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DARP: Distance-Aware Relay Placement in WiMAX Mesh Networks

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Abstract— **The emerging WiMAX technology (IEEE 802.16) is the fourth generation standard for low-cost, high-speed and long-range wireless communications for a large variety of civilian and military applications. IEEE 802.16j has introduced the concept of mesh network model and a special type of node called Relay Station (RS) for traffic relay for Subscriber Stations (SSs). A WiMAX mesh network is able to provide larger wireless coverage, higher network capacity and Non-Line-Of-Sight (NLOS) communications. This paper studies a Distance-Aware Relay Placement (DARP) problem in WiMAX mesh networks, which considers a more realistic model that takes into account physical constraints such as channel capacity, signal strength and network topology, which were largely ignored in previous studies. The goal here is to deploy the minimum number of RSs to meet system requirements such as user data rate requests, signal quality and network topology. We divide the DARP problem into two sub-problems, *LOwer-tier Relay Coverage* (LORC) Problem and *Minimum Upper-tier Steiner Tree* (MUST) Problem. For LORC problem, we present two approximation algorithms based on independent set and hitting set, respectively. For MUST problem, an efficient approximation algorithm is provided and proved. Then, an approximation solution for DARP is proposed and proved which combines the solutions of the two sub-problems. We also present numerical results confirming the theoretical analysis of our schemes as the first solution for the DARP problem.**

Keywords: **WiMAX mesh network; relay station placement; approximation algorithm; hitting set; independent set.**

I. Introduction

The emerging WiMAX technology (IEEE 802.16 [29]–[32]) is the fourth generation (4G) standard for low-cost, high-speed and long-range wireless communications for a large variety of civilian and military applications. WiMAX uses large chunks of spectrum (10-20 MHz or more), and delivers high bandwidth (up to 75 Mbps). Despite the high bandwidth promised by WiMAX, there are several challenges to improving the network throughput. The first challenge is to eliminate or reduce *coverage holes*. Because of high path-loss, and shadowing due to obstacles such as large buildings, trees, tunnels, etc., there would be some spots with poor connectivity, which we call coverage holes. This leads to degradation in overall system throughput. Another key design challenge is *range extension*. At times, it is required to provide wireless connectivity to an isolated area outside the reach of the nearest Base Station (BS). To solve the coverage holes and range extension problems, adding more base stations would be an easy choice. However, given the high cost of deploying BSs, such a solution could be an overkill, and too expensive [1]. In such contexts, relay stations (RSs) are a cost-effective alternative. Recently, IEEE $802.16j$ [31] has been proposed to enhance the existing standard IEEE $802.16e$ [30], which introduces the concept of mesh network model and a special type of node, *relay station* for traffic relay for Subscriber Stations (SSs). RSs act as MAC-layer repeaters to extend the range of the base station. An RS decodes and forwards MAC-layer segments unlike a traditional repeater which merely amplifies and retransmits PHY-layer signals. Hence, an RS may use a different modulation coding scheme for reception and forwarding of a MAC segment. A WiMAX mesh network is illustrated in Fig. 1, which is able to provide larger wireless coverage, higher network capacity and Non-Line-Of-Sight (NLOS) communications [29]. This model is especially suitable for some application scenarios, such as broadband Internet access and emergency communications.

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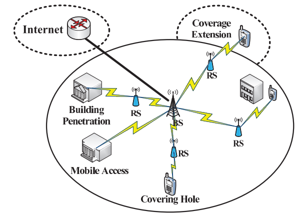


Fig. 1. A WiMAX mesh network

This paper studies the RS placement problem for the models where the locations of SSs are known and the placement of RSs can be controlled to meet data rate, link capacity or signal quality requirements. Related work is introduced in Section II. In Section III, we define the Distance-Aware Relay Placement (DARP) problem. In Section IV, approximation schemes are presented and proved for the DARP problem. Numerical results are provided in Section V, which is followed by the conclusion of the work in Section VI.

II. Related Work

Relay station placement has been an active research topic in wireless networks, especially in wireless sensor networks. By using RSs, one could deploy a network at a lower cost than using only (more expensive) BSs to provide wide coverage while delivering a required level of service to users [8],

[13], [14], [25]. Relay node placement problems are usually classified into two classes: single-tiered (both relay nodes and sensor nodes can relay traffic) and two-tiered (sensor nodes cannot relay traffic). In [15], Lin and Xue proved the single-tiered placement problem with $R = r$ and $K = 1$ is NP-hard, where R, r and K denote the transmission range of relay nodes, the transmission range of sensor nodes, the connectivity requirement respectively. A 5-approximation algorithm was presented to solve the problem. The authors also designed a *steinerization* scheme which has been used by many later works. Better constant factor approximation algorithms for the cases where $R \geq r$ and/or $K > 1$ have been presented in [16], [28]. In [5], a 3.11-approximation algorithm was presented. The authors also proved that one-tier version admits no Polynomial Time Approximation Scheme (PTAS), assuming $P \neq NP$. For the *two-tiered placement*, under the assumption that $R \geq 4r$, a 4.5-approximation algorithm was provided in [24]. Lloyd and Xue [16] relaxed the assumption and presented a $(5+\epsilon)$-approximation algorithm for the problem with $R \geq r$ and $K = 1$. [5] improved the approximation algorithm in [16] by providing a PTAS. A $(10+\epsilon)$-approximation algorithm has been presented in [28] for the case where $R \geq r$ and $K = 2$. In [10], the authors studied a fault-tolerant relay placement problem in heterogeneous sensor networks, where target nodes have different transmission radii. However, the work still assumed that the transmission range of relay nodes is the same.

Beside minimizing the number of placed RSs, some work has been done on placement with physical constraints, such as energy consumption and network lifetime. Hou *et al.* studied the energy provisioning problem for a two-tiered wireless sensor network [12]. Besides provisioning additional energy on the existing nodes, they consider deploying relay nodes (RNs) into the network to mitigate network geometric deficiency and prolong network lifetime. In [26], Hassanein *et al.* proposed three random relay deployment strategies for connectivity-oriented, lifetime-oriented and hybrid deployment. In [20], Pan *et al.* studied base station placement to maximize network lifetime. Along this line, [22] considered joint base station placement and data routing strategy to maximize network lifetime. The same group studied using mobile base stations to prolong sensor network lifetime in [23].

