

Forces on Ground Vehicles

ES100

March 29, 1999

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Topics

- Reducing the airfoil lift data (CLFIT.M)
- Making decisions, part I
- Ground vehicle in steady motion
- Some vehicle geometry
- Lift and drag
- Vehicle wind tunnel testing
- Rolling resistance
- Making decisions, part II
- Coast-down tests; reducing data (COAST.M)

Making Decisions I

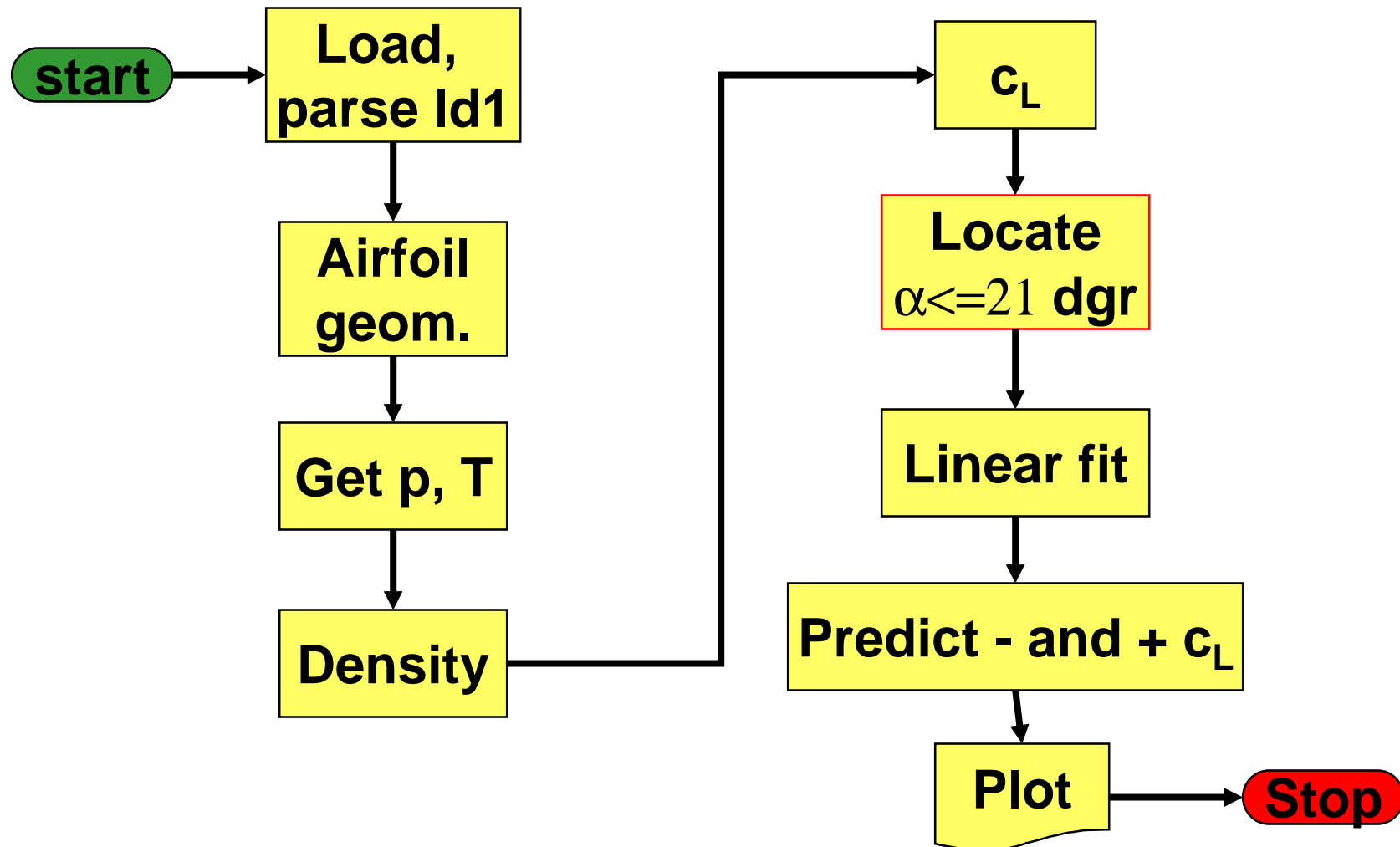
- Relational operators

Find where the **angle of attack** is **not greater than 21 degrees**. Then fit a straight line to the data.

$z = (\text{angle} \leq 21)$

- Array z has 1's where true, zero's where false

CLFIT.M I



CLFIT.M II

% Load and parse the data file

```
load a:\ld1.txt
```

```
V = ld1(:,4);      % vector of air speeds  
                    % (m/s)
```

```
L = ld1(:,2);      % vector of lifts (N)
```

```
D = ld1(:,3);      % vector of drags (N)
```

```
angle = ld1(:,1);  % vector of angles of  
                    % attack (N)
```

% Airfoil dimensions

```
c = 0.10152;      % m
```

```
b = 0.254;        % m
```

```
S = b*c;          % planform area (m2)
```

CLFIT.M III

% Get pressure and temperature

```
HmmHg = input( 'Enter air pressure  
              (mm Hg): ' );
```

```
p = mmhg2pa( HmmHg ); % air pressure  
                        % (N/m^2)
```

```
Tc = input( 'Enter temperature (C): ' );
```

```
T = tk( Tc ); % convert to  
             % absolute (K)
```

% Calculate air density

```
rho = airden ( p, T );
```

CLFIT.M IV

```
% Calculate the lift coefficients  
q = .5*rho*v.^2; % dynamic pressures  
                    % (N/m^2)  
cL = L./(S*q);  
  
% Locate angles <= 21 degrees  
  
        z = ( angle <= 21 );  
  
fprintf( '\nThe array z:\n %g\n', z)  
limit_index = sum(z); %count the 1's
```

