Topology Control in Multihop Wireless Networks with Multi-beam Smart Antennas

Brendan Mumey, Ivan Judson, Jian Tang, Yun Xing

***Abstract*—Smart (directional) antennas can be deployed in a wireless network to enhance the capacity of the network by forming one or multiple beams only towards intended receivers. In this paper, we focus on the topology control approach for efficient communications in a wireless network with multi-beam smart antennas. This approach pre-computes an antenna pattern for each node such that an efficient network topology can be formed for future communications. The corresponding optimization problem is formally defined as the Sector Selection Problem (SSP). First, we present a Mixed Integer Linear Programming (MILP) formulation to provide optimal solutions. Then we present a Linear Programming (LP) rounding based algorithm for the SSP. Extensive simulation results show that the proposed algorithm provides close-to-optimal performance and outperforms a Minimum Spanning Tree (MST) based algorithm and the k nearest neighbors algorithm in terms of both network capacity and fairness.**

***Index Terms*– Multihop wireless networks, smart antennas, topology control and cross-layer design**

I. Introduction

Compared to a conventional omni-directional antenna, which wastes most of its energy in directions where there is no intended receiver, a smart (directional) antenna offers a longer transmission range and lower power consumption by forming one or multiple beams only toward intended receivers.

There are primarily two approaches to exploit the benefits of smart antennas for mesh networking: the cross-layer approach [1] and the topology control approach [2]. With the cross-layer approach, a joint antenna pattern assignment and scheduling solution is provided to switch beams to communicate to different neighbors at a fast time scale (e.g., on a per-timeslot basis). However, the topology control approach pre-computes an antenna pattern for each node such that a certain network topology can be formed for future communications. Using this approach, antenna beam switching is conducted in a slower time scale (in the order of seconds or more) at the potential expense of performance. Compared to the cross-layer approach, the major advantage to using the topology control approach is that it is purely a link layer solution that does not require any modifications to a standard MAC protocol. Hence, it can be easily implemented in a system using Commercial-Off-The-Shelf (COTS) and standard protocols. The topology control approach is the focus of this paper, which may lead to performance comparable to the cross-layer approach with a carefully designed algorithm and full consideration for link capacity. In this paper, we study topology control in multihop wireless networks with multi-beam smart antennas with the objective of maximizing network capacity. Our contributions are summarized as follows:

1) We formally define the corresponding optimization problem as the Sector Selection Problem (SSP).

2) We present a Mixed Integer Linear Programming (MILP) formulation to provide optimal solutions.

3) We present an effective Linear Programming (LP) rounding based algorithm for the SSP.

4) We present extensive simulation results to show that the proposed algorithm provides close-to-optimal performance and yields good solutions in terms of both capacity and fairness compared to alternative approaches including a Minimum Spanning Tree (MST) based algorithm and the k nearest neighbors algorithm.

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II. Related Work

Smart antennas have received tremendous research attentions. Topology control with directional antennas has been studied in [2]–[8]. In one of the first works on this topic [2], Kumar *et al.* presented a topology control approach to effectively using directional antennas with legacy MAC protocols, which uses multiple directional antennas on each node and orients them appropriately to create low interference topologies while maintaining network connectivity. They showed via empirical studies that this approach can reduce interference significantly and improve network capacity without increasing stretch factors to any appreciable extent. In [3], the authors considered the problem of power-efficient topology control with switched beam directional antennas, taking into account their non-uniform radiation pattern within the beamwidth. In [4], Huang and Shen presented several heuristic algorithms for topology control with multi-beam directional antennas, and showed that compared to the omnidirectional topology control approach, the proposed algorithms provide equivalent performance in terms of the probability distribution of the number of symmetric neighbors in their resulting topologies, but can reduce hop count, save power and provide symmetric links. The authors of [5] presented a bandwidth-guaranteed topology control algorithm for TDMA-based ad hoc networks with sectorized antennas. In a recent paper [6], the authors introduced a measurement-based optimization framework for topology control in dense 802.11 networks using sectorized