Comprised of small form factor low-cost relays, associated with specific BSs, the main advantages of the WiMAX relay network model are increased coverage and capacity enhancement [17]. RSs are expected to have significantly lower complexity than 802.16e BSs. In [13], an optimal scheme was proposed to find the location of a single RS and resource allocation for all the SSs. In [14], the authors introduced a novel dual-relay architecture, where each SS is connected to the BS via exactly two active RSs through the decode-and-forward scheme. They proposed a two-phase heuristic algorithm to solve the dual-relay RS placement problem. The authors of [27] divided the network into clusters. Then in each cluster, Integer Linear Programming (ILP) formulation was proposed to select the locations for BSs and RSs from a set of given positions. Recently, a new dual-relay coverage architecture was proposed for 802.16j Mobile Multi-hop Relay-based (MMR) networks [13], [14], where each subscriber station (SS) is covered by two RSs. [13] assumed that only one RS is placed in each cell. ILP formulation was applied to find an optimal placement of RS which can maximize the cell capacity in terms of user traffic rates. In [14], assuming a uniform distribution on user traffic demand, the authors studied how to determine the RSs' locations from a set of predefined candidate positions. To the best of our knowledge, this paper is the first work that studies the multi-hop relay node placement considering individual channel capacity constraints in WiMAX mesh networks.

III. Problem Statement

According to IEEE $802.16j$ [32], a WiMAX mesh network is composed of a BS, SSs and a set of RSs. An RS can relay traffic for SSs and the BS, and an SS does not have the routing and traffic relay capabilities. This communication scenario is worth studying since in the near future, there may exist a large number of simple WiMAX terminals (SSs) in need of network connections, just like the current WiFi terminals. As suggested by the WiMAX standard [30], [31], a tree rooted at the BS is usually constructed to support packet forwarding in a WiMAX mesh network. In the tree, all SSs must be leaf nodes and only the RSs can serve as intermediate (non-leaf) nodes connecting the SSs with the BS. By placing the RSs in the network, we actually construct a tree structure and a routing strategy for the WiMAX network. It has been shown that RS placement has a significant impact on network performance [13], [19].

A. Network and Relay Models

In this paper, IEEE 802.16j Mobile Multi-hop Relay (MMR) network is used as the model for the network infrastructure. As proposed in the IEEE 802.16j standard [32], an 802.16j radio link between a BS and an RS or between a pair of RSs is a *relay link*. Concatenation of k consecutive relay links $(k \geq 1)$ between the BS and the designated access RS forms a relay path. Compared to the BS, an RS has a significantly simpler hardware and software architecture, and hence a lower cost. An RS merely acts as a link layer repeater, and therefore does not require a wired backhaul. Furthermore, an RS needs not perform complex operations such as connection management, hand-offs, scheduling, etc. Also, an RS typically operates at a much lower transmit power, and requires lower-MAC and PHY layer stack. All these factors lead to a much lower cost of an RS, and thus, relay networks are evolving as a low-cost option to fill coverage holes and extend range in many scenarios.

The major goals of deploying relay stations are to improve coverage in geographic areas that are severely shadowed from the BS, to extend the range of a BS, and to improve the link data rate and network throughput. IEEE $802.16j$ [32] defines three types of RSs whose functions are to relay traffic between an SS and a BS, including Fixed Relay Station (FRS), Nomadic Relay Station (NRS), and Mobile Relay Station (MRS). An FRS is a relay station that is permanently installed at a fixed location. An NRS is a relay station that is intended to function at a fixed location for periods of time comparable to a user session. An MRS is a relay station that is intended to function while in motion. In this work, we consider static SSs such as McDonald's, gas stations, and grocery stores. Thus, we will study a *static network planning* problem, i.e., finding where to place a minimum number of relay nodes such that certain performance requirements can be satisfied. Therefore, we focus on FRSs and NRSs in this work.

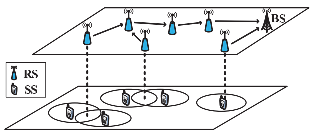


Fig. 2. Two-tiered relay model

In this paper, we study the *two-tiered* relay station placement problem which is particularly suitable for WiMAX-based mesh networks. The two-tiered network model divides the network into two tiers, as shown in Fig. 2.

All the SSs form the lower tier, each of them is covered by at least one RS, through which each SS can relay its traffic co-operatively to the BS. Meanwhile, following the WiMAX mesh network convention, all the RSs and the BS are connected on the upper tier to enable two-hop or multi-hop relay capability.

B. Distance-Aware Relay Station Placement

Because user data rate requests, channel capacity as well as the LOS effect should be carefully taken into account for the RS placement, we will study an RS placement problem satisfying each SS's data rate request, which has not been well addressed before. Note that previous studies on relay node placement have mainly focused on coverage and connectivity.

Definition 1 (Feasible coverage)**.** Let s_i be a fixed SS with known location, and b_i be its data rate request (in terms of *bps*). An RS r_m is said to provide *a feasible coverage* for s_i if the channel capacity of the link (in terms of *bps*) between s_i and r_m is sufficient for the data rate request of s_i. In other words, the capacity of link (s_i, r_m) is no less than b_i. □

Two kinds of placement scenarios are defined in WiMAX standards: *two-hop relay* and *multi-hop relay*. According to the IEEE $802.16j$ [32], supporting 2-hop relay is mandatory but supporting multihop relay (more than 2) is optional. In this paper, we study the *multihop relay* for the RS placement, while most previous work studies the *two-hop relay* model.

It is well-known that the capacity of a wireless connection is highly related to the Euclidean distance between its two end nodes [4]. If the two-ray ground path loss model is considered (which is generally used for modeling the large scale signal strength over the distance of several kilometers that use tall towers as well as for LOS micro-cell channels) [13], the power level at the receiver P_r is given as

$$P_r = P_t G_t G_r h_t^2 h_r^2 d^{-\alpha} \tag{3.1}$$

where P_t is the transmission power, and G_t/G_r and h_t/h_r are the gains and heights of transmitter antenna and receiver antenna, respectively. d is the Euclidean distance between the transmitter and the receiver, and α is the attenuation factor, which depends on the environment and typically varies in a range of $2-4$ for the terrestrial propagation. Then the signal-to-noise ratio (*SNR*) at receiver is $\mathit{SNR}_r = P_r/N_0$, where N_0 is the thermal noise power at the receiver which is normally a *constant*. Based on Shannon's theorem, the link capacity is given by $W \log(1 + \mathit{SNR}_r)$, where W is spectrum bandwidth. Therefore, when the noise N_0 is constant, the received signal quality, and consequently *the channel capacity, are determined by the received signal strength* P_r. The problem we study is a special case where the channel capacity of each link between each SS and its corresponding RS is *decided by the received power at the receiver* P_r.

Based on formula (3.1), for each RS or SS, its transmitter/receiver gain is set to be fixed. We can see that for a pair of transmitter and receiver, the signal received at the receiver is decided by *the distance between the pair*. Consequently, the channel capacity of the transmission between an SS and its covering RS is decided by the distance between these two stations. Therefore, *the data rate request* b_i *of each SS* s_i *can be translated into an equivalent problem with requirement of distance between* s_i *and its covering RS*.

