Power Efficient Broadcasting and Multicasting in Wireless Networks with Directional Antennas

Jian Tang, Guoliang Xue and Weiyi Zhang
Department of Computer Science and Engineering
Arizona State University
Tempe, AZ 85287-8809.
Email: {jian.tang, xue, weiyi.zhang}@asu.edu.

Abstract— **Broadcasting and multicasting packets in a power efficient way is a critical task in wireless ad hoc networks. In a recent paper [6], Li *et al.* study the Minimum Energy Broadcast (MEB) routing problem in a wireless ad hoc network where every node has an omni-directional antenna and a fixed transmission power level. We extend their work to wireless networks with directional antennas in this paper. We formulate the Minimum Power Multicasting/Broadcasting using Directional Antennas (PMDA/PBDA) problems. For each problem, we present an approximation algorithm with worst-case approximation ratio $O(\log^2 n)$, where n is the number of nodes in the network. We also present several effective heuristics to solve the problems, the Shortest Path Tree (SPT) heuristic, the Directed Minimum Spanning Tree (DMST) heuristic and a greedy heuristic. Simulation results are presented to show the performance of our algorithms.**

Keywords: Wireless ad hoc networks, power efficient broadcasting and multicasting, directional antennas.

I. Introduction

Multicasting in wireless ad hoc networks are quite different from that in wired networks. Power efficiency is the major concern for wireless ad hoc routing because wireless nodes are generally powered by batteries which can not last too long if operated at high power levels. In addition, radio is a broadcast medium, i.e., all receivers within the transmission range of a transmitter can receive transmitted packets. This property is very useful for wireless multicasting since one transmission can deliver a packet to multiple receivers. This property is known as Wireless Multicast Advantage (WMA) in [12]. The authors show that we can substantially reduce total transmission power for a multicast request if we carefully construct a multicast tree with WMA in mind. They present a simple but very efficient greedy heuristic, the Broadcast Incremental Power (BIP), to solve the minimum power broadcasting problem and slightly modify it for the multicasting problem. Recently, the authors of [6] study power efficient broadcasting in a static wireless ad hoc network where every node has an omni-directional antenna and a fixed transmission power level. They formulate a new optimization problem, *Minimum Energy Broadcast (MEB)* routing problem, which seeks a broadcast tree with minimum total energy cost. They prove that the MEB problem is NP-hard and propose three heuristic algorithms to solve it. They also show that the approximation ratio of their node weighted Steiner tree based heuristic is $(1+2\ln(n-1))$. In addition, several other algorithms for power efficient multicasting or broadcasting have been presented in [1], [7], [10].

This research was supported in part by ARO grant W911NF-04-1-0385 and NSF grant CCF-0431167.The information reported here does not reflect the position or the policy of the federal government.

A directional antenna can concentrate RF energy in the direction where the transmission needs to be made. Since transmission power is proportional to beam width, using directional antennas can significantly save transmission power. We employ a well known sectored antenna model ([8], [9]) and model a wireless ad hoc network using a communication graph. We formulate the *Minimum Power Multicasting/Broadcasting using Directional Antennas (PMDA/PBDA)* problems and discuss their complexities. Based on a graph transformation, we propose a directed Steiner minimum tree based approximation algorithm for both problems whose approximation ratio is bounded by $O(\log^2 n)$, where n is the number of wireless nodes. We also propose two fast heuristics, the *Shortest Path Tree (SPT)* heuristic and the *Directed Minimum Spanning Tree (DMST)* heuristic, whose running time are $O(n^2 + Kn\log(Kn))$ and $O(K^2n^2)$ respectively, where K is the number of sectors in one node. In addition, we present a simple greedy heuristic which is able to compute a broadcast tree without transforming the given communication graph. By applying a pruning procedure, we can construct a multicast tree accordingly. Power efficient broadcasting/multicasting with directional antennas has also been studied in [4], [13]. However, all of their algorithms are heuristics without any performance guarantee. To our best knowledge, this is the first paper proposing an approximation algorithm with a provably good performance ratio.