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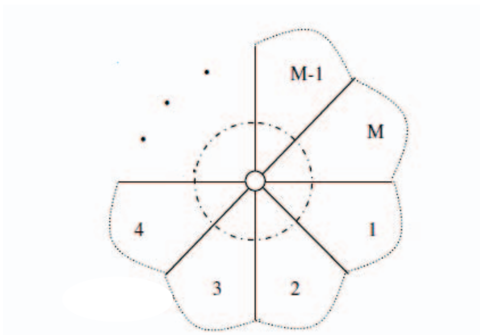

Fig. 1. A multi-beam antenna

antennas. Topology control with omni-directional antennas have been extensively studied in the literature [9], [10].

We summarize the differences between our work and these related works as follows: 1) Topology control with directional antennas and that with omni-directional antennas [9], [10] are significantly different due to directional beam patterns. 2) Most of previous research on directional antennas was focused on single-beam antennas [2], [3], [5], [6], [8]. However, we consider multi-beam smart antennas, which makes the corresponding topology control problems much harder. 3) Different from [4], we aim at maximizing network capacity instead of minimizing power consumption.

III. System Model

We consider a multihop wireless network composed of n nodes. Each node v_i is equipped with an adaptive directional antenna that can form beams in any of M different sectors (see Figure 1). Each sector that is activated (turned on) creates a beam of width $\frac{360}{M}$ degrees.

We assume that the data rates available to a node depends on the number of sectors it has activated. Let $c^a_{i,j} \geq 0$ be the maximum data rate that can be supported by the link (v_i, v_j) assuming v_i has activated a sectors in total, including the sector in which v_j falls in. Typically, $c^a_{i,j}$ depends on the transmit power at v_i, the distance between v_i and v_j, the operating frequency, and maybe other factors. If the free space path loss model [11] is considered, then the Signal to Noise Ratio (SNR) at node v_j for a transmission over link (v_i, v_j) is

$$\text{SNR}_{ij} = \frac{P_t G^a_t G_r \lambda^2}{(4\pi)^2 d^{\alpha}_{ij} N_0} \tag{1}$$

where G^a_t is the transmitter gain assuming that v_i has activated exactly a sectors, G_r is the receiver antenna gain (omni-directional reception), d_{ij} is the distance between v_i and v_j, λ is the wavelength and N_0 is the background noise power. α is the path loss exponent and is usually between 2 and 4. We assume a fixed transmit power P_t, operating frequency and receiver gain G_r, so SNR_{ij} varies only with the number of sectors activated by v_i and the transmission distance. Practically, if a radio is capable of Adaptive Modulation and Coding (AMC), the maximum link data rate c^a_{ij} is given by a discrete step increasing function of SNR at the receiver (instead of the continuous Shannon's function). A set of SNR thresholds, and the corresponding modulation indices and maximum data rates (link capacities) specified by IEEE 802.16e [12] is given in Table II, which was used for our simulations. For each $1 \leq a \leq M$, we compute the transmission range for a node using a sectors as

$$\left(\frac{P_t G^a_t G_r \lambda^2}{(4\pi)^2 \text{SNR}_{\min} N_0}\right)^{\frac{1}{\alpha}},$$

where $\text{SNR}_{\min}$ is the minimum SNR threshold.

IV. Problem Formulation

We are interested in the problem of choosing which antenna sectors each node should activate. We will assume that the power available to each activated sector for transmission depends on the number of sectors activated. Thus, there is a trade-off between activating additional sectors to increase the number of directions that a node can use to reach other nodes and the transmission rates achievable to those nodes. We formally define the optimization problem as follows.