Definition 2 (Distance-Aware Relay Placement (DARP) Problem)**.** Given a WiMAX mesh network with a BS and a set of SSs $S = \{s_1, s_2, \ldots, s_n\}$, let $\mathcal{D} = \{d_1, d_2, \ldots, d_n\}$ be the distance requirement set for the SSs. The DARP problem seeks a *minimum number* of RSs R such that:

1) Providing feasible coverage for each $s_i \in S$. In other words, s_i is covered by at least one RS or the BS within distance d_i
2) Each placed RS has enough data rate to relay traffic for each SS or another RS that it covers for relay. □

In addition to the feasible coverage of the SSs in the lower-tier (Condition 1), to ensure that all the packets from SSs can transmit to the BS, we need to consider the connectivity of the placed RSs and the BS through possible multi-hop relays in the upper-tier (Condition 2). It is worth noting that we only study *providing enough data rate for each individual SS or RS* in this work. How to schedule the SSs and RSs to satisfy the total throughput of each SS is an important and related topic, but is out of the scope of this work.

IV. Approximation Solution for DARP

When all the distance requirements are the same ($d_1 = d_2 = \ldots = d_n$), DARP becomes the 2tRNP problem in [16], which was proved to be *NP*-hard [7]. Thus, DARP is *NP*-hard. Given the hardness of the problem, it is not possible to find a polynomial time optimal solution for DARP unless *P* = *NP* [7]. Therefore, the best solution we can expect are polynomial time approximation algorithms. To solve the DARP problem, we divide the problem into two sub-problems and conquer them one by one. First, we focus on the lower tier and aim to find a minimum set of RSs for feasible coverage of the SSs. Next, we move onto the upper tier and provide distance-constrained connections between RSs and the BS. We discuss each sub-problem in the following.

A. Lower-tier Coverage of Subscriber Stations

In the first step, we need to solve the coverage sub-problem in the lower tier, which seeks to use the minimum number of

RSs to guarantee that each SS is covered feasibly.

Definition 3 (LOwer-tier Relay Coverage (LORC) Problem)**.** Given a WiMAX mesh network with a BS and a set of subscriber stations $S = \{s_1, s_2, \ldots, s_n\}$, let $\mathcal{D} = \{d_1, d_2, \ldots, d_n\}$ be the distance requirement set for the SSs. The LORC problem seeks a *minimum number* of relay stations R that provides feasible coverage for each $s_i \in S$. In other words, s_i is covered by at least one RS or the BS within distance d_i. □

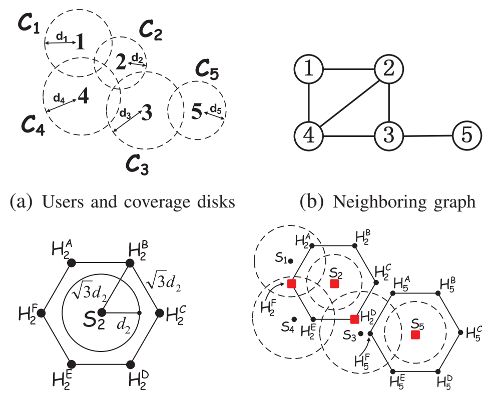
When all the SSs have the same distance requirement, it is easy to see that the coverage problem is equivalent to the GeoDC problem [6], which seeks a minimally sized set of disks (of prescribed radius) covering all points in a Euclidean plane [11]. Therefore, with a special case being *NP*-hard, we can see that LORC is also *NP*-hard.

1) Maximal Independent Set Based Approximation Solution: Our first solution is based on the following observations. First, to provide a feasible coverage for an SS s_i with distance requirement d_i, it is easy to see that an RS must be placed *in* or *on* a disk centered at s_i with radius d_i. We denote such a disk by the *feasible coverage disk* for s_i. Second, for any two SSs, if their feasible coverage disks intersect with each other, then they can be covered by one RS in the intersection area. We called such two SSs are *neighbors*. Similarly, if multiple SSs are all neighbors with each other (a *clique*), all these SSs can be covered by one RS. Based on above observations, we present a *simple and provably good* solution in Algorithm 1.

Algorithm 1 LORC-MIS(S, $\mathcal{D}$)

```
1: Construct sets of disks C and C'; C ← ∅; C' ← ∅
2: for s_i ∈ S do
3:     Calculate its feasible coverage disk 𝒞_i; C ← C ⋃ {𝒞_i}.
4: end for
5: while C ≠ ∅ do
6:     Find s_min, the SS with the minimum distance requirement
       d_min; 𝒞_min is the feasible coverage disk of s_min;
7:     Construct a regular hexagon ℋ_min centered at s_min with side
       length √3 d_min;
8:     Construct a point set P = {6 vertices of ℋ_min, s_min};
9:     C' = {𝒞_min};
10:    for 𝒞_i ∈ C do
11:        if 𝒞_i intersects with 𝒞_min then
12:            C' ← C' ⋃ {𝒞_i};
13:        end if
14:    end for
15:    while C' ≠ ∅ do
16:        Choose the point v ∈ P which covers most disks in C';
17:        Place an RS r at the location of v; R ← R ⋃ {r};
18:        Remove all the disks covered by r from C and C';
19:    end while
20: end while
21: return R
```

Let us use an example in Fig. 3 to illustrate our algorithm. For SSs s_1, s_2, s_3, s_4, and s_5 in Fig. 3(a), we first calculate their respective feasible coverage disks $\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3, \mathcal{C}_4$, and $\mathcal{C}_5$ (Lines 2 - 4), shown by the circles around each node. Figure 3(b) demonstrates the neighboring relationship between two nodes if two disks intersect each other. We first select disk $\mathcal{C}_2$ which has the smallest radius (Line 6). Next, as shown in Fig. 3(c), we construct a regular hexagon $\mathcal{H}_2$ for $\mathcal{C}_2$, and then have set P including 7 possible positions $\{\mathcal{H}_2^A, \mathcal{H}_2^B, \mathcal{H}_2^C, \mathcal{H}_2^D, \mathcal{H}_2^E, \mathcal{H}_2^F, s_2\}$ to place RSs (Lines 7 - 8). From Line 9 to Line 14, we calculate $C' = \{\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3, \mathcal{C}_4\}$, which are the disks that will be covered in this step. In Fig. 3(d), we first select $\mathcal{H}_2^F$ to place an RS because it can cover most (two) disks (SSs), $\mathcal{C}_1$ and $\mathcal{C}_4$ (Line 16 to Line 18). Then, two disks $\{\mathcal{C}_2, \mathcal{C}_3\}$ are left in C' to be covered. Following same process, an RS is place on s_2, and another is placed on $\mathcal{H}_2^D$ to cover these two disks. At this time, after removing SSs $1, 4, 2$, and 3, $C = \{\mathcal{C}_5\}$, which is not empty. Similarly, we construct a regular hexagon $\mathcal{H}_5$ for $\mathcal{C}_5$, place an RS at the center of $\mathcal{H}_5$, and remove SS 5. The solution uses four RSs $R = \{s_2, s_5, \mathcal{H}_2^F, \mathcal{H}_2^D\}$ to cover all SSs.