CLFIT.M V

```
% Fit a straight line to the data subset
```

```
L = polyfit( angle(1:limit_index),...  
            cL(1:limit_index), 1 );
```

```
fprintf( '\nThe slope is %g ;  
the intercept is %g \n\n', L(1), L(2) )
```

```
% Predict cL at - and + angles
```

```
x = [-angle(limit_index)-6:1:...  
     angle(limit_index)+6];
```

```
y = polyval( L, x );
```


CLFIT.M VI

```
% Plot cL and cL fit vs. angle of attack  
plot ( angle, cL, 'square', x, y, '-' );  
legend ('lift coefficient',...  
        '10-point linear fit', 0 );  
xlabel ('angle of attack (degrees)');  
ylabel ('lift coefficient');  
title ('\Lift Coefficient and Linear Fit vs.  
        Angle of Attack')  
grid
```

Demonstrate CLFIT.M

- Note: the prediction

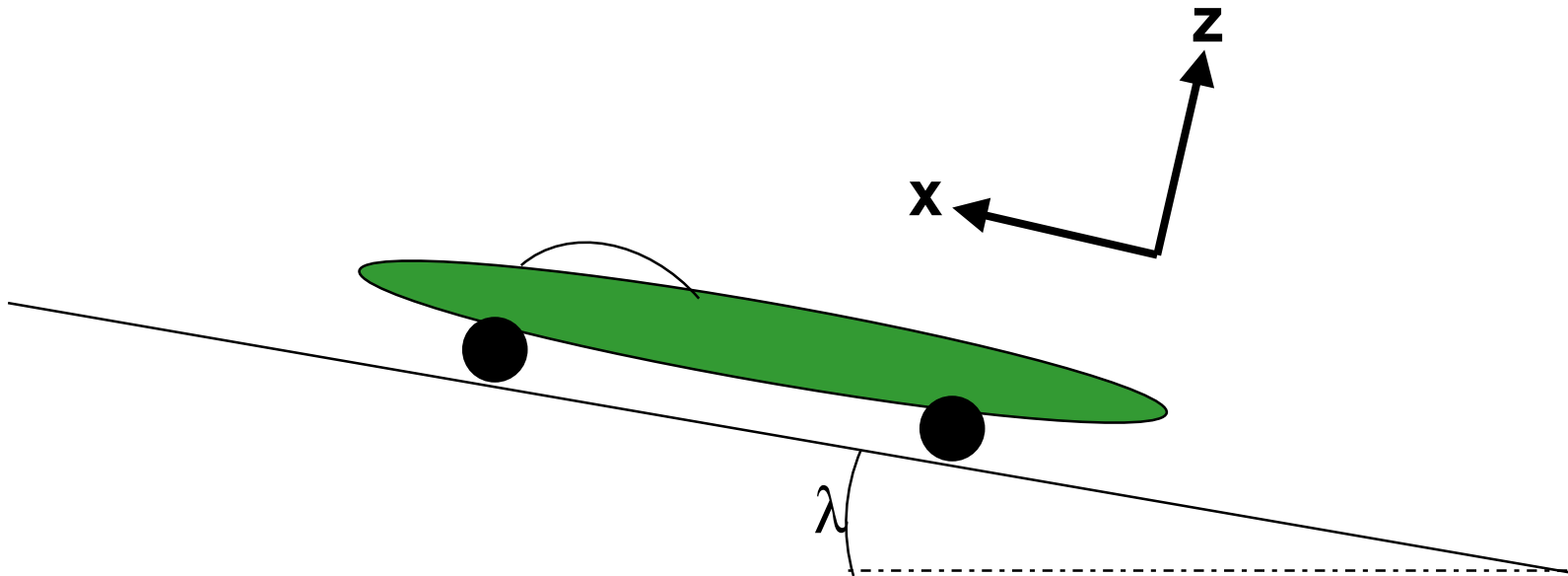
$$c_L(-\alpha) = -c_L(\alpha)$$

is true.