Definition 1 (SSP)**.** *Given n network nodes $\{v_1, \ldots, v_n\}$, each with M sectors, the* **Sector Selection Problem (SSP)** *is to select an active set of sectors $A_i \subset [1, M]$ for each node v_i such that (1) the network is connected, (2) links (u, v) are bidirectional, and (3) the total capacity of the network is maximized.*

In this problem, our objective is to maximize the summation of link capacities since this summation gives the *maximum (possible) capacity* (note that the actual network capacity may depend on many other factors such as the MAC protocol). It may be argued that maximizing the total link capacity may lead to unfairness, however we will show that our SSP algorithm offers good performance in terms of both maximum capacity and fairness via simulation results.

A. MILP Formulation

In this section, we present the following MILP formulation of the SSP, that can be used to provide optimal solutions.

Variables:
$s_{u,k} \in \{0, 1\}$: indicates whether u activates sector k
$x^a_{u,v} \in \{0, 1\}$: indicates whether link (u, v) is active and u uses a sectors
$z^a_u \geq 0$: used for constraining the $x^a_{u,v}$ variables (real)
$f_{u,v} \geq 0$: the amount of flow on the edge (u, v) (real)

Objective:

$$\max \; T_{total} = \sum_{u,v,a} c^a_{u,v} x^a_{u,v} \tag{2}$$

subject to the following constraints (indices vary over their entire domains, unless otherwise noted). We also choose one node arbitrarily to be the *source* vertex s. The purpose of s is explained below.

$$\sum_a x^a_{u,v} \le 1 \tag{3}$$

$$x^a_{u,v} = 0, \quad \text{if } c^a_{u,v} = 0 \tag{4}$$

$$\sum_a x^a_{u,v} = \sum_a x^a_{v,u} \tag{5}$$

$$x^a_{u,v} \le s_{u,s(u,v)} \tag{6}$$

$$z_u^{|A|-1} \le |A| - \sum_{k\in A} s_{u,k}, \quad A \subset [1, M], |A| > 1 \tag{7}$$

$$x^a_{u,v} \le z^a_u \tag{8}$$

$$f_{u,v} \le 1 \quad (u,v) \in E \tag{9}$$

$$f_{u,v} \le n \sum_a x^a_{u,v}, \quad (u,v) \in E \tag{10}$$

$$\sum_{\{(u,s)\in E\}} f_{u,s} + 1 = \frac{1}{n} + \sum_{\{v:(s,v)\in E\}} f_{s,v} \tag{11}$$

$$\sum_{\{u:(u,v)\in E\}} f_{u,v} = \frac{1}{n} + \sum_{\{w:(v,w)\in E\}} f_{v,w}, \quad v \in V \setminus \{s\} \tag{12}$$

In this formulation, constraint (3) ensures that for each link (u, v), the transmitting node u must fix the number of sectors, a, it is using. Constraint (4) ensures that links (u, v) cannot be used if u, when using a sectors, cannot provide any capacity to v. Constraint (5) guarantees that each link will be bidirectional in the solution (if (u, v) is operational then (v, u) must also be operational). The constraint (6) requires that if the link (u, v) is used, then u must activate the sector containing the node v. Constraints (7) and (8) work together to ensure that if $x^a_{u,v} = 1$ then the node u must not have more than a sectors activated. This is done by enumerating all subsets of u's sectors of size two or greater, e.g. if any two sectors are activated then u cannot use a link of the form $x^1_{u,v}$, etc. We note that there are $2^M - M - 1$ such constraints. The flow constraints are used to ensure network connectivity. In particular, one node is designated as the source node s and there must be connectivity from s to all other nodes. We view each node $v \in V$ as able to implicitly absorb $\frac{1}{n}$ units of flow. We assume that there is 1 unit of flow originating at the source node and that the source node is able to absorb $\frac{1}{n}$ of this flow and must ship the remainder to the other $n - 1$ nodes to be absorbed. In particular, (11) stipulates that the net flow out of the source node is $1 - \frac{1}{n}$ and (12) says that the flow into any other node equals the flow absorbed at the node plus the outgoing flow from the node. Constraint (9) ensures that the flow on each link is at most 1. Constraint (10) requires that if a link (u, v) has positive flow, then it must be active since $\sum_a x^a_{u,v} \ge \frac{1}{n} f_{u,v}$. The $\frac{1}{n}$ factor is used to reduce the influence of which node was chosen to be the source vertex s. Together, the transmission and flow constraints guarantee that the MILP solution meets both the network connectivity and bidirectional links requirements of the SSP and the objective function ensures the solution has maximum total capacity.