The rest of this paper is organized as follows. We describe the system model and formally define the PMDA/PBDA problems in Section II. Then, we present our approximation and heuristic algorithms in Section III. Simulation results are presented in Section IV. We conclude the paper in Section V.

II. Problem Formulation

A. System Model

We consider a wireless ad hoc network with directional antennas in which there are n wireless nodes $v_1, v_2, \ldots, v_n$ deployed in the Euclidean plane. It is assumed that the

0-7803-8939-5/05/$20.00 (C) 2005 IEEE

locations of all nodes are known. We use $d(v_i, v_j)$ to denote the Euclidean distance between nodes v_i and v_j. We assume that all wireless nodes are static or move very slowly.

We use a similar antenna model as in [8], [9]. We assume that each wireless node in the network is equipped with a single radio transceiver which is able to receive signals from all directions. Every transceiver has K directional antenna elements, each of which spans an angle of $2\pi/K$ radians and maintains fixed non-overlapping beam direction and transmission range. The radio coverage area of an antenna element is a sector. Since each antenna element corresponds to a unique sector, we will use the term sector and antenna element interchangeably in the following. We will label them from 1 to K for each node in the counter clockwise direction. A wireless node can switch any antenna element on or off at a time. Transmission using some sector means the corresponding antenna element is turned on for transmission. Transmission ranges of different antenna elements of a wireless node are assumed to be the same, all equal to a fixed value which will not change regardless of how many antenna elements are used. However, different nodes may have different transmission ranges. If all antenna elements in a wireless node are turned on, their characteristics are exactly the same as a single ommi-directional antenna.

We follow a well known power dissipation model used in [13]. The minimum power needed by node v to transmit for a distance r ($r \leq R_{max}$, where R_{max} is the maximum transmission range of node v) using a directional antenna with beam width θ is $C \times \frac{\theta}{2\pi} \times r^{\alpha}$, where C is a positive constant α is also a constant, typically between 2 and 4. According to our antenna model, a wireless node uses the same power in all active sectors (the sectors whose corresponding antennas are turned on) when transmits simultaneously in multiple sectors. Therefore for a given node v with a fixed transmission range R_v, the power required for using k sectors is given as

$$P(v, k) = C \times \frac{k}{K} \times R_v^{\alpha} \ . \qquad (1)$$

We will use a directed graph $G(V, E)$ to model the wireless ad hoc network under consideration. The vertices of G correspond to the n nodes in the ad hoc network. There is a directed edge from vertex u to v in G if and only if $d(u, v) \leq R_u$. We call this graph the *communication graph*.

B. Problem Formulation

Let us be given a communication graph $G(V, E)$ and a multicast request ρ consisting of a source node $v_s \in V$ and a set of destination nodes $\{v_{d_1}, v_{d_2}, \ldots, v_{d_{M-1}}\} \subseteq V$.

Definition 1: A multicast tree for ρ is a directed tree T in G such that there is a directed path p_i in T from v_s to v_{d_i} for $i = 1, 2, \ldots, M-1$.

When a multicast request arrives, we need to construct a multicast tree along which packets will be transmitted. If multiple children of a node v in the multicast tree T lies in a single sector j of node v, then only one transmission at node v along sector j needs to be made because of WMA and the total power cost is $P(v, 1)$ according to Equation 1. For a given node v on a multicast tree T, the number of sectors

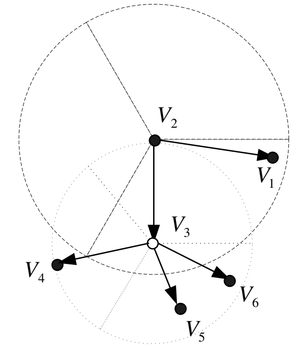


Fig. 1. A sample multicast tree T.

required to be on for maintaining T will be denoted by n_v^T, which can be easily computed since we assume that locations of wireless nodes are known *a priori*. Obviously, $n_v^T = 0$ if v is a leaf node.

Definition 2: The **total power of a multicast tree** T, denoted by $P(T)$, is the sum of the transmit power required at all nodes of the multicast tree: $P(T) = \sum_{v \in T} P(v, n_v^T)$.

