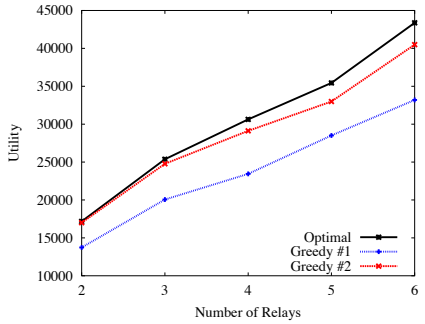

Fig. 4. Scenario 2: Performance VS. the number of RSs (m)

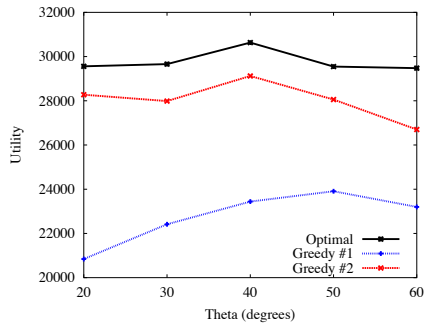

Fig. 5. Scenario 3: Performance VS. beamwidth (θ)

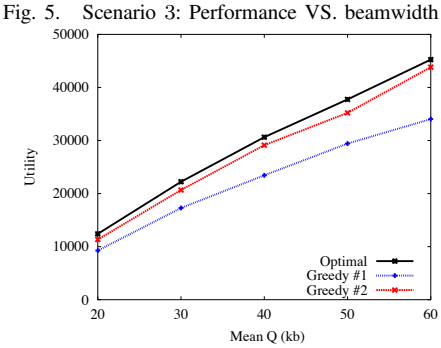

Fig. 6. Scenario 4: Performance VS. mean queue length (μ)

and then presented two two greedy approaches for the BS-RAP, one of which, BS-RAP-Greedy1, was shown to have an approximation ratio of $\frac{1}{2\lceil 2\pi/\theta_1 \rceil}$. These algorithms are simple, easy to implement and scale well to larger network instances. We also considered the related problem BSP and presented the BS-DP algorithm to solve it optimally in polynomial time. The simulation results show that both proposed BS-RAP algorithms provide good performance, with the BS-RAP-Greedy2 algorithm better than 85% of optimal in all cases.

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