V. Proposed Algorithm

We present an effective heuristic algorithm to solve the SSP based on LP rounding. We begin with an observation that the $x^a_{u,v}$ variables can be relaxed to be real-valued in the MILP from Section IV-A, provided the $c^a_{u,v}$ constants are non-increasing with a:

Lemma 1. *If $c^1_{u,v} \ge \ldots \ge c^M_{u,v}$ for all $(u, v) \in E$, then relaxing the $\{x^a_{u,v}\}$ to be real-valued and adding additional constraints of the form $0 \le x^a_{u,v} \le 1$ does not change the optimal objective value of (2).*

Proof: The objective value of the original MILP cannot exceed that of the relaxed version. Suppose $\{x^a_{u,v}\}$ are part of an optimal relaxed solution and suppose (u, v) is some edge such that $\sum_a x^a_{u,v} > 0$. We observe that constraint (3) must be tight, since the right hand sides of constraints (6) and (8) are integer values. In particular, let $a' = \text{argmin}_a\, z^a_u > 0$. If we set $x^{a'}_{u,v} = 1$ and $x^a_{u,v} = 0$ for $a \ne a'$, this cannot decrease the objective value and remains feasible. It follows that an optimal solution to the relaxed LP can be found with integral $\{x^a_{u,v}\}$ values and so the original and relaxed MILPs have the same optimal objective value. ■

By Lemma 1, we can solve the relaxed version of the MILP instead of the original. The idea of our algorithm is to further relax the MILP to a LP formulation in which the sector usage variables $\{s_{u,k}\}$ are also relaxed to be real-valued, with additional constraints of the form $0 \le s_{u,k} \le 1$. The algorithm proceeds in two phases. In the first phase, the network connectivity constraints are satisfied by ensuring that a solution contains a spanning tree of the network. The approach used is similar to Prim's algorithm for finding a minimum spanning tree in a graph: an edge (u, v) is added to the spanning tree if nodes u and v are currently disconnected and the addition of the link (u, v) most improves the objective. If link (u, v) is selected, u and v's components are merged and we add the explicit constraint $s_{u,s(u,v)} = 1$ to the LP. In this way, sectors in the network are gradually forced to be fully activated. After phase one, if the LP remains feasible, the network will be connected. At this point, all sectors that were not explicitly turned on in phase one, are set to be off. We consider this as a baseline integer solution. In phase two, we greedily try to improve the baseline solution by checking to see if any facing sector pairs, with at least one sector of pair unactivated, can be both activated to realize a gain in the overall network capacity. If so, we choose the pair with the greatest capacity improvement and activate these sectors. We repeat this until no further improvements are found.

A. Computational Complexity

We note that the time complexity of the algorithm is polynomial. This because the LPs that are solved are polynomial in size and can be solved in polynomial time. Step 2 requires solving $n-1$ such LPs. A simple upper bound on the length of L in Step 4 is $n(n-1)/2$ since each pair of nodes may or may not yield an unused facing pair of sectors in L. L shrinks by 1

Algorithm 1 SSP-LPR

Step 1 Let P be the fully relaxed version of the MILP with the $\{x^a_{u,v}\}$ and $\{s_{u,k}\}$ real-valued;
for $i=1$ **to** n: set cpnt$(v_i)=i$; **endfor**

Step 2 **for** $i=1$ **to** n-1
Solve P;
Compute

$$(u,v) = \underset{\{(u,v)|\text{cpnt}(u)\neq\text{cpnt}(v)\}}{\text{argmax}} \sum_a c^a_{u,v} x^a_{u,v}$$