Fig. 1 shows a multicast tree T with source node v_2 and destination node set $\{v_1, v_4, v_5, v_6\}$. In order to transmit a packet from the source node to all destination nodes along T, the source node v_2 needs to turn on sector 3 (which covers nodes v_1 and v_3) and node v_3 needs to turn on sectors 2 and 3. In this case, $P(T) = P(v_2, 1) + P(v_3, 2)$.

Definition 3: A multicast tree T is said to be a **minimum power multicast tree** if $P(T)$ is minimum among all multicast trees for ρ. The **Minimum Power Multicasting using Directional Antennas (PMDA)** problem seeks a minimum power multicast tree for ρ.

Note that broadcasting is a special case of multicasting where the set of destinations consists of $V \setminus \{v_s\}$. Therefore, we can define **broadcast tree**, **minimum power broadcast tree** and **Minimum Power Broadcasting using Directional antennas (PBDA)** problem similarly. We will solve the PBDA problem separately since broadcasting is very important by itself and it is sometimes easier to present algorithms for the PMDA problem after the presentation of algorithms for the PBDA problem.

By restricting the number of sectors K in every wireless node to be 1, the PBDA problem becomes the Minimum Energy Broadcast (MEB) problem which has been shown to be NP-hard in [6]. Hence, we conclude that the PBDA problem is also NP-hard. Since the PBDA problem is a special case of the PMDA problem, the PMDA problem is NP-hard as well. In the following section, we will present approximation and heuristic algorithms to solve both PBDA and PMDA problems.

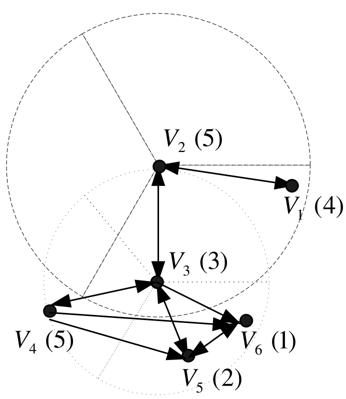
III. Proposed Algorithms

In this section, we present directed Steiner tree based approximation algorithms and several heuristics. Our approximation algorithm is based on a graph transformation which is also used by two of our heuristics.

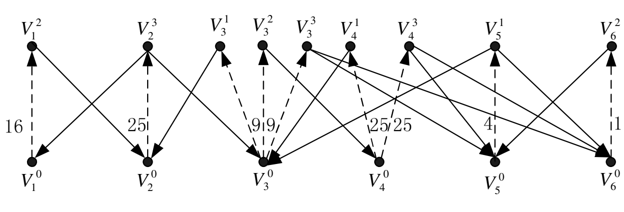
A. Directed Steiner Tree based Algorithms

Previously proposed tree construction algorithms [2], [3], [5] can not be employed directly to solve our problems since their optimization goal is to find a spanning tree or a Steiner tree with minimum total edge weight while the communication graph G is not an edge weighted graph. However, we can transform the original graph $G(V, E)$ to an auxiliary graph $AG(V_A, E_A)$ with weights on its edges. This transformation is critical to our approximation and heuristic algorithms. Therefore we describe this transformation first.

Let v_i be a node in V with transmission range R_{v_i}. We say v_i covers v_j in sector k if v_j is located in sector k of v_i. We use Fig. 2(a) to illustrate this. The number beside every node indicates its transmission range, e.g. Node v_1 has a transmission range of 4. In sector 2, v_1 covers node v_2. However, v_1 does not cover any other nodes in either sector 1 or sector 3. Similarly, v_2 covers both v_3 and v_1 in sector 3. In sector 1, v_3 covers v_2. In sector 2, v_3 covers v_4. In sector 3, v_3 covers v_5 and v_6. In sector 1, v_4 covers v_3. In sector 3, v_4 covers v_5 and v_6. v_5 covers v_3 and v_6 in sector 1, and v_6 covers v_5 in sector 2.



(a) The communication graph $G(V, E)$



(b) The auxiliary graph $AG(V_A, E_A)$

Fig. 2. A graph transformation from G to AG.