Ground Vehicle in Steady Motion

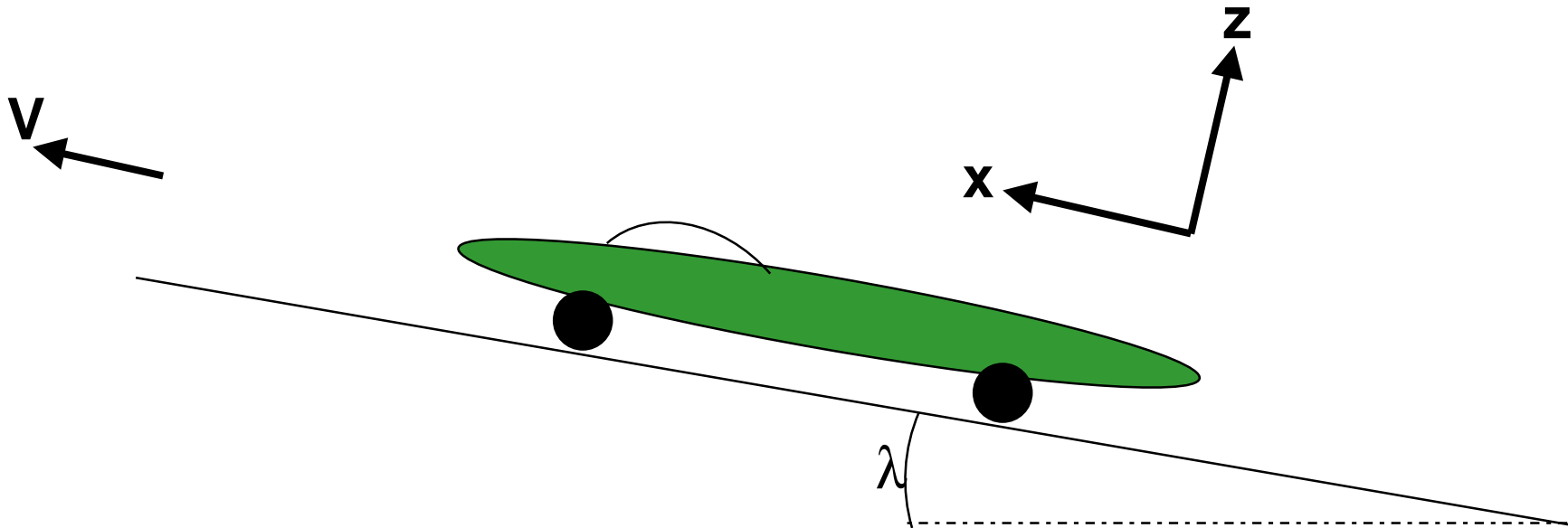
Ground Vehicle in Steady Motion

- Neglect transverse forces and moments



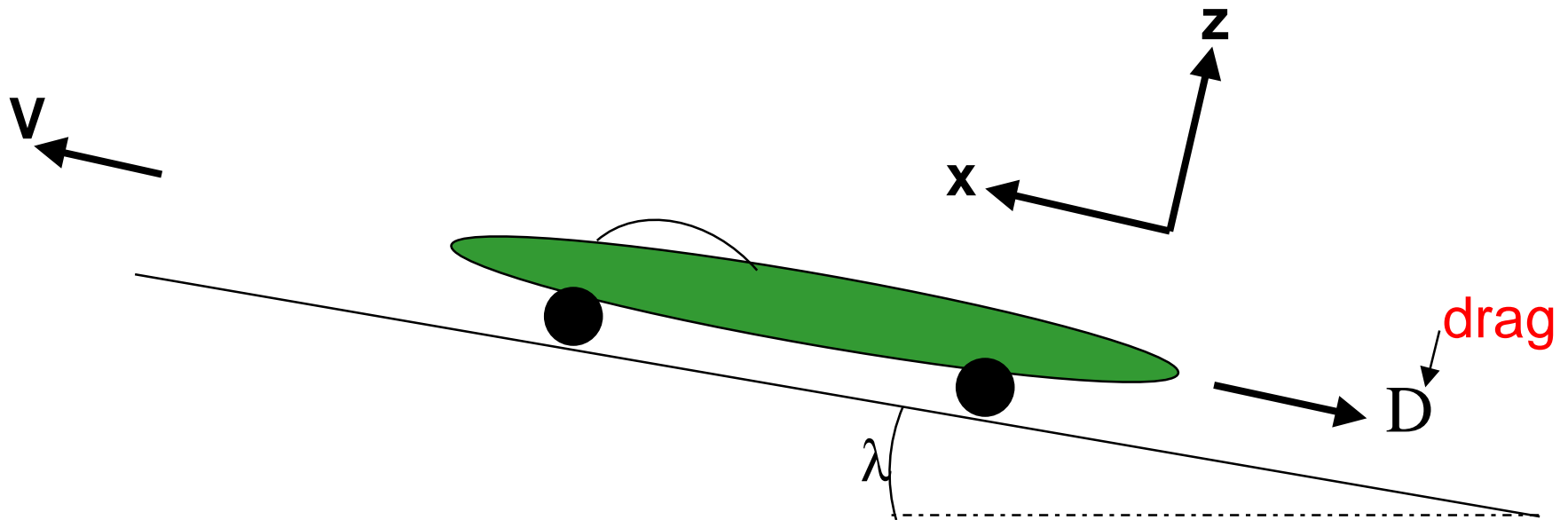
Ground Vehicle in Steady Motion

- Neglect transverse forces and moments