Add $\{s_{u,s(u,v)}=1, s_{v,s(v,u)}=1\}$ to P;
Merge cpnt(u) with cpnt(v);
endfor

Step 3 **for** $s_{u,k} \in P$ unconstrained by Step 2
Add $\{s_{u,k}=0\}$ to P;
endfor
Let $L=\{((u_l,k_l),(u_r,k_r))\}$ be all facing sector pairs such that $s_{u_l,k_l}=0$ or $s_{u_r,k_r}=0$;

Step 4 **do**
improvement = **FALSE**;
Compute $((u_l,k_l),(u_r,k_r))=$
argmax$_L$ obj$(P+\{s_{u_l,k_l}=1, s_{u_r,k_r}=1\})$;
if obj$(P+\{s_{u_l,k_l}=1, s_{u_r,k_r}=1\})>$ obj(P)
Let $P=P+\{s_{u_l,k_l}=1, s_{u_r,k_r}=1\}$;
Update L;
improvement = **TRUE**;
endif
while (improvement);

pair each iteration. So, in the worst case Step 4 makes $O(n^4)$ LP solve calls but in practice the number is much smaller as instances up to $n=30$ can be run in a few minutes on a fast workstation.

VI. Simulation Results

In this section, we present simulation results to show the performance of the proposed algorithm. In our simulations, n wireless nodes were randomly deployed within a square 30km on a side. We calculated SNRs using the free space path loss model [11] as described in Section III. The transmitter antenna gain of each node was set to $\frac{360}{\theta}G_o$, where G_o is the gain of an antenna working in the omni-directional mode. In addition, each node was assumed to be able to receive signals from all directions, i.e., the receiver antenna gain of each node was set to G_o. The values of those parameters relevant to the propagation model and other related parameters were set according to Table I.

As described before, the link capacity is given by a discrete step increasing function. A set of SNR thresholds, and the corresponding modulation indices and link capacities specified by IEEE 802.16e [12] are given in Table II.

It should be obvious that the placement of the nodes in the 30km region, the number of nodes (n), and the number of sectors per node (M) affect the performance of the solutions.

TABLE I
Common Simulation Settings

Omni-directional antenna gain G_o	2dB;
Operating frequency	5.8GHz;
Path loss exponent	2;
Transmit power of each RS (P_t)	1W;
Noise power	-174dBm/Hz;
Channel bandwidth	10MHz;

TABLE II
SNR vs. Link Capacity

SNR Threshold (dB)	Modulation Index	Link Capacity (Mbps)
10	QPSK $1/2$	10
14.5	16QAM$1/2$	20
17.25	16QAM $3/4$	30
21.75	64QAM $2/3$	40
23	64QAM $3/4$	45

In our simulation scenarios we change either the value of n or M while keeping the other value fixed. We evaluate the performance of the proposed algorithm, i.e., the SSP-LP rounding algorithm (labeled as SSP-LPR), a Minimum Spanning Tree Approach (labeled, SSP-MST), and a K-Nearest Neighbors based algorithm (labeled as SSP-kNN) in terms of the summation of link capacities (i.e. maximum capacity) and the well-known Jain's fairness index [13],

$$f(r_1,r_2,\cdots,r_n)=\frac{\left(\sum_{i=1}^n r_i\right)^2}{n\sum_{i=1}^n (r_i)^2},$$

where r_i is the capacity of each edge in the network. Further we compute the fairness into and out of each node to verify the results. Jain's fairness index is the most commonly used metric for evaluating the performance of resource allocation algorithms in terms of fairness. In the first scenario, we compared the proposed algorithms against the optimal solutions given by solving the MILP in small cases. In the other scenarios, we compared the proposed algorithm in terms of both metrics on large input cases. We summarize our simulation scenarios in the following and present the corresponding simulation results in Figs. 2–4. Each number in these figures is an average over 10 runs, each with a different randomly generated network.