For each node $v_i \in V$, the graph AG contains a vertex $v_i^0 \in V_A$ and a subset of $\{v_i^1, v_i^2, \ldots, v_i^K\}$: $v_i^k \in V_A$ if and only if wireless node v_i covers at least one other wireless node in sector k. We call a node v_i^k $(0 < k \leq K)$ a virtual node. The directed edges of AG and their corresponding weights are defined as follows. Suppose that wireless node v_i covers wireless node v_j in sector k. Then there is a directed edge $(v_i^0, v_i^k) \in E_A$ whose weight, denoted by $c(v_i^0, v_i^k)$, is assigned to be $P(v_i, 1)$. There is also a directed edge $(v_i^k, v_j^0) \in E_A$ with weight 0. We call the edge (v_i^0, v_i^k) an *intra-node edge* and the edge (v_i^k, v_j^0) an *inter-node edge*. Fig. 2(b) illustrates the graph AG corresponding to G in Fig. 2(a). For simplicity, the zero-weight edges are not labelled and weights of other edges are illustrated by $R_{v_i}^2$ in this example, as taken out the factor of $\frac{C}{K}$ does not change the problem that we are studying. Take $v_4 \in V$ as an example. We have the vertex $v_4^0 \in V_A$ and the vertices v_4^1 and v_4^3 in V_A. The other vertices in V_A are obtained similarly. Since the transmission range of v_4 is 5, the weight of edge (v_4^0, v_4^1) is 25. So is the weight of edge (v_4^0, v_4^3). Since node v_4 covers node v_3 in sector 1, there is a zero-weight edge (v_4^1, v_3^0) in E_A. Since node v_4 covers nodes v_5 and v_6 in sector 3, there are zero-weight edges (v_4^3, v_5^0) and (v_4^3, v_6^0) in E_A. The other edges are constructed similarly.

Suppose the communication graph G has n vertices and m edges. Then the auxiliary graph has at most $((K+1)n)$ vertices and at most $(Kn + m)$ edges. The graph transformation can be accomplished in $O(Kn + m)$ time.

Both PBDA problem and the PMDA problem in the communication graph G become the *Directed Steiner Minimum Tree (DSMT)* problem in the auxiliary graph AG. Our approximation algorithm based on the DSMT algorithm of [2] is presented in the following. For a node v on the tree, we use $parent_v$ to denote its parent and CH_v to denote its children list. T, T_A in Algorithm 1 are node sets.

Algorithm 1 DSMT based Algorithm for PMDA

step_1 Construct the auxiliary graph AG of G.

step_2 Run a DSMT algorithm of [2] on AG to find a directed Steiner tree T_A rooted at v_s^0 and spanning all nodes in $\{v_{d_1}^0, v_{d_2}^0, \ldots, v_{d_{M-1}}^0\}$.

step_3 **forall** $v_i^j \in T_A (0 < j \leq K)$
  forall $v_k^0 \in CH_{v_i^j}$
    $T = T \cup \{v_k\}$
    $parent_{v_k} = v_i$
    $CH_{v_i} = CH_{v_i} \cup \{v_k\}$
  endforall
endforall

Theorem 1: Algorithm 1 correctly construct a multicast tree connecting the given source node to all destination nodes and its total power is within a factor of $O(\log^2 n)$ of the total power of the minimum power multicast tree.

PROOF. It follows from the construction of the auxiliary graph that computing a minimum power multicast tree for ρ in G is equivalent to computing a minimum cost multicast tree with source node v_s^0 and destination nodes $\{v_{d_1}^0, v_{d_2}^0, \ldots, v_{d_{M-1}}^0\}$ in AG. **Step_2** computes an approximation to the minimum cost multicast tree in AG. **Step_3** constructs the equivalent multicast tree in G with equal cost. Since the DSMT algorithm of [2] achieves a worst-case approximation ratio of $O(\log^2 M)$, where M is the number of nodes in the multicast group and $M \leq n$. Algorithm 1 is an approximation algorithm for the PMDA problem with approximation ratio bounded by $O(\log^2 n)$. □

B. Heuristics based on Shortest Path Tree and Directed Minimum Spanning Tree

Since the DSMT algorithms of [2] has a worst-case time complexity of $O(((Kn)^{3\log n})$ on AG, our approximation algorithm has a high time complexity. In this subsection, we describe two fast heuristics. One of them is based on Shortest Path Tree (SPT) and the other is based on Directed Minimum Spanning Tree (DMST). Both of them make use of the auxiliary graph AG.