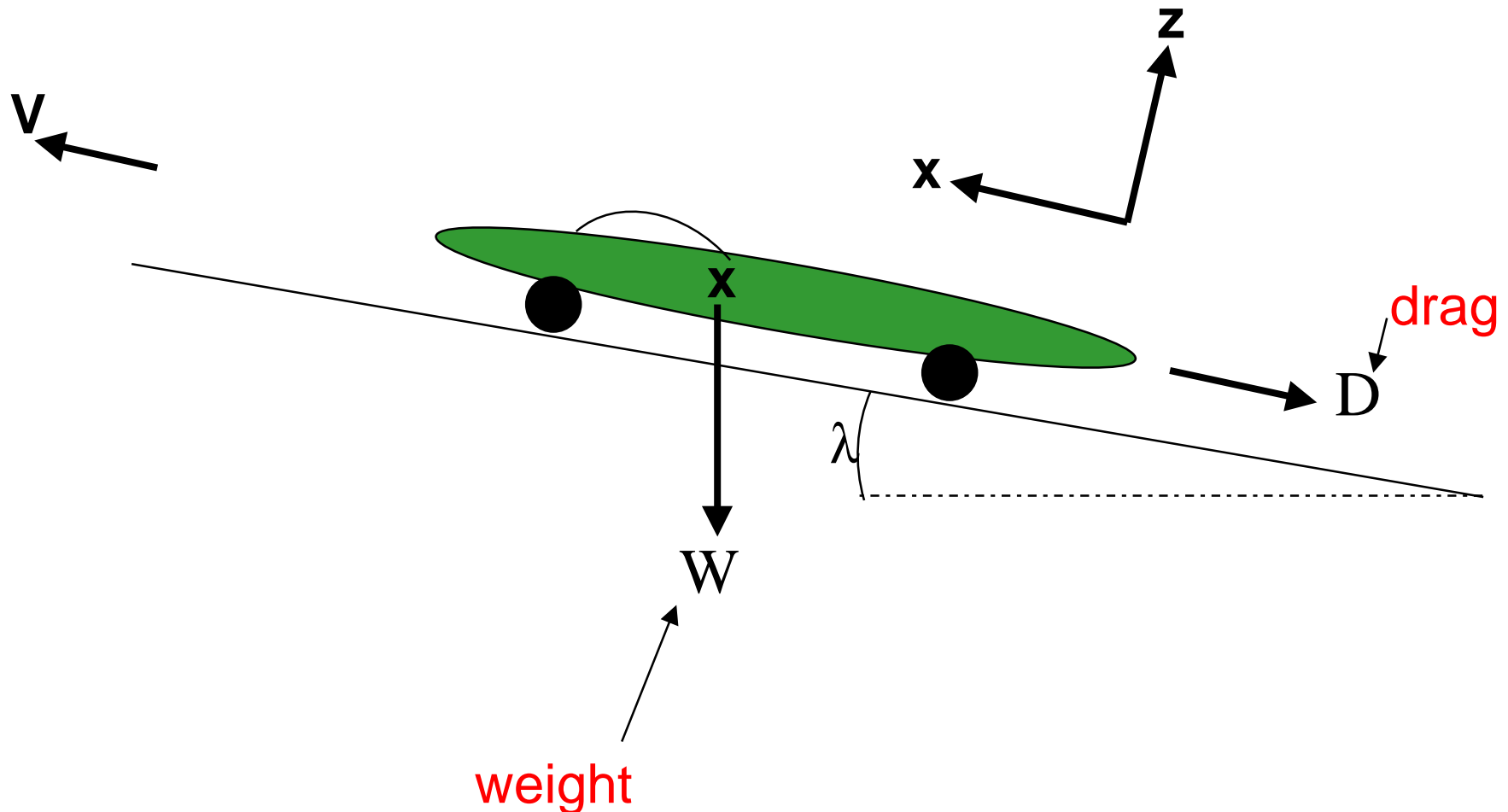
Ground Vehicle in Steady Motion

- Neglect transverse forces and moments



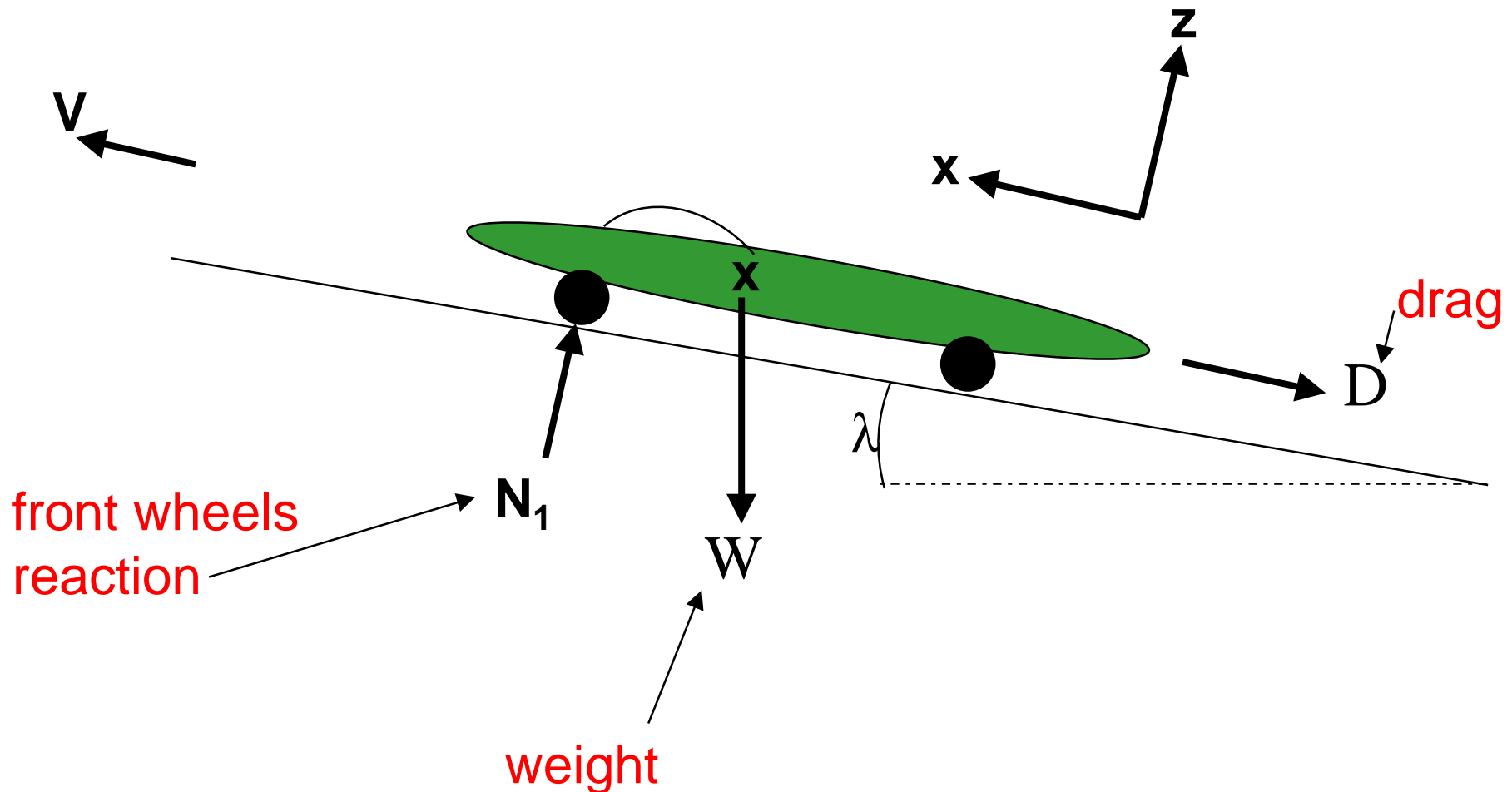
Ground Vehicle in Steady Motion

- Neglect transverse forces and moments