(a) Users and coverage disks (b) Neighboring graph

(c) Regular hexagon (d) Relay placement

Fig. 3. Illustration of LORC-MIS

Next, we aim to prove that our RS placement algorithm is actually a 7-approximation solution for the LORC problem.

Lemma 1. Given an SS s_{min}, which has the smallest radius d_{min}, and a set $\mathcal{N}_{s_{min}}$ including all the neighboring SSs of s_{min}, $\mathcal{N}_{s_{min}} \cup \{s_{min}\}$ can be covered by *at most* 7 RSs. □

Proof: For any neighbor SS o of s_{min}, assuming its distance requirement is $d_o(\geq d_{min})$, without loss of generality, o is located in the region of angle $\angle \mathcal{H}_{s_{min}}^A s_{min} \mathcal{H}_{s_{min}}^B$, where $\mathcal{H}_{s_{min}}^A$ and $\mathcal{H}_{s_{min}}^B$ are two adjacent vertices of hexagon $\mathcal{H}_{s_{min}}$. Now we want to prove that there must exist a point $v \in \{\mathcal{H}_{s_{min}}^A, \mathcal{H}_{s_{min}}^B, s_{min}\}$ that can cover SS o. In other words, v is in or on disk $\mathcal{C}_o$, the feasible coverage disk of o.

Let us use the auxiliary graphs in Fig. 4 to illustrate our proof. For the simplicity, we use a, b, and s to denote $\mathcal{H}_{s_{min}}^A, \mathcal{H}_{s_{min}}^B$, and s_{min}, respectively. We first construct perpendiculars at node a on line $s - a$, and at node b on line $s - b$, respectively. The two perpendiculars intersect at point q. Based on geometrical properties, we can derive that $\angle asq = \angle bsq = 30°$ and $\angle aqs = \angle bqs = 60°$.

Without loss of generality, we assume that o lies in the region of $\angle asq$. Now we need to prove that o can be covered by an RS on a or s. In Figs. 4(a) and 4(b), we draw a circle $\mathcal{C}_a$ centered at a with radius d_{min}. There are two cases to consider.

CASE 1 - o is in or on disk $\mathcal{C}_s$ or disk $\mathcal{C}_a$: As shown in Fig. 4(a), if o is in or on disk $\mathcal{C}_a$, we can easily see that $|so| \leq d_{min} \leq d_o$. Therefore, s is in or on the disk of $\mathcal{C}_o$ and can cover SS o. Similarly, if o is in (on) the disk $\mathcal{C}_a$, then $|ao| \leq d_{min} \leq d_o$. Thus, a is in (on) the disk of $\mathcal{C}_o$ and covers o.

CASE 2 - o is NOT in or on disk $\mathcal{C}_s$ and disk $\mathcal{C}_a$: As shown in Fig. 4(b), if o is in or on neither $\mathcal{C}_s$ nor $\mathcal{C}_a$, we need to prove that it can be covered by a. To prove it, we need two auxiliary lines; One connects s and o. This line intersects with disk $\mathcal{C}_s$ at point p. The other connects s and q, which intersects with disk $\mathcal{C}_s$ at point l. Since $\angle aql = \angle aqs = 60°$, and

$$|ql| = |sq| - |sl| = 2d_{min} - d_{min} = d_{min} = |qa|$$

it is easy to see that triangle $\triangle alq$ is an equilateral triangle. Thus, $\angle alq = \angle laq = 60°$. It is easy to see that

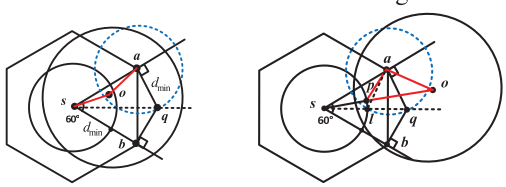
$$\angle apo \leq \angle alq = \angle laq \leq \angle pao$$

Therefore, in $\triangle apo$, we have $|ao| \leq |po|$. Meanwhile, we have

$$|sp| + |po| = |so| \leq d_{min} + d_o$$

Given $|sp| = d_{min}$, we know that $|po| \leq d_o$. And consequently, $|ao| \leq d_o$, and a could be used to cover o.

Combing both cases, any neighbor o of s can be covered by either s or one of the six vertices of hexagon $\mathcal{H}_s$. Therefore, at most 7 RSs can cover s and all of its neighbors. ■



(a) o in/on disk $\mathcal{C}_s$ or $\mathcal{C}_a$ (b) o not in/on disks $\mathcal{C}_s$ and $\mathcal{C}_a$

Fig. 4. Auxiliary graphs for proof

Theorem 1. Algorithm 1 is a 7-approximation for the LORC problem. More specifically, if the number of the RSs returned by Algorithm 1 is denoted by $|R|$, we have $|R| \leq 7 \cdot |OPT_L|$, where OPT_L is an optimal solution for LORC. □

Proof: At Line 6 in Algorithm 1, each time we select a remaining uncovered SS with the minimum distance requirement, and try to cover it as well as all of its neighboring SSs. From Lemma 1, we know that, at each time, at most 7 RSs are needed to cover a selected SS and all its neighbors. Assume that the total number of SSs selected in this step is L, then the total number of RSs will be no more than $7L$.

Meanwhile, using the neighboring graph in Fig. 3(b), we can see that these L nodes form a ***maximal*** *independent set* of the graph. Denote a ***maximum*** *independent set* by $\mathcal{M}$ with size M, it is obvious that *no any two or more nodes in $\mathcal{M}$ can be covered by one RS*. In other words, each node in $\mathcal{M}$ needs one RS to exclusively cover itself. Thus, M RSs have to be placed to cover the nodes in $\mathcal{M}$. Since $\mathcal{M}$ is a subset of G, in order to cover all the nodes in G, it is easy to see that at least M RSs are needed. Therefore, we have $|OPT_L| \geq$ M. Consequently, the number of RS placed by Algorithm 1 is

$$|R| \leq 7 \cdot L \leq 7 \cdot \mathsf{M} \leq 7 \cdot |OPT_L|$$

Therefore, Algorithm 1 is a 7-approximation. ■

2) Hitting Set Based Approximation Solution: In this section, we want to improve our solution by exploring the geometric structure of the problem. Our solution is based on the relationship between LORC and the well-known *hitting set* problem.