In the SPT heuristic, we first construct the auxiliary graph AG from G, as in **Step_1** of our approximation algorithm. Instead of computing an approximation to the Steiner minimum tree, we use Dijkstra's algorithm to find a shortest path tree. If the connection request is a multicast request rather than a broadcast request, we prune the tree by repeatedly deleting leaf nodes that do not correspond to a destination node. Since the construction of AG takes $O(Kn + m)$ time, Dijkstra's algorithm takes $O(Kn + m + Kn\log(Kn))$ time and m is at most $n(n-1)$, the SPT heuristic has a worst-cast time complexity of $O(n^2 + Kn\log(Kn))$. The number of sectors in a node, K, is usually a small integer ($K \le 10$ [8], [9]) and m is normally much less than $n(n-1)$. Therefore the SPT heuristic runs in $O(m + n\log n)$ time in practice.

In the DMST heuristic, we first construct the auxiliary graph AG. Then we use the algorithm proposed in [3], [5] to compute a directed minimum spanning tree in the auxiliary graph AG. If the connection request is a multicast request rather than a broadcast request, we prune the tree by repeatedly deleting leaf nodes that do not correspond to a destination node. This heuristic has a worst-cast time complexity of $O(K^2n^2)$, which is dominated by the computation of a DMST ([11]).

C. Greedy Heuristic

In this section, we present a greedy heuristic which is able to solve PBDA problem without transforming the graph G. By appending a pruning procedure, we can use it to solve PMDA problem as well.

The basic idea of this method is an extension of the greedy scheme for the *Set Cover* problem and the greedy heuristic proposed in [6]. However, neither of the above methods can be applied directly to solve our problems.

For a network of nodes with multiple sectors, we consider the effective *coverage of a sector* in each iteration, not the coverage of a node. We use b_i^k to denote sector k of node v_i and use B to denote the set of sectors not yet used but whose owner node, $o(b)$, is already on the tree. We use T to denote the set of nodes which have been added on the broadcast tree, U_b to denote the set of nodes within sector b but not yet on the tree, and B_i to denote the set of sectors whose owner node is v_i. For each sector $b \in B$, the effective coverage is computed using Equation 2.

$$e(b) = |U_b|/P(o(b), 1) \tag{2}$$

In the above equation, the denominator is the incremental power needed for using sector b. In each iteration, a sector from B with the maximum effective coverage will be selected and those nodes covered by it will be added onto the broadcast tree. We formally present our greedy algorithm for PBDA problem as Algorithm 2.

Algorithm 2 Greedy Heuristic for PBDA

```
step_1 T = {v_s}; B = B_s;
step_2 while (|T| < n)
          select b ∈ B with maximum e(b);
          T = T ∪ U_b;
          B = B \ {b};
          forall v_i ∈ U_b
            B = B ∪ B_i;
            parent_{v_i} = o(b);
            CH_{o(b)} = CH_{o(b)} ∪ {v_i};
          endforall
        endwhile
```

In each iteration, the set of selected nodes U_b will choose $o(b)$ as their parent node on the tree T, where b is the selected sector. In this way, the broadcast tree T can be constructed. All selected sectors will be turned on to enable transmission along the tree.

Theorem 2: Algorithm 2 correctly computes a broadcast tree with the given root v_s on G. The worst-case running time of Algorithm 2 is $O(Kn^2)$.

PROOF. **Step_1** makes sure the tree has root v_s and **Step_2** keeps growing a tree until it spans all nodes in G. In the while-loop, there are at most n iterations. For each iteration, finding the sector with maximum effective coverage takes $O(Kn)$ time since we have at most (Kn) sectors. Updating the set B takes $O(n)$ time as well. Hence, the worst-case running time of Algorithm 2 is $O(Kn^2)$. □

In order to solve the PMDA problem, we can repeatedly prune the non-destination leaf nodes from the computed broadcast tree to obtain a corresponding multicast tree.

