Acceleration-invariance of hyperbolic frequency modulated pulse compression

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Abstract

It is well known that hyperbolic frequency modulated waveform is Doppler-invariant. However it suffers severe distortion when the velocity of the target is not constant. In this paper we demonstrate that the acceleration of the target results in a frequency shift which is the source of the signal distortion under the assumption that the acceleration is constant and along the direction of the velocity. Therefore the frequency-shifted version of the matched filter can be applied to eliminate the mismatch between the reflected signal and the matched filter caused by the acceleration of the target. An example of rectangular envelope hyperbolic frequency modulated pulse is presented to illustrate the effect of the acceleration on signal distortion and a bank of filters with selected value of frequency shift are applied to the distorted waveform to improve the compression of the pulse. The simulation also illustrates that the noise performance of the pulse compression is not affected by the frequency shift scheme.

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1. Introduction

In order to increase range capacity with a limited peak power in the transmitter, pulse compression technique is applied in many pulsed radar systems. Narrow pulse provides a number of advantages over wide pulse, including better range resolution and range accuracy and better performance under cluttered environment. However, the sensitivity of radar system, thus the maximum detection range, depends on the transmitted power, which is proportional to the length of the pulse, therefore for a narrow pulse the higher level of peak power is required to compensate. The peak power, however, is always limited for the transmitter due to the reliability and safety issues associated with the high-voltage system. Pulse compression is a method which combines the high energy of a long pulse width with the high resolution of a short pulse width. In the original system design [1], the transmitted signal is modulated by a linear chirp, and then the received signal is passed through a matched filter which is matched to the transmitted waveform. The output of the matched filter will be an extremely narrow pulse with a large peak value, thus the transmitted pulse is compressed in time domain [2]. The disadvantage of the linear frequency modulation is when the Doppler effect exists, i.e., when the relative velocity between the radar and the target is relatively large compared with the velocity of signal propagation and is not negligible, the received signal is Doppler distorted and does not match with the matched filter.
This mismatch results in the performance degradation of linear FM pulse compression system discussed by Ramp et al. [3], Cook et al. [4], and Rubin et al. [5]. In [6], Kelly and Wishner parameterize the returned signal and modify the matched filters and ambiguity functions to compensate the Doppler effect. Their work is extended by Rihaczer [7] to apply the complex notation and to take into account the Doppler-caused amplitude changes in the returned signal. In [8], Kramer uses different reference signals (lines with different slope in \( f-t \) plane) for matched filter to eliminate the mismatch. In the other hand, this problem has also been addressed by utilizing the Doppler-invariant waveforms. In [9,10], Kroszczynski shows the hyperbolic frequency modulated waveform is insensitive to target velocity, and it is interesting to know that some kinds of bats use the similar signal for echolocation purpose [11,12]. The design and implementation of matched filter for hyperbolic frequency modulated waveform are discussed by Rhodes [13] and Altes and Skinner [14]. However, if target velocity is not a constant, similar to the Doppler effect on linear frequency modulated waveform, the received waveform will mismatch with the matched filter and the output is again degraded. In this paper we assume the target acceleration is a constant and demonstrate that the constant acceleration of the moving target leads to both phase and frequency shifts in hyperbolic frequency modulated waveform. The degradation of the output comes from the frequency shift, which depend on the acceleration of the target. If we shift the frequency response of the matched filter exact amount caused by the acceleration of the target and take it as the new matched filter, the mismatch between the reflected waveform and the matched filter will be eliminated.

In Section 2, we derive the distortion of the hyperbolic frequency modulated waveform caused by the constant acceleration of the moving target. Some numerical examples are presented in Section 3, followed by the conclusion in Section 4.

### 2. Effect of Constation acceleration on hyperbolic frequency modulated waveform

If a point target has a relative constant velocity with respect to the radar, the returned signal is Doppler-distorted. This Doppler-distorted signal can be viewed as a function of two variables: a time delay caused by the distance and a Doppler scale caused by the relative velocity between the radar and the target. Assuming both transmitted and returned signals have the same energy, then the returned signal can be express as [6]

\[
r(t) = \sqrt{s} x[s(t - \tau)],
\]

(1)

where \(x(t)\) is the transmitted waveform, \(\tau\) is the propagation time delay, and \(s\) is the Doppler factor:

\[
\tau = \frac{2D}{c},
\]

(2)

\[
s = \frac{c - v}{c + v} \approx 1 - \frac{2v}{c},
\]

