Lab3 solving differential equations

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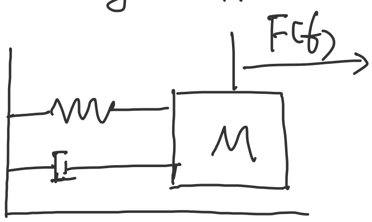
Solving Differential Equation



Governing Eqn.

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

$$\Downarrow$$

$$2\zeta\omega_n = \frac{c}{m}$$
$$\omega_n^2 = \frac{k}{m}$$

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{1}{m}F(t) \quad ①$$

→ Sinusoidal Forcing $F(t) = F_0 \sin(\omega t)$

Suppose $x = X\sin(\omega t - \delta)$ ← The expression can be expanded

$x = X\cos(\delta)\sin(\omega t) - X\sin\delta\cos(\omega t)$ ← Since X and δ will be constants, Let $A = X\cos(\delta)$ and $B = X\sin(\delta)$ and substitute back into Eqn.

$x = A\sin(\omega t) - B\cos(\omega t)$ ← The expression can be differentiated twice

$$\begin{cases} \dot{x} = \omega A\cos(\omega t) + \omega B\sin(\omega t) \\ \ddot{x} = -\omega^2 A\sin(\omega t) + \omega^2 B\cos(\omega t) \end{cases}$$

← Now that $x, \dot{x}, \ddot{x}$ have been found, they can now be input back into Eqn.①

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{1}{m}F(t)$$

$$\frac{F_0}{m}\sin(\omega t) = -\omega^2 A\sin(\omega t) + \omega^2 B\cos(\omega t) + 2\zeta\omega_n\omega A\cos(\omega t) + 2\zeta\omega_n\omega B\sin(\omega t) + A(\omega_n)^2\sin(\omega t) - B(\omega_n)^2\cos(\omega t)$$

↑ Above, notice that each of the terms has either a $\cos(\omega t)$ or $\sin(\omega t)$ term that is multiplied by some other factors. Since the left hand side of the equation contains a $\sin(\omega t)$ term, but not a $\cos(\omega t)$ term, it can be stated that →

$$\cancel{\frac{F_0}{m}} \sin(\omega t) + 0\cos(\omega t) = \frac{F_0}{m}\sin(\omega t)$$

$$\begin{cases} \frac{F_0}{m} = -\omega^2 A + 2\zeta\omega_n\omega B + A(\omega_n)^2 \\ 0 = \omega^2 B + 2\zeta\omega_n\omega A - B(\omega_n)^2 \end{cases} \leftarrow$$

Since ζ and ω_n will be either known or calculated, A and B can be solved for in terms of the other variables.

$$A = \frac{F((\omega_n)^2 - \omega^2)}{m(((\omega_n)^2 - \omega^2) + (2\zeta\omega_n\omega)^2)}$$

$$B = -\frac{2F\zeta\omega_n\omega}{m(((\omega_n)^2 - \omega^2) + (2\zeta\omega_n\omega)^2)}$$

$$X = \sqrt{A^2 + B^2} = \frac{F_0}{m}\frac{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}}{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}$$

$$X = \frac{F_0/m}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}}$$

$$\tan\zeta = \frac{B}{A} = \frac{2\zeta\omega_n\omega}{(\omega_n^2 - \omega^2)}$$