IV. PERFORMANCE EVALUATION

In this section, we evaluate the performance of our algorithms via simulations. We consider static wireless networks with nodes randomly located in a 750×750 region. The constant C and α in Equation 1 are set to 0.0001 and 2 respectively in all simulations. Every node has a fixed transmission range given by a uniformly distributed random integer number. We fix the difference between the maximum range and the minimum range to 100 and the mean transmission range (R) to 250. We compare the performance of Greedy heuristic, SPT heuristic, DMST heuristic and we will use the *Average Total Power (ATP)* of computed trees as the performance metric.

The following 3 system parameters will influence the performance: network size, i.e., the number of nodes in a network (n), multicast group size, i.e., the number of nodes in a multicast request (M), and the number of sectors in each node (K). In each simulation scenario, we fix 2 parameters and vary the other one to observe its effects on the performance. Each entry in the tables reported here is the average over 100 runs. In each run, we randomly deploy n nodes in the given region and randomly generate a source node v_s and $(M-1)$ destination nodes as the multicast request. For broadcasting cases, only a source node is randomly generated and M is equal to n.

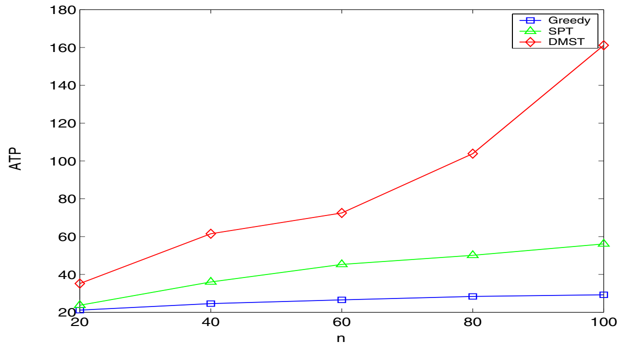


Fig. 3. $M = n, K = 3$

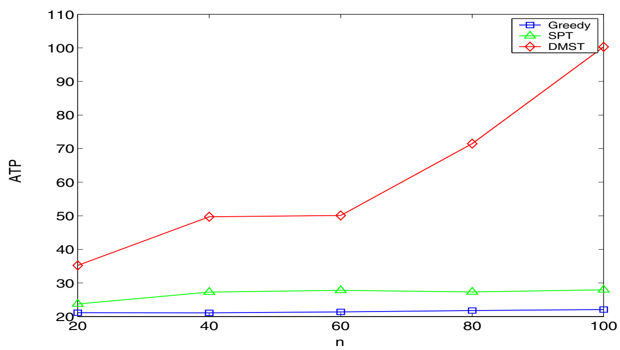


Fig. 4. $M = 20, K = 3$

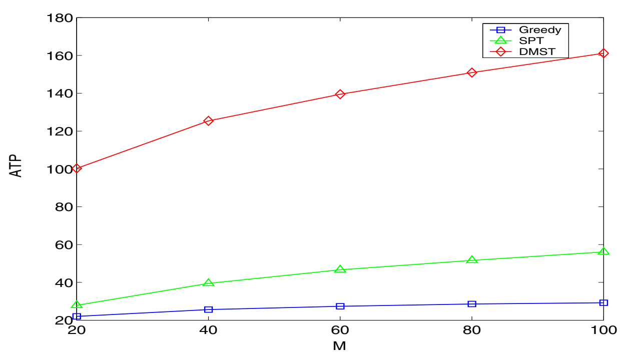


Fig. 5. $n = 100, K = 3$

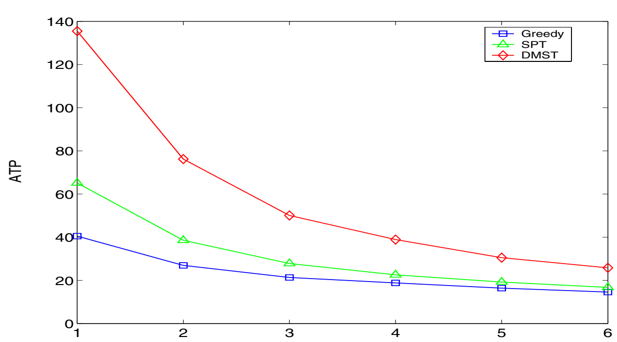


Fig. 6. $n = 60, M = 20$

We make the following observations from the simulation results.