(3)

where \(D\) is the distance between the source and the target, \(c\) is the velocity of the signal propagation and \(v\) is the relative velocity between the source and the target. Now let us assume the target has a constant acceleration along the direction of the velocity, the scaling factors is no longer a constant, but a linear function of time. Thus, Eq. (1) can be expanded to this constant acceleration case with the scaling factor changed to [15]

\[
s \approx 1 - \frac{2v}{c} - \frac{a}{c} t.
\]

(4)

The hyperbolic frequency modulated pulse can be written as

\[
x(t) = A(t) \exp \left[ j \frac{2\pi}{k} \log(1 + kf_0 t) \right],
\]

(5)

where \(k\) is a constant factor and \(f_0\) is the starting carrier frequency. The instantaneous frequency is the derivative of the phase term inside the cosine function:

\[
f(t) = \frac{d}{dt} \left[ j \frac{1}{k} \log(1 + kf_0 t) \right] = \frac{1}{f_0 + kt},
\]

(6)

which is a hyperbolic function of time, so it is called hyperbolic frequency modulation.
To simplify the derivation, Eq. (5) is rewritten as

\[ x(t - t_0) = A(t - t_0) \cos \left( \frac{2\pi}{k} \log(kf_0t) \right), \]  

(7)

where \( t_0 = 1/kf_0 \). Substituting Eq. (7) into Eq. (1), the reflected signal is

\[ r(t - t_0) = \sqrt{s} A[s(t - t_0 - \tau)] \exp \left[ j \frac{2\pi}{k} \log(kf_0s(t - \tau)) \right] \]

\[ = \sqrt{s} A[s(t - t_0)] \exp \left[ j \frac{2\pi}{k} \log(kf_0(t - \tau)) + j \frac{2\pi}{k} \log(s) \right] \]

\[ = \sqrt{s} A[s(t - t_0)] \exp \left[ j \frac{2\pi}{k} \log(kf_0(t - \tau)) \right] \exp \left[ j \frac{2\pi}{k} \log(s) \right]. \]  

(8)

Comparing with the carrier chirp signal, the dilation on the envelope \( A(t) \) can be negligible. Thus, Eq. (8) can be rewritten as

\[ r(t - t_0) = \sqrt{s} x(t - t_0 - \tau) \exp \left[ j \frac{2\pi}{k} \log(s) \right]. \]  

(9)

Substituting Eq. (4) into Eq. (9), we get

\[ r(t - t_0) = \sqrt{s} x(t - t_0 - \tau) \exp \left[ j \frac{2\pi}{k} \log \left( 1 - \frac{2v}{c} - \frac{a}{c} (t - \tau) \right) \right]. \]  

(10)

For small value of acceleration \( a \), the first term \( \sqrt{s} \) is approximately a constant. Also we have the assumption

\[ \log(1 + x) \approx x \quad \text{for} \quad x \ll 1. \]  

(11)

Substituting Eq. (11) into Eq. (10), we have

\[ r(t - t_0) = \sqrt{s} x(t - t_0 - \tau) \exp \left[ j \frac{2\pi}{k} \left( - \frac{2v}{c} - \frac{a}{c} t \right) \right] \]

\[ = \sqrt{s} x(t - t_0 - \tau) \exp \left[ 2\pi j \left( \frac{-a}{kc} (t - \tau) - \frac{2v}{kc} \right) \right] \]

\[ = \sqrt{s} x(t - t_0 - \tau) \exp \left[ 2\pi j (\Delta f(t - \tau) + \Delta \phi) \right], \]  

(12)

where \( \Delta f = -a/kc \) is the frequency shift and \( \Delta \phi = -2v/kc \) is the phase shift. From Eq. (12) we can see that the reflected signal can be approximated by a delayed, frequency-shifted and phase-shifted version of the transmitted signal. The phase shift will be ignored by the matched filter output. For the special case that the acceleration is zero, the frequency shift term is disappear thus the hyperbolic frequency modulated waveform is Doppler-invariant. For non-zero acceleration value, we can construct a matched filter by shifting the frequency response of the transmitted signal to match the reflected waveform. Normally the acceleration value is not known. In this case we may also select a number of \( \Delta f's \) and use a set of filter bank to cover the exact frequency response of the reflected signal and use it to detect the actual acceleration of the target.

