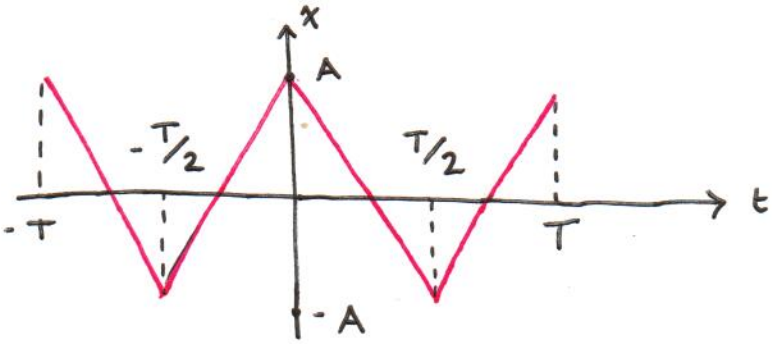
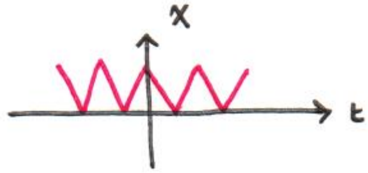
Triangle Wave Example



t = time
T = period
A = amplitude

- Symmetric about $t = 0 \Rightarrow$ this tells us that the function is <u>even</u>
  - $\therefore$ Can assume $b_n = 0$
- When $n = 0$, wave oscillates around $x = 0$
  - Gives us an average value of 0
    - Can assume $a_0 = 0$
- However, if: $x(t) =$  then $a_0 \neq 0$

- Remember,

$$a_n = \frac{2}{T}\int_T X_e(t)\cos(n\omega_0 t)\,dt$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} X_e(t)\cos(n\omega_0 t)\,dt = \frac{4}{T}\int_0^{T/2} X_e(t)\cos(n\omega_0 t)\,dt$$

$$X_e(t) = A - \frac{4A}{T}t \quad \text{(Based on geometry of wave)}$$

Plug $X_e(t)$ into a_n

$$a_n = \frac{4}{T}\int_0^{T/2}\left(A - \frac{4A}{T}t\right)\cos(n\omega_o t)\,dt$$

$$a_n = \frac{4A}{T}\left[\underbrace{\int_0^{T/2}\cos(n\omega_o t)\,dt}_{①} \underbrace{- \frac{4}{T}\int_0^{T/2} t\cos(n\omega_o t)\,dt}_{②}\right]$$

- Execute Integrals (plug in: $\omega_o \cdot T = 2\pi \Rightarrow \omega_o = \frac{2\pi}{T}$)

① $$\int_0^{T/2}\cos(n\omega_o t)\,dt = \frac{\sin(n\omega_o t)}{n\omega_o}\Bigg|_0^{T/2} = \frac{\sin(n\omega_o T/2)}{n\omega_o}$$

$$= \boxed{\frac{T\sin(n\pi)}{2\pi n}}$$

② $$-\frac{4}{T}\int_0^{T/2} t\cos(n\omega_o t)\,dt$$

* Must use integration by parts

$u = t \qquad dv = \cos(n\omega_o t)\,dt$

$du = dt \qquad v = \frac{\sin(n\omega_o t)}{n\omega_o}$

$$\int u\,dv = uv - \int v\,du$$

- Finish Integral

$$\left[w/\ \omega_o = \frac{2\pi}{T}\right] \longrightarrow \frac{T^2\sin(n\pi)}{4\pi n} + \frac{T^2\cos(n\pi)}{4\pi^2 n^2} - \frac{T^2}{4\pi^2 n^2}$$

- Use Half-Angle Identity: $\left(\sin^2(x) = \frac{1}{2}(1-\cos(2x))\right)$

$$= \frac{-T^2\left(2\sin\left(\frac{\pi n}{2}\right)^2 - \pi n\sin(\pi n)\right)}{4\pi^2 n^2}$$

- Multiply by $-\frac{4}{T}$ and combine

$\sin(n\pi) = 0$!

$$a_n = \frac{4A}{T}\left(\frac{T\cancelto{0}{\sin(n\pi)}}{2\pi n} + \frac{4}{T}\,\frac{T^2\left(2\sin^2\left(\frac{\pi n}{2}\right) - \cancelto{0}{\pi n \sin(n\pi)}\right)}{4\pi^2 n^2}\right)$$

$$a_n = \frac{8A\sin^2\left(\frac{n\pi}{2}\right)}{\pi^2 n^2}$$

- When $n = 0, 1, 2, \ldots$ $\sin^2\left(\frac{n\pi}{2}\right) = 0, 1, 0, 1, \ldots$

which equals : $\frac{1-(-1)^n}{2}$

$$\therefore \boxed{a_n = 4A\,\frac{1-(-1)^n}{\pi^2 n^2}}$$

$$\boxed{X_T(t) = \sum_{n=1,3,5\ldots}^{\infty} 4A\,\frac{1-(-1)^n}{\pi^2 n^2}\cos\left(n\,\frac{2\pi}{T}\,t\right)}$$

★ a_n only comes out to be $\neq 0$ when n is <u>ODD</u>