Definition 4 (Hitting set problem). Given a set $S = \{e_0, e_1, \ldots, e_m\}$ and a collection of sets $C = \{S_i \mid 0 \leq i \leq n\}$, where S_i is a set of elements $S_i = \{e_j \mid 0 \leq j \leq m\}$, a sub-set $S' \in S$ which contains at least one element from each subset S_i in C is a *hitting set*. A hitting set with the smallest size is the *minimum hitting set*. □

For example, given $S = \{0, 1, 2, 3, 4\}$ and a collection of sets $C = \{\{0, 1\}, \{1, 2, 3\}, \{3, 4\}\}$, a minimum hitting set is $\{1, 3\}$. Finding a minimum hitting set is a *NP*-hard problem [7]. There exist efficient approximation algorithms [3], [21] for the hitting set problem, and a PTAS for geometric hitting set problem [18].

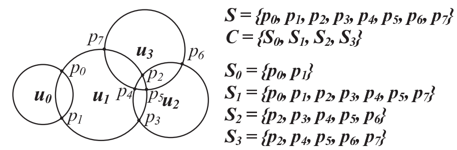


Fig. 5. Relationship between LORC and minimum hitting set

To see how to translate LORC into an equivalent hitting set problem, we use an example in Fig. 5 for illustration. 4 users u_0, u_1, u_2 and u_3 are to be covered. u_0 and u_1 are neighbors, $\mathcal{C}_{u_0}$ and $\mathcal{C}_{u_1}$ intersect on points p_0 and p_1. u_2 and u_1 are neighbors, $\mathcal{C}_{u_1}$ and $\mathcal{C}_{u_2}$ intersect on points p_2 and p_3. C_{u_3} intersects with $\mathcal{C}_{u_1}$ and $\mathcal{C}_{u_2}$ at p_5, p_7 and p_4, p_6, respectively. It is easy to see that by placing an RS on $\{p_0,\ p_1\}$, user u_0 will be covered. Similarly, an RS on any location from $\{p_2, p_3, p_4, p_5, p_6\}$ will cover u_2. Therefore, we construct an instance of hitting set from an instance of LORC by giving set $S = \{p_0, p_1, p_2, p_3, p_4, p_5, p_6, p_7\}$ and a collection of sets $C = \{S_0, S_1, S_2, S_3\}$, while $S_0 = \{p_0, p_1\}$, $S_1 = \{p_0, p_1, p_2, p_3, p_4, p_5, p_7\}$, $S_2 = \{p_2, p_3, p_4, p_5, p_6\}$, and $S_3 = \{p_2, p_4, p_5, p_6, p_7\}$, respectively. If we find a minimum hitting set $\{p_0, p_2\}$, then by placing RSs at p_0 and p_2, we will cover all the users $\{u_0, u_1, u_2, u_3\}$.

Algorithm 2 LORC-HS (S, $\mathcal{D}$)

```
1:  for any two SSs s_i and s_j do
2:      Calculate the feasible covering disks C_i and C_j;
3:      if C_i and C_j intersect with each other then
4:          Assume the intersection points are p and q (or p = q);
5:          I = I ∪ {p, q};
6:      end if
7:  end for
8:  for each SS s_i do
9:      H_i = ∅;
10:     for each point p ∈ I is in or on disk C_{s_i} do
11:         H_i = H_i ∪ {p};
12:     end for
13: end for
14: Construct a set H = {H_0, ..., H_n};
15: H_min ⇐ Solve a minimum hitting set problem MHS(H);
16: Place an RS on each point p ∈ H_min;
```

Based on our observation, we present a simple Hitting Set Based algorithm to solve the LORC Problem. This algorithm is formally presented by Algorithm 2.

In Algorithm 2, we first find the set (I) of all possible RS locations, which are the intersection points of covering disks (Line 1 - Line 7). Then, we construct an instance of hitting set problem from the instance of LORC. For each s_i, we try to find a corresponding set H_i in the instance of hitting set, which includes all the positions that can cover s_i (from Line 8 to Line 13). Note that the geometric hitting set problem admits a PTAS [18]. It is easy to see that Algorithm 2 could return a $(1+\epsilon)$-approximation scheme for LORC.

B. Upper-Tier Connectivity of Relay Stations

Besides the coverage of the SSs in the lower tier, another important requirement for RS placement is the connectivity between the RSs and the BS, which promises the connections from SSs to the BS. After the coverage stage, in the upper tier, if the BS and the covering RSs are all connected, then we already have a solution. However, if they are not connected, which is more typical given the large range of a WiMAX cell, we need to study the problem of how to connect the BS and the RSs. The basic idea of providing connectivity is to add more RSs for *multi-hop* relay. In the upper tier, we aim to construct a tree-topology, where BS is the root, all the coverage RS placed for the lower tier SSs are the leaf nodes, and the newly added RSs will be the intermediate nodes on the tree. If we regard the coverage RSs and the BS as target points, then the upper-tier connection problem is related to the well-known constrained Steiner tree problem [7], [15].

Definition 5 (Minimum Upper-tier Steiner Tree (MUST) problem)**.** Given $X = \{x_1, x_2, \ldots, x_n\}$ be the set of n target points (which are the BS and coverage RSs placed for SSs in LORC), MUST seeks a constrained steiner tree $\mathcal{T}$ spanning the set X of target points and a set of *minimum* additional steiner points (new RSs to be placed) such that:

- *Each tree edge length should be no more than D_i, which is the feasible distance requirement for each RS r_i* □

It is easy to see that MUST is *NP*-hard given that its special case in [15] is *NP*-hard. In this paper, we will design an efficient approximation algorithm for the MUST problem based on a concept known as *steinerization*, which was first introduced in [15]. The biggest challenge is that the newly placed RSs will have *various* distance requirements. Most previous work assume that all the SSs have the same transmission range, and all the RSs have the same transmission range [15], [16], [28]. One close work is in [10], in which the authors studied to provide the single-tiered 2-connectivity placement for all the terminals that have different transmission ranges. However, they still assumed that RSs share the same range.

To solve MUST, the first challenge is *how to decide the distance requirement for each RS*, which is affected by the SSs or RSs being covered. In order to guarantee the data rate of each SS, for each RS r_i, the link capacity between r_i and its parent node on the tree $\mathcal{T}$ cannot be lower than the one between r_i and any child of its. The definition of distance requirement of RS is formally given in the following.