Scenario 1: Change n from 8 nodes to 16 nodes with a step size of 2. Fix $M=8$.
Scenario 2: Change n from 10 nodes to 30 nodes with a step size of 5. Fix $M=8$.
Scenario 3: Change M from 4 sectors to 12 sectors with a step size of 2. Fix $n=20$.

The following observations can be made from the simulation results:

1) From Fig. 2, we can see that the maximum capacity values given by the proposed algorithm closely track the optimal values. The average difference is only 3.61% for the SSP-LPR algorithm.

2) From Figs. 3–4, we see that when the number of nodes in the network varies, the SSP-LPR algorithm outperforms the SSP-MST by approximately a factor of two and it outperforms

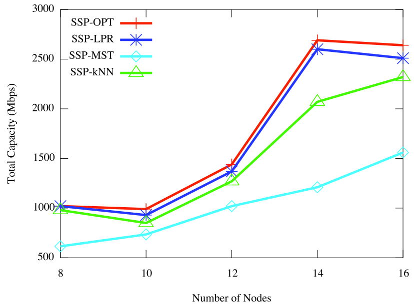

Fig. 2. Scenario 1: The proposed algorithm VS. optimal

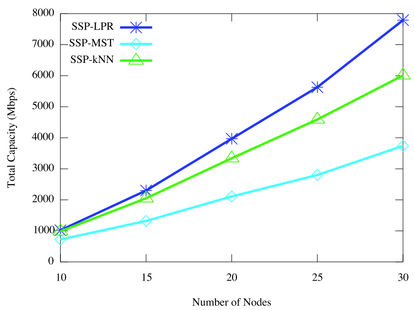

Fig. 3. Scenario 2: Performance VS. the number of nodes in the network.

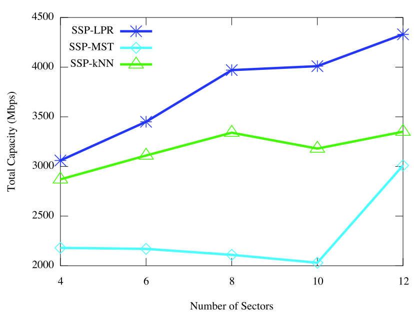

Fig. 4. Scenario 3: Performance VS. the number of sectors in each node.

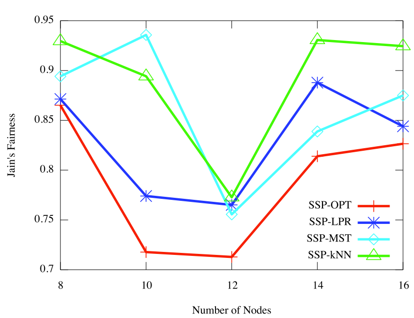

Fig. 5. Average Jain's Fairness for each of the algorithms.

the SSP-kNN algorithm by an average of 30%. As the number of sectors per antenna is increased the SSP-LPR outperforms the SSP-MST algorithm by an average of 30% and the SSP-kNN by an average of 23%, however this performance shows increasing gains for the SSP-LPR algorithm as the number of sectors grows.

3) Although the goal of the proposed algorithm is to maximize the network capacity, maximizing capacity can lead to poor fairness. In Scenario 1, we computed the fairness of both incoming and outgoing node capacities. The average of these values for for each algorithm is shown in Fig. 5. All algorithms are relatively fair (fairness > 0.7), with SSP-kNN and SSP-MST providing the best average fairness.

VII. Conclusions

In this paper, we have studied the topology control approach for efficient communications in wireless relay networks with smart antennas. The corresponding optimization problem was formally defined as the SSP. We first presented an MILP formulation to provide optimal solutions. Then we presented a new LP rounding algorithm. It has been shown by extensive simulation results that the proposed algorithm provides close-to-optimal performance and is superior to several alternative approaches in terms of both network capacity and fairness.

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