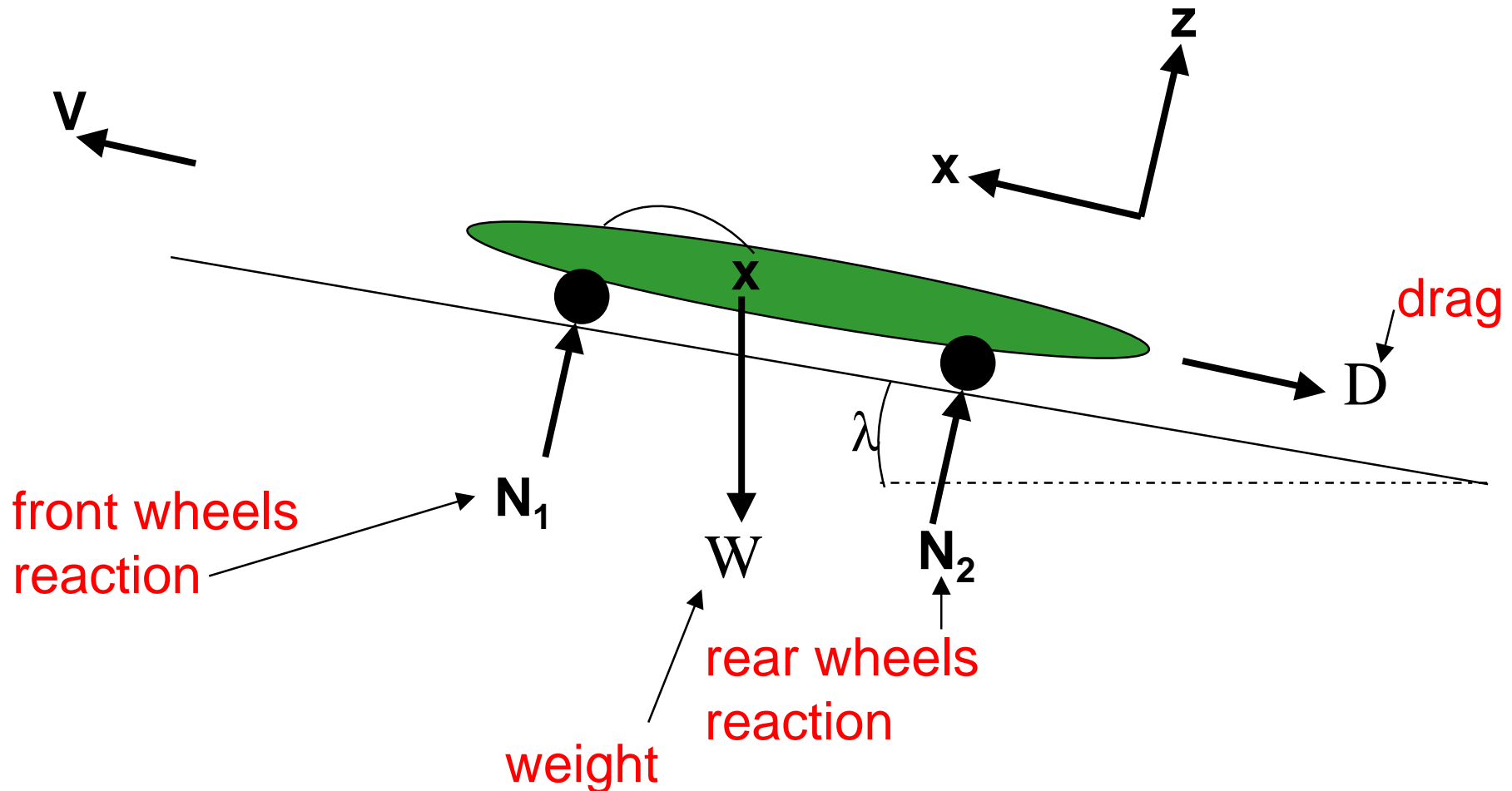
Ground Vehicle in Steady Motion

- Neglect transverse forces and moments



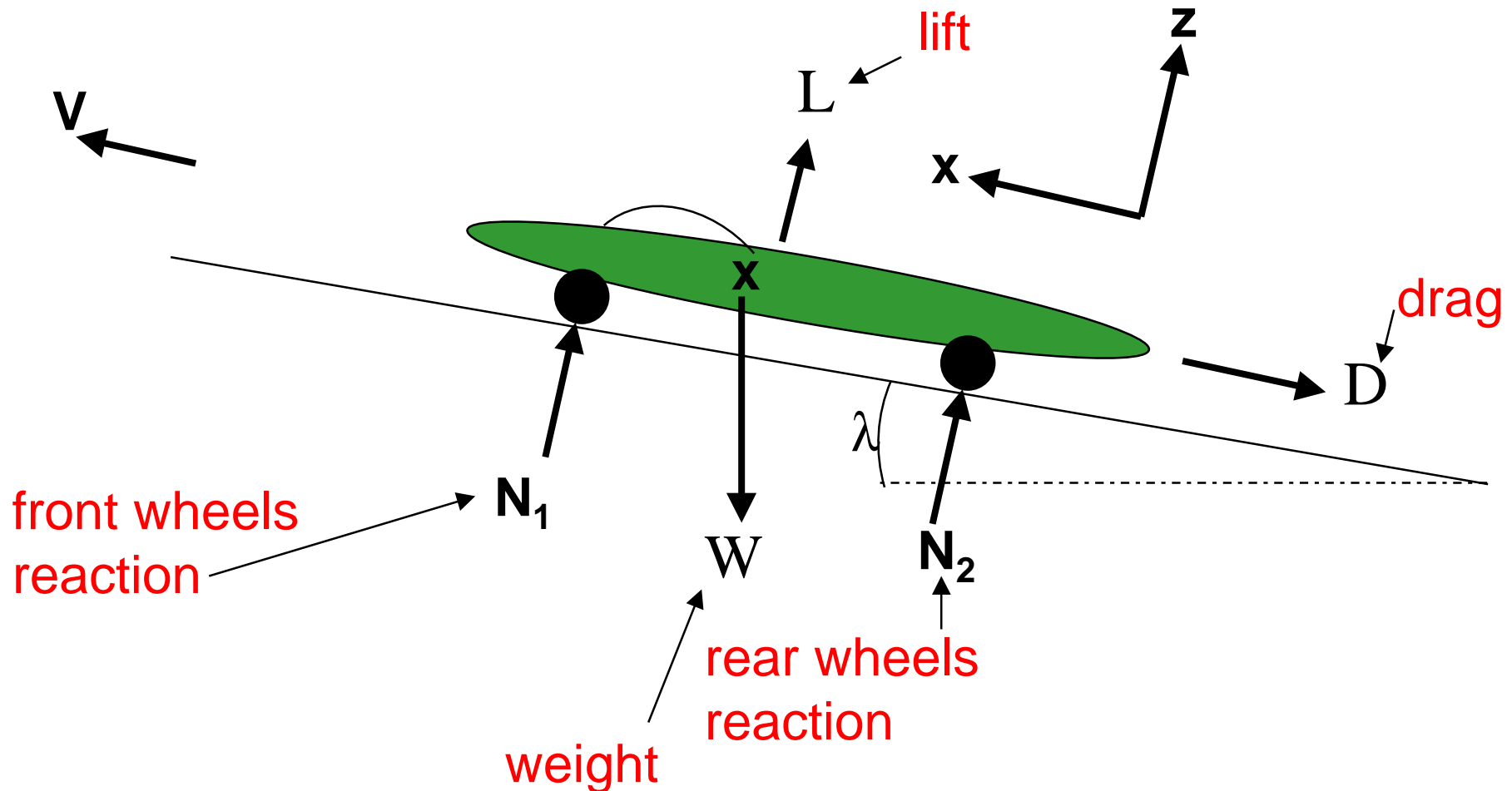
Ground Vehicle in Steady Motion

- Neglect transverse forces and moments