Definition 6 (Distance Requirement of RS)**.** For each RS r_i, D_i, the distance requirement of r_i, which represents the maximum feasible distance between r_i and its parent on tree $\mathcal{T}$, equals to the minimum distance requirement of all its children. In other words, $D_i = \min_{k \in \mathcal{T}_i} d_k$, where $\mathcal{T}_i$ is the sub-tree of $\mathcal{T}$ rooted at r_i. □

An example is shown in Fig. 6 for demonstration. RS_1 covers two SSs A and B, whose distance requirements are 16 and 15, respectively. Therefore, the distance requirement of RS_1 is 15, which can guarantee that the data rate requirements of A and B can be satisfied. Similarly, RS_2 has its own distance requirement of 18. For RS_3, its distance requirement is 15, which is the smallest among D_{RS_1} and D_{RS_2}. It is worth noting that this work studies *ensuring data rate for each individual* SS or RS. How to satisfy the throughput of SSs/RSs will be handled by network resource allocation and scheduling research. It is an important and related topic, but is out of the scope of this work.

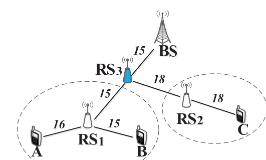


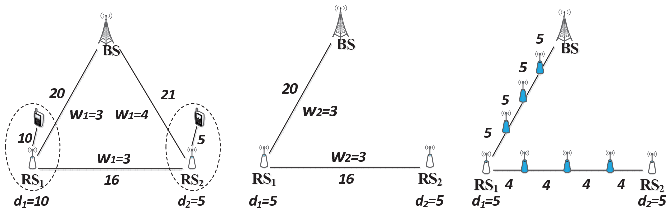
Fig. 6. Example of distance requirements of RSs

With the approach to deciding distance requirements, our solution for MUST is listed in Algorithm.3.

Algorithm 3 MUST (X, $\mathcal{D}$)

1: Construct a *complete* graph $G = (X, E)$; $d_{min} = \min_{i \in S} d_i$;
2: **for** each edge $e(x_i, x_j)$ **do**
3: Assign weight $w_1(x_i, x_j) = \lceil \frac{\|e(x_i,x_j)\|}{d_{min}} \rceil - 1$ on the edge;
4: **end for**
5: Find a minimum spanning tree $\mathcal{T}_{mst}$ of G with BS as the root;
6: **for** each RS x_i **do**
7: Calculate the distance requirement $D_i = \min_{x_j \in \mathcal{T}_i} d_j$;
8: **end for**
9: **for** each RS x_i and its parent x_i^p on $\mathcal{T}_{mst}$ **do**
10: $w_2(x_i^p, x_i) = \lceil \frac{\|e(x_i^p,x_i)\|}{D_i} \rceil - 1$;
11: Place $w_2(x_i^p, x_i)$ RSs on edge $e(x_i^p, x_i)$ separating the edge into $\lceil \frac{\|e(x_i^p,x_i)\|}{D_i} \rceil$ parts with each one having feasible distance;
12: **end for**

Let us use an example in Fig. 7 to illustrate our solution. The network includes BS, RS_1 and RS_2. First, we construct a undirected complete graph in Fig. 7(a) (Line 1). We then assign edge weight $w_1(e) = \lceil \|e\|/d_{min} \rceil - 1$ on each edge e (Lines 2 - 4), where d_{min} is the minimum distance requirement among all the nodes, which is 5 in the example. The distances of edges (BS, RS_1), (BS, RS_2) and (RS_1, RS_2) are 20, 21 and 16, respectively. The corresponding weights of these edges are 3, 4 and 3. Next in Line 5, a minimum spanning tree is constructed in Fig. 7(b). Now we have the parent-child relationship between nodes. For example, RS_1 is the parent of RS_2. Based on the parent-child relation, now D_i for each RS r_i has to be updated (Lines 6 - 8). For example, RS_1 has to reduce its distance requirement to 5 to ensure the service for its child RS_2. Next, we need to re-calculate the edge weight $w_2(e)$ for each tree edge e (Line 10), shown in Fig. 7(b), and then place RSs accordingly (Line 11), shown in Fig. 7(c).



(a) Complete graph (b) Minimum spanning tree (c) MUST Solution

Fig. 7. Illustration of MUST

Theorem 2. Algorithm 3 finds a $\frac{8 \cdot d_{max}}{d_{min}}$-approximation for the MUST problem. In other words, let R_M be the set of RSs placed by our solution and OPT_M be an optimal solution for MUST, we have

$$|R_M| \leq \frac{8d_{max}}{d_{min}}|OPT_M|$$

where d_{min} and d_{max} denote the *minimum* and *maximum* distance requirements from SSs, respectively. □

Proof: We assume that each edge needs at least one RS placed (if no RS is needed on an edge, then we simply ignore this edge because it has no affect on the solution). First, if we consider a *special case*, MUST(X, d_{min}) (denoted by *ST*) that all users have the same distance requirement d_{min}, then *ST* is a *Minimum Steiner Tree* problem studied in [2], [15]. We denote R_{ST}, OPT_{ST} as our scheme and an optimal solution for the *ST* problem, respectively.

Given OPT_M, an optimal solution for MUST$(X, \mathcal{D})$, instead of placing RSs with distance D_i, we place RSs with distance d_{min} on the same tree structure. Then we will have a *feasible solution*, denoted by OPT'_M, for *ST*. The number of RSs placed on each edge e changes from $\lceil \frac{\|e\|}{D_i} \rceil - 1$ to $\lceil \frac{\|e\|}{d_{min}} \rceil - 1$. Therefore,

$$|OPT_{ST}| \leq |OPT'_M| = \frac{\lceil \frac{\|e\|}{d_{min}} \rceil - 1}{\lceil \frac{\|e\|}{D_i} \rceil - 1}|OPT_M|$$

Let $\|e\| = \alpha_i D_i + \beta_i = \alpha_{min} d_{min} + \beta_{min}$, where $\alpha_i, \alpha_{min} \geq 1$, $\beta_i < D_i$ and $\beta_{min} < d_{min}$. We have

$$\frac{\alpha_{min}}{\alpha_i} = \frac{D_i}{d_{min}} + \frac{\beta_i - \beta_{min}}{\alpha_i d_{min}} \tag{4.1}$$

CASE 1: If $\beta_i > 0$

$$\frac{|OPT_{ST}|}{|OPT_M|} \leq \frac{\lceil \frac{\|e\|}{d_{min}} \rceil - 1}{\lceil \frac{\|e\|}{D_i} \rceil - 1} = \frac{\lceil \alpha_{min} + \frac{\beta_{min}}{d_{min}} \rceil - 1}{\lceil \alpha_i + \frac{\beta_i}{D_i} \rceil - 1} \leq \frac{\alpha_{min}}{\alpha_i}$$