- The Greedy heuristic performs the best at all different cases. The SPT heuristic always outperforms DMST heuristic. The DMST heuristic performs very poorly, especially in those cases where the network size or the multicast group size is relatively large. We find out that directed spanning trees constructed by DMST heuristic in the auxiliary graph usually have much more layers and less leaf virtual nodes than those computed by the SPT heuristic. Then when we transform those back to the corresponding ones on the original communication graph, many non-zero cost intra-node edges can not be removed, i.e, more sectors needs to be turned on, because they are required to connect some downstream nodes. It will dramatically increase total power of the tree.
- From Fig. 6, we note that using directional antennas can substantially reduce power consumption compared with using omni-directional antennas $(K = 1)$. Moreover, the more sectors we have in one wireless node, the less power we need to spend for routing no matter which algorithms are used.

V. Conclusions

In this paper, we have studied power efficient broadcast and multicast routing in wireless ad hoc networks with directional antennas. We formally formulate the Minimum Power Multicasting/Broadcasting (PMDA/PBDA) problems. We have presented an approximation algorithm with a worst-case approximation ratio of $O(\log^2 n)$ and effective heuristic algorithms for the problems. Simulation results show that our Greedy heuristic performs the best at all cases and that we can save power by having more sectors at each node.

References

[1] M. Cagalj, J. P. Hubaux, C. Enz, Minimum-energy broadcast in all-wireless networks: NP-completeness and distribution issues, *Proceedings of ACM Mobicom'2002*, pp. 172–182.

[2] M. Charikart, C. Chekurit, T. Cheungs, *et al.*, Approximation algorithms for directed steiner problems, *Proceedings of ACM-SIAM SODA'1998*, pp. 192–200.

[3] Y. J. Chu, T. H. Liu, On the shortest arborescence of a directed graph, *Science Sinica*, Vol. 14(1965), pp. 1396–1400.

[4] Q. Dai, J. Wu, Construction of power efficient routing tree for ad hoc wireless networks using directional antenna *Proceedings of ICD-CSW'2004*, pp. 718–722.

[5] J. Edmonds, Optimum branchings, *Journal of Research of the National Bureau of Standards*, Vol. 71B(1967), pp. 233–240.

[6] D. Li, X. Jia, H. Liu, Energy efficient broadcast routing in static ad hoc wireless networks, *IEEE Transactions on Mobile Computing*, Vol. 3(2004), pp. 144-151.

[7] W. Liang, Constructing minimum-energy broadcast trees in wireless ad hoc networks, *Proceedings of IEEE MobiHoc'2002*, pp. 112–122.

[8] C. Jaikaeo, C-C Shen, Multicast communication in ad hoc networks with directional antennas, *Proceedings of ICCCN'2003* pp. 385–390.

[9] A. Nasipuri, S. Ye, J. You, R.E. Hiromoto, A MAC protocol for mobile ad hoc networks using directional antennas, *Proceedings of IEEE WCNC'2000*, pp. 1214–1219.

[10] J. Tang, G. Xue, W. Zhang, Energy efficient survivable broadcasting and multicasting in wireless ad hoc networks, *Proceedings of IEEE Milcom'2004*.

[11] R. E. Tarjan, Finding optimum branchings, *Networks*, Vol. 7(1977), pp. 25–35.

[12] J. E. Wieselthier, G. D. Nguyen and A. Ephremides, On construction of energy-efficient broadcast and multicast trees in wireless networks, *Proceedings of IEEE Infocom'2000*, pp. 585–594.

[13] J. E. Wieselthier, G. D. Nguyen, A. Ephremides, Energy-aware wireless networking with directional antennas: the case of session-based broadcasting and multicasting, *IEEE Transactions on Mobile Computing*, Vol. 1(2002), pp. 176–191.