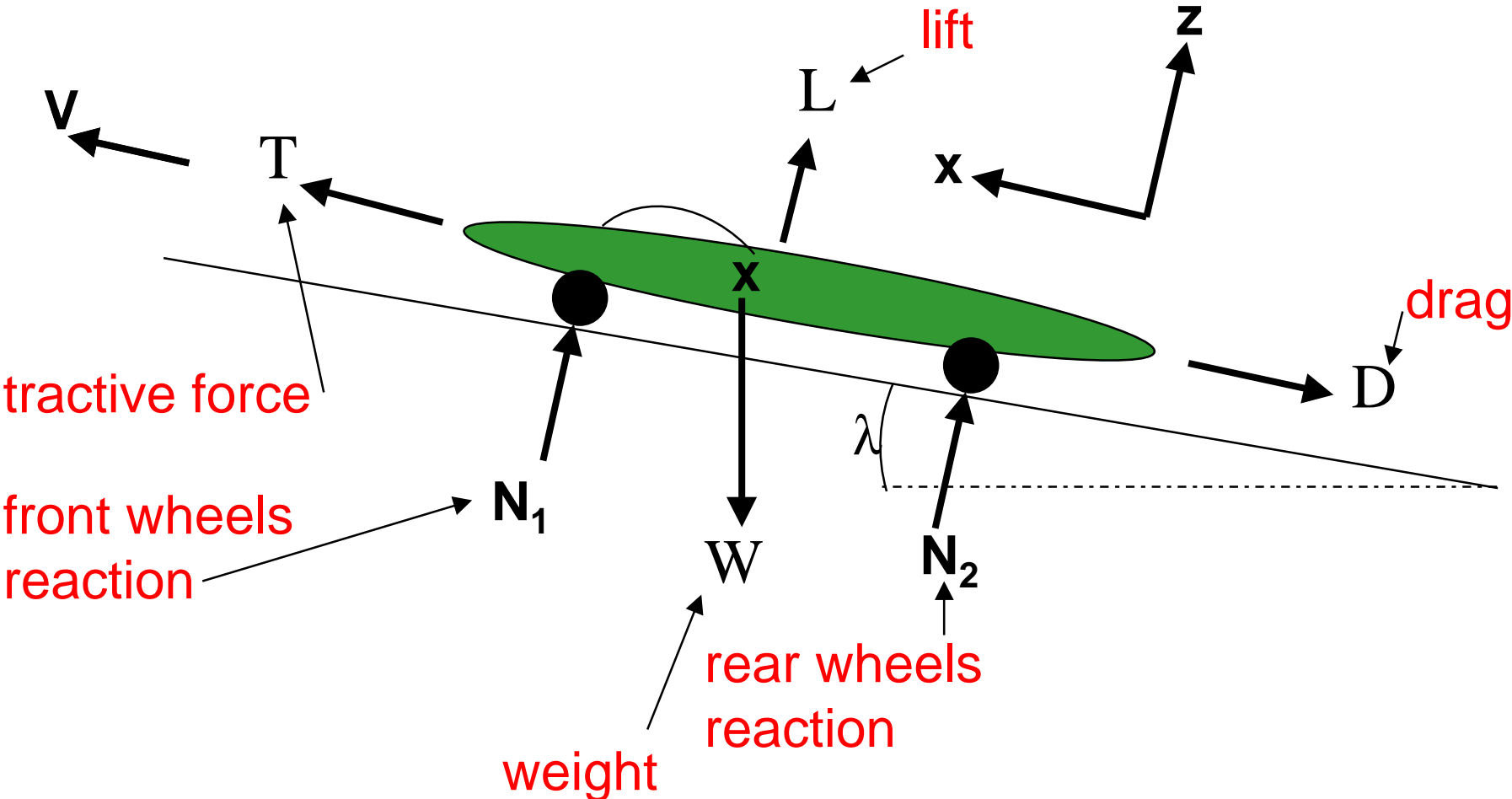
Ground Vehicle in Steady Motion

- Neglect transverse forces and moments



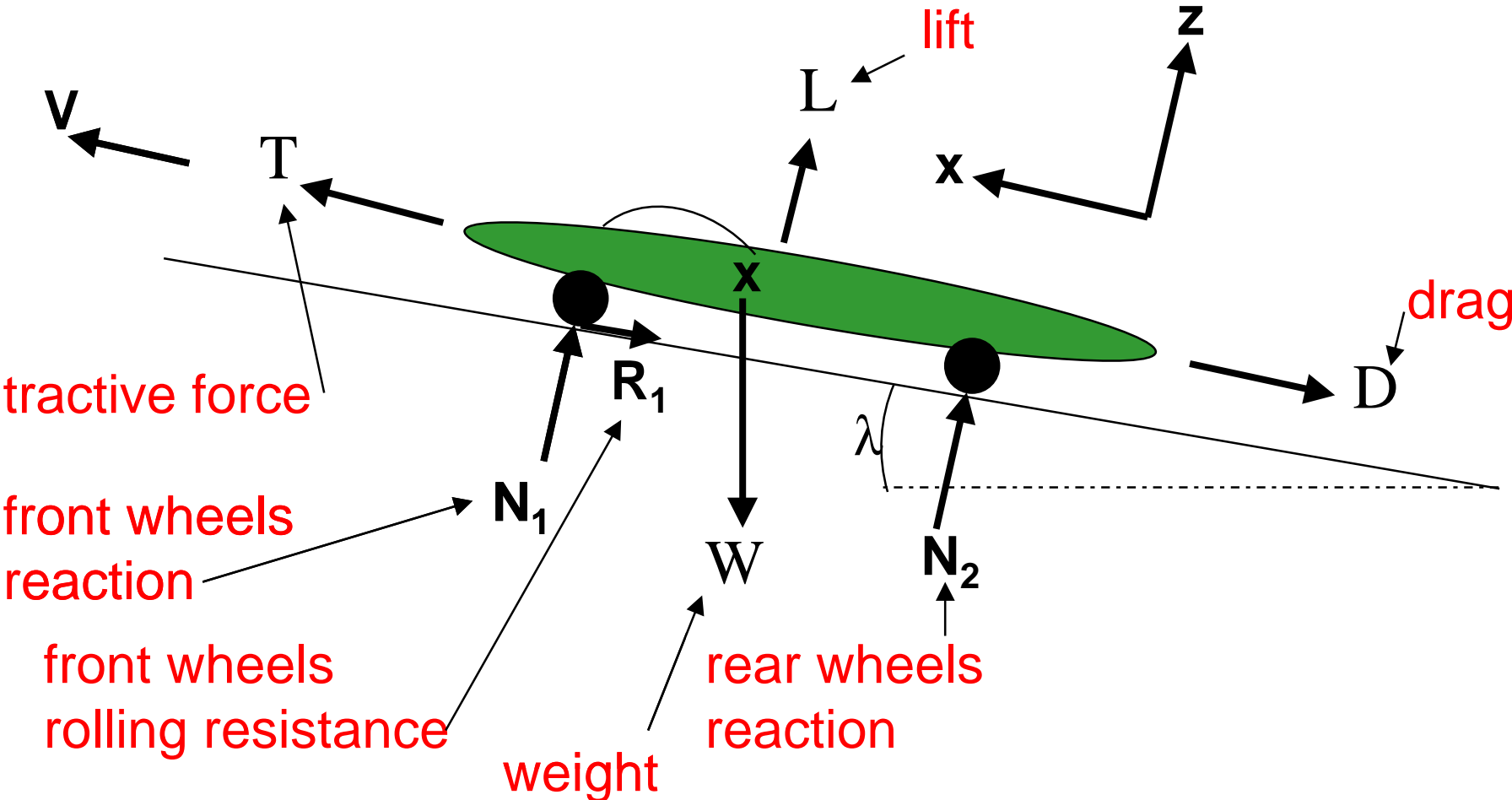
Ground Vehicle in Steady Motion

- Neglect transverse forces and moments



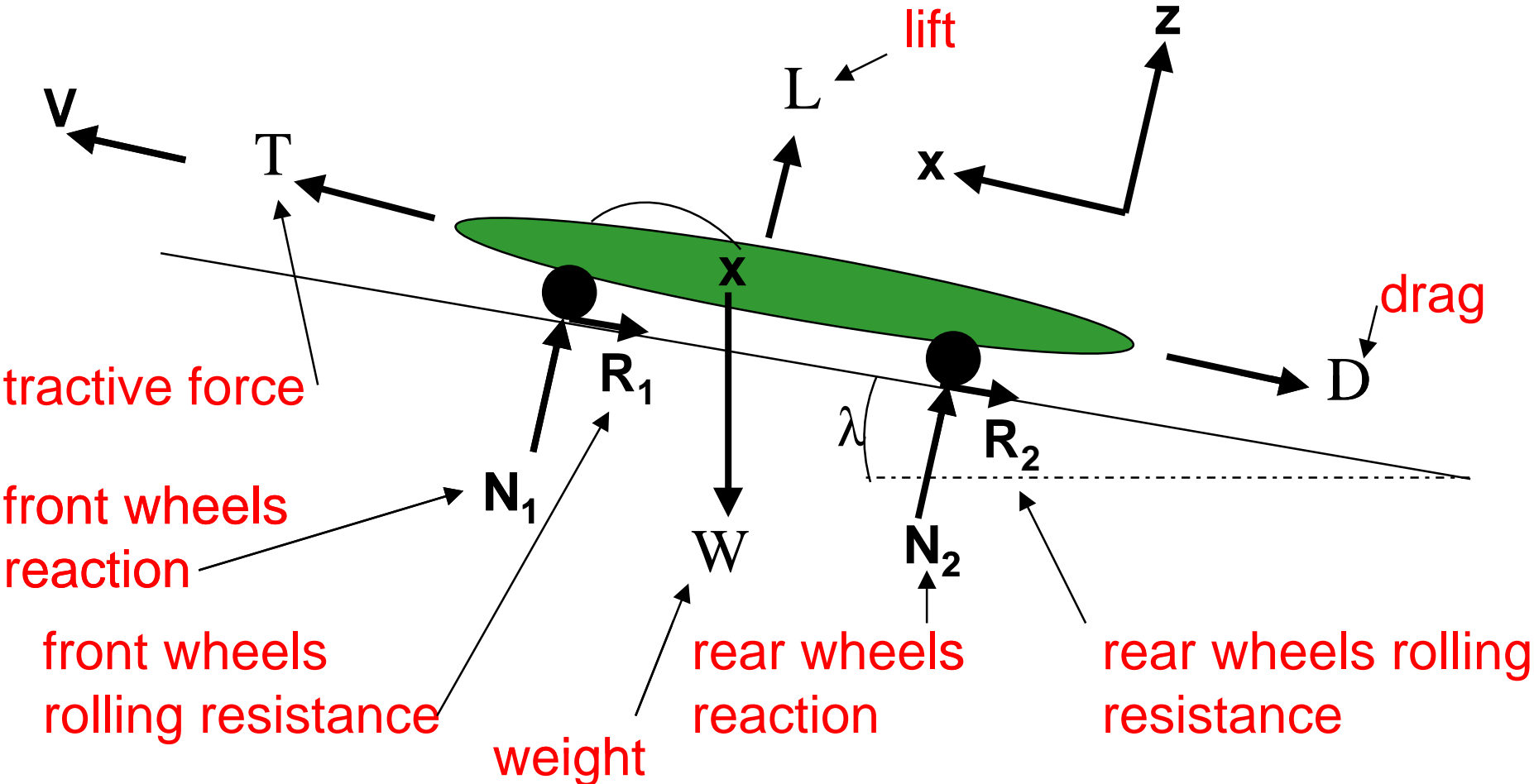