If $\beta_i \leq \beta_{min}$, then based on Equation (4.1)

$$\frac{\alpha_{min}}{\alpha_i} \leq \frac{D_i}{d_{min}} \leq \frac{d_{max}}{d_{min}}$$

If $\beta_i > \beta_{min}$, then based on Equation (4.1)

$$\frac{\alpha_{min}}{\alpha_i} \leq \frac{D_i}{d_{min}} + \frac{\beta_i}{\alpha_i d_{min}}$$

Because $\alpha_i \geq 1$ and $\beta_i < D_i$, therefore

$$\frac{\alpha_{min}}{\alpha_i} \leq \frac{D_i}{d_{min}} + \frac{D_i}{d_{min}} \leq 2\frac{d_{max}}{d_{min}}$$

CASE 2: If $\beta_i = 0$

$$\frac{|OPT_{ST}|}{|OPT_M|} \leq \frac{\lceil \frac{\|e\|}{d_{min}} \rceil - 1}{\lceil \frac{\|e\|}{D_i} \rceil - 1} = \frac{\lceil \alpha_{min} + \frac{\beta_{min}}{d_{min}} \rceil - 1}{\lceil \alpha_i + \frac{\beta_i}{D_i} \rceil - 1} \leq \frac{\alpha_{min}}{\alpha_i - 1}$$

Note that we only consider *the edges that have at least one RS placed*, then $\alpha_i - 1 \geq 1$. Consequently, $\frac{1}{\alpha_i - 1} \leq \frac{2}{\alpha_i}$. So we have

$$\frac{|OPT_{ST}|}{|OPT_M|} \leq 2\frac{\alpha_{min}}{\alpha_i}$$

Based on Equation (4.1), we have

$$\frac{|OPT_{ST}|}{|OPT_M|} \leq 2\left(\frac{D_i}{d_{min}} + \frac{\beta_i - \beta_{min}}{\alpha_i d_{min}}\right)$$

Because $\beta_i = 0$ and $\alpha_i \geq 2$, therefore

$$\frac{|OPT_{ST}|}{|OPT_M|} \leq 2\frac{D_i}{d_{min}} \leq 2\frac{d_{max}}{d_{min}}$$

Combining **CASE 1** and **CASE 2**, we have

$$|OPT_{ST}| \leq \frac{2d_{max}}{d_{min}}|OPT_M| \tag{4.2}$$

For *ST* problem [2] [15], we have $|R_{ST}| \leq 4|OPT_{ST}|$. Combining with formula (4.2), we know that

$$|R_{ST}| \leq 4|OPT_{ST}| \leq \frac{8d_{max}}{d_{min}}|OPT_M|$$

It is easy to see that R_{ST} is a feasible solution for MUST$(S, X, \mathcal{D})$. Note that R_M and R_{ST} use the same minimum spanning tree topology $\mathcal{T}_{mst}$, with different distance requirement set D_i and d_{min}. Therefore,

$$|R_M| \leq |R_{ST}| \leq \frac{8d_{max}}{d_{min}}|OPT_M|$$

This completes our proof. ■

C. *Approximation Algorithm for* DARP *Problem*

With the approximation solutions for LORC and MUST, we can present an approximation algorithm for the DARP problem, which is listed in Algorithm 4.

Algorithm 4 DARP(S, $\mathcal{D}$)

```
X ← LORC(S, D); //LORC-MIS or LORC-HS can be applied
for each x_i ∈ X do
    Among all the SSs covered by x_i, pick up the s_i having the smallest distance requirement;
    Place an RS z_i on the location of s_i; Z = Z ∪ {z_i};
end for
Y ← MUST(Z, D);
return X ⋃ Y ⋃ Z.
```

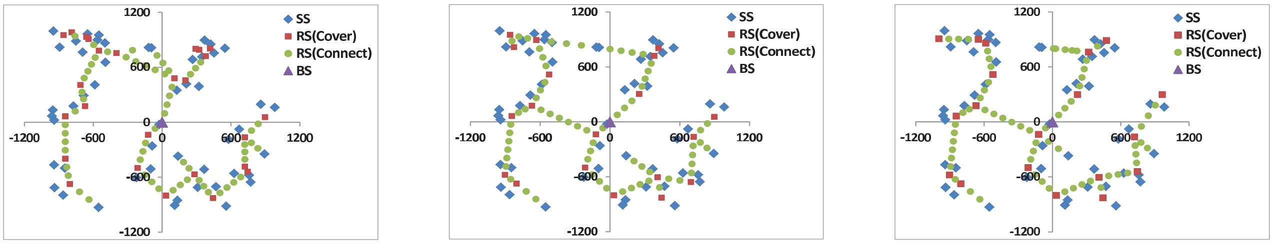
Theorem 3. The set of relay stations produced by Algorithm 4 is a $(2\alpha + \beta)$-approximation for the DARP problem, where α is the approximation ratio of solution for LORC, and β is the approximation ratio of solution for MUST. □

Proof: Denote *OPT* as the optimal solution for the DARP$(S, \mathcal{D})$ problem, O_L and O_M as the optimal solution (the minimal number of RSs) for the LORC$(S, \mathcal{D})$ and the MUST$(Z, \mathcal{D})$ subproblems, respectively. It is easy to see that *OPT* is also a feasible solution for the LORC$(S, \mathcal{D})$ problem, then we have

$$|O_L| \leq |OPT|$$

If we provide an α-approximation solution S_{cover} for LORC, then we have

$$|S_{cover}| \leq \alpha|O_L| \leq \alpha|OPT|$$



(a) MIS + MUST (b) HS + MUST (c) ILP + MUST

Fig. 8. Illustration of tree topology for 50 SSs

Note that *OPT* is an optimal solution for DARP($S, \mathcal{D}$), it would be a feasible solution to MUST($S, \mathcal{D}$). Since $Z \subseteq S$, *OPT* must be a feasible solution for MUST($Z, \mathcal{D}$). Therefore, we have

$$|O_M| \leq |OPT|$$

If we provide a β-approximation solution S_{con} for MUST, it is easy to see that

$$|S_{con}| \leq \beta |O_M| \leq \beta |OPT|$$

Since our solution is $S_{cover} \cup S_{con} \cup Z$, and $|Z| = |X| = |S_{cover}|$, the number of placed RSs is

$$|S_{cover} \cup S_{con} \cup Z| \leq |S_{cover}| + |S_{con}| + |Z| = 2|S_{cover}| + |S_{con}|$$

Hence, we have

$$|S_{cover} \cup S_{con} \cup Z| \leq 2|S_{cover}| + |S_{con}|$$
$$\leq 2\alpha \cdot |OPT| + \beta \cdot |OPT| = (2\alpha + \beta) \cdot |OPT|$$

This completes the proof of the theorem. ■

Note that we provide a general framework for the DARP problem. Within the framework, for given requirements on running times and performances (e.g., k-approximation), we can provide various approximation algorithms to solve the two sub-problems and consequently provide different approximation solutions to the DARP problem. For example, using LORC-MIS and MUST solutions, we provide a fast $(14 + 8\frac{d_{max}}{d_{min}})$-approximation for the DARP problem. While using LORC-HS and MUST, a solution with better approximation ratio can be found in a much longer time.