3. Numerical example

In this section we use a rectangular envelop modulated pulse to show the effect of the acceleration of the point target on the hyperbolic frequency modulated waveform, and how the output of the matched is improved by shifting the frequency response of the matched filter. The duration of the transmitted pulse is 100 µs, the carrier frequency for hyperbolic chirp is from 2 to 20 MHz, the propagation time delay \( \tau = 20 \) µs. The transmitted pulse modulated by the hyperbolic chirp is shown in Fig. 1. Figure 2 is the frequency spectrum of the transmitted pulse. It can be shown that unlike spectrum of the widely used linear chirp which has a constant value over the frequency band, the frequency spectrum of hyperbolic chirp is a decreasing function with a long tail. Figure 3 shows the output of the matched filter when both the velocity and the acceleration of the target are zero, with the maximum peak value normalized to 1.
Fig. 1. Hyperbolic frequency modulated rectangular pulse.

Fig. 2. Spectrum of the hyperbolic frequency modulated pulse.

Fig. 3. Compressed pulse with $v = 0$ and $a = 0$.

Fig. 4. Compressed pulse with $v = 0.001c$ and $a = 0$.

Fig. 5. Compressed pulse with $v = 0.001c$ and $a = 0.001v$.

Fig. 6. Compressed pulse with $v = 0.001c$ and $a = 0.01v$. 
The outputs of the matched filters for different Doppler factors are all normalized to the output without the Doppler effect to keep the same scale. The compressed pulses given the velocity of the target is 0.001c and the acceleration is 0, 0.001v, and 0.01v are plotted in Figs. 4, 5, and 6, respectively. Here c is the velocity of the signal propagation and v is the initial velocity of the target. From these three figures we can see that the transmitted waveform is compressed very well when the velocity of the target is constant. The output of the matched filter degrades dramatically even there is only a small value of acceleration exist, and becomes worse when the acceleration increases. Figures 7, 8, and 9 show the output of the matched filter when the acceleration is 0.01v and \( \Delta f \) is equal to 0.01, 0.02, and 0.04 MHz, respectively. Here \( \Delta f \) is the frequency shift value of the matched filter from the frequency response of the transmitted waveform. Figures 7 and 8 indicate that when we shift the frequency response of the matched filter, the performance of the pulse compression becomes better and better. At a certain optimum \( \Delta f \) (in this example the value 0.022 MHz) the system achieves the best performance. This optimum frequency shift is roughly equal to \( -a/kc \). Figure 9 shows that if we continue increase \( \Delta f \), the result starts getting worse. Therefore we can use a bank of matched filters with slightly changed \( \Delta f \) values to estimate the acceleration of the moving target and achieve the best compression ratio.

Next we want to test the influence of this shift in frequency domain to the noise performance of the hyperbolic frequency modulation technique. We assume the reflected signal from a accelerating target is affected by a certain
Fig. 10. Noise performance of HFM pulse compression (SNR = 0 dB). (a) With and (b) without acceleration of target and frequency shift of matched filter.

Fig. 11. Noise performance of HFM pulse compression (SNR = −10 dB). (a) With and (b) without acceleration of target and frequency shift of matched filter.

amount of noise and is passed through the frequency shift version of the matched filter. Figures 10, 11, and 12 show the simulation results when the additive white Gaussian noise (AWGN) with 0, −10, and −20 dB, are applied respectively. For comparison, we also plot the results for the reflection signal without acceleration and the matched filter without frequency shift under the same SNR in each of the figures. The following three figures clearly indicate that the frequency shift scheme does not scarify the noise performance of the hyperbolic frequency modulated pulse compression. Both the matched filters with frequency shift and without frequency shift perform equally under three levels of SNR. They all perform very well when SNR is 0 dB. There are some undesired small side peaks for both of them when SNR is −10 dB, and they all start to break down when the SNR is around −20 dB. From the results we can also conclude that the noise performance of hyperbolic frequency modulated pulse compression is very impressive. The SNR bound is around −20 dB, that means the small signal can be picked up from the reflected waveform even the big noise is added during the transmission.
4. Conclusion

Hyperbolic frequency modulated waveform is Doppler-invariant but suffers from the distortion caused by the acceleration of the moving target. Numerical example indicates even a small value of acceleration will degrade the performance of pulse compression significantly. A filter bank method is presented by utilizing the fact that a constant acceleration leads to a time delay as well as phase and frequency shift of the hyperbolic frequency modulated waveform. From the numerical example, the output of the matched filter reaches the largest improvement when at the point of optimum frequency shift, which is a function of the acceleration of the target and the chirp rate. The simulation also indicates that the hyperbolic frequency modulated pulse compression has a very good noise performance and which is not affected by the frequency shift of matched filter. The better performance of the filter bank method under acceleration distortion makes it very useful in detecting moving object with a constant acceleration along the direction of the velocity.

References


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