V. Numerical Results

In this section, we present numerical results to confirm the effectiveness of our solutions. We implemented both the *7-approximation* solution and *hitting set* based scheme for LORC, which are denoted as MIS and HS in the figures, respectively. The solution for MUST was also implemented. All our simulation runs were performed on a 2.8 GHz Linux PC with 1G bytes of memory. As in [24], [28], SSs were uniformly distributed in a square playing ground. One base station was deployed at the center of the field. All the figures illustrate the average of 10 test runs for various scenarios.

First, we illustrate the tree topologies generated by our solutions in Fig. 8. With distance requirements randomly distributed in [100,150], 50 SSs were deployed in a 2000×2000 sq. unites. For LORC, we presented MIS, HS, and an optimal solution using Integer Linear Programming (ILP) solved by Gurobi Optimizer [9]. From Fig. 8, we observed that ILP placed the smallest number of RSs. HS not only deployed similar number of RSs, but also generated similar tree topology with the one generated by ILP.

Next, we test performances, in terms of the *number of RSs placed* and the *running times*, of our solutions. Figs. 9 and 10 present the results using two different playing fields with different network density. In both cases, *four* metrics were tested to compare *the number of coverage RSs* (LORC), *the number of connectivity RSs* (MUST), *number of total deployed RSs* (DARP), and *running time*.

Figs. 9(a) and 10(a) showed that ILP always provide the best results for LORC, and HS provided a solution that is close to ILP and better than MIS. Meanwhile, the solutions found by MIS were always less than 3 times of the one found by ILP, which confirms our theoretical analysis.

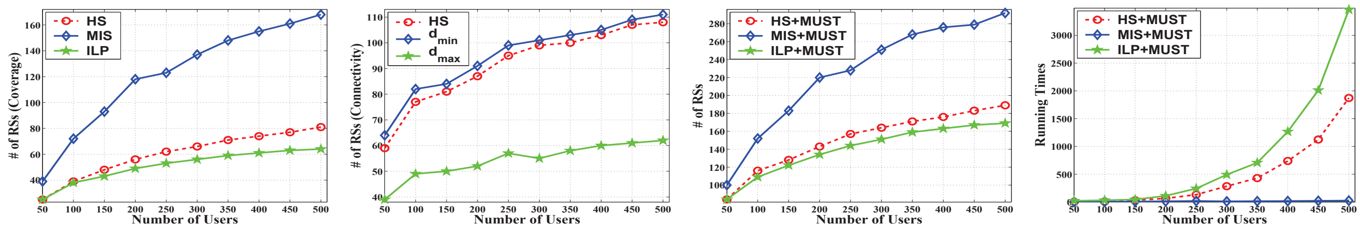
In Fig. 9(b), using HS as the coverage solution, we tested the performance of MUST in terms of providing connectivity RSs. Since there are no previous algorithms for MUST, and that optimal solutions are difficult to obtain, we implemented two special cases for comparison: placement with the same distance requirements d_{min} and d_{max}, respectively. The corresponding results are presented in Fig. 9(b). As expected, our solution performs between these two spacial cases. We observed that the number of connectivity RSs found by MUST was less than the one with requirement d_{min}, and is *no more than 4 times* of the one found by the case with d_{max}. Similar results can be found in Fig. 10(b).

Fig. 9(c) illustrated the performance of DARP, which provided the total number of RSs placed. First, we noticed that the number of RSs increased as the number of SSs increased. ILP, the best solution for LORC, seemed to provide best overall solution. And HS+MUST performed better than MIS+MUST. It seems that *the coverage RS placement has important effects on the overall placement performance*.

Fig. 9(d) demonstrated the running time performances. We can see that MIS had much better running time than HS and ILP, which makes it to be the best solution for large number of users. Similar trends were found in Fig. 10(d) with different network density.

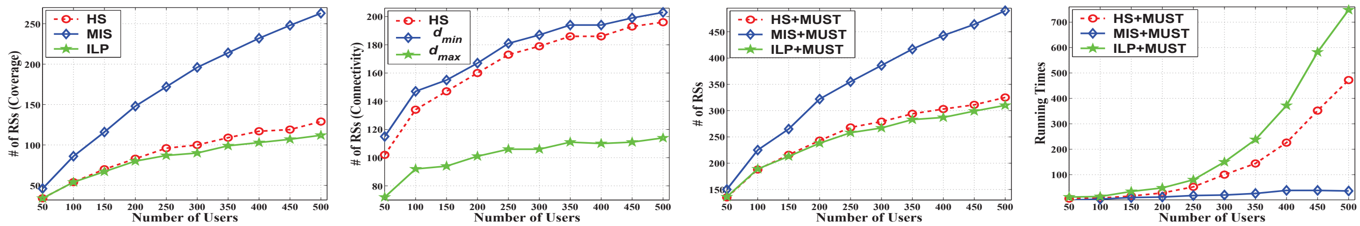
VI. Conclusions

In this work, we studied the Distance-Aware Relay Placement (DARP) problem, which seeks the multi-hop relay node placement with channel capacity constraint, in WiMAX mesh networks. We divided this problem into two sub-problems, LOwer-



(a) Coverage RSs for LORC (b) Connectivity RSs for MUST (c) Overall performances for DARP (d) Running times

Fig. 9. 2000×2000 playing field



(a) Coverage RSs for LORC (b) Connectivity RSs for MUST (c) Overall performances for DARP (d) Running times

Fig. 10. 3000×3000 playing field

tier Relay Coverage (LORC) problem and Minimum Upper-tier Steiner Tree (MUST) problem. For LORC problem, we proposed two approximation algorithms. For the MUST problem, we presented a minimum spanning tree based steinerization scheme, and proved this solution is an $8\frac{d_{max}}{d_{min}}$-approximation scheme. Then we presented an approximation framework of DARP by combining the solutions of the sub-problems. Numerical results confirmed our theoretical analysis.

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