Ground Vehicle in Steady Motion

- Neglect transverse forces and moments



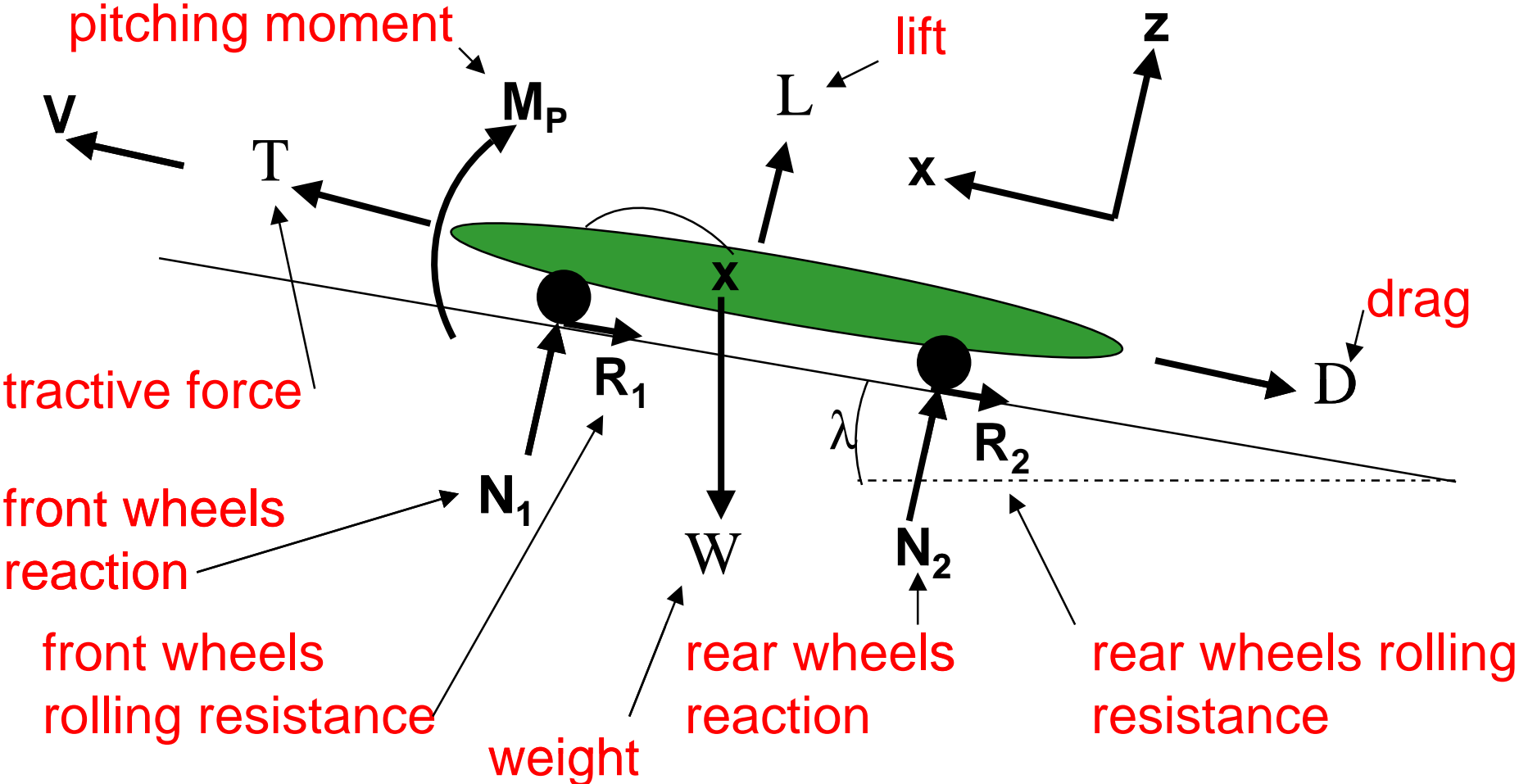
Ground Vehicle in Steady Motion

- Neglect transverse forces and moments



Ground Vehicle in Steady Motion

- Neglect transverse forces and moments



Ground Vehicle in Steady Motion

Summing Forces in x- and z-directions:

$$\sum F_x = T - R - D - W \sin \lambda = 0$$

$$\sum F_z = L + N - W \cos \lambda = 0$$

$$\begin{aligned} R &= R_1 + R_2 \\ N &= N_1 + N_2 \end{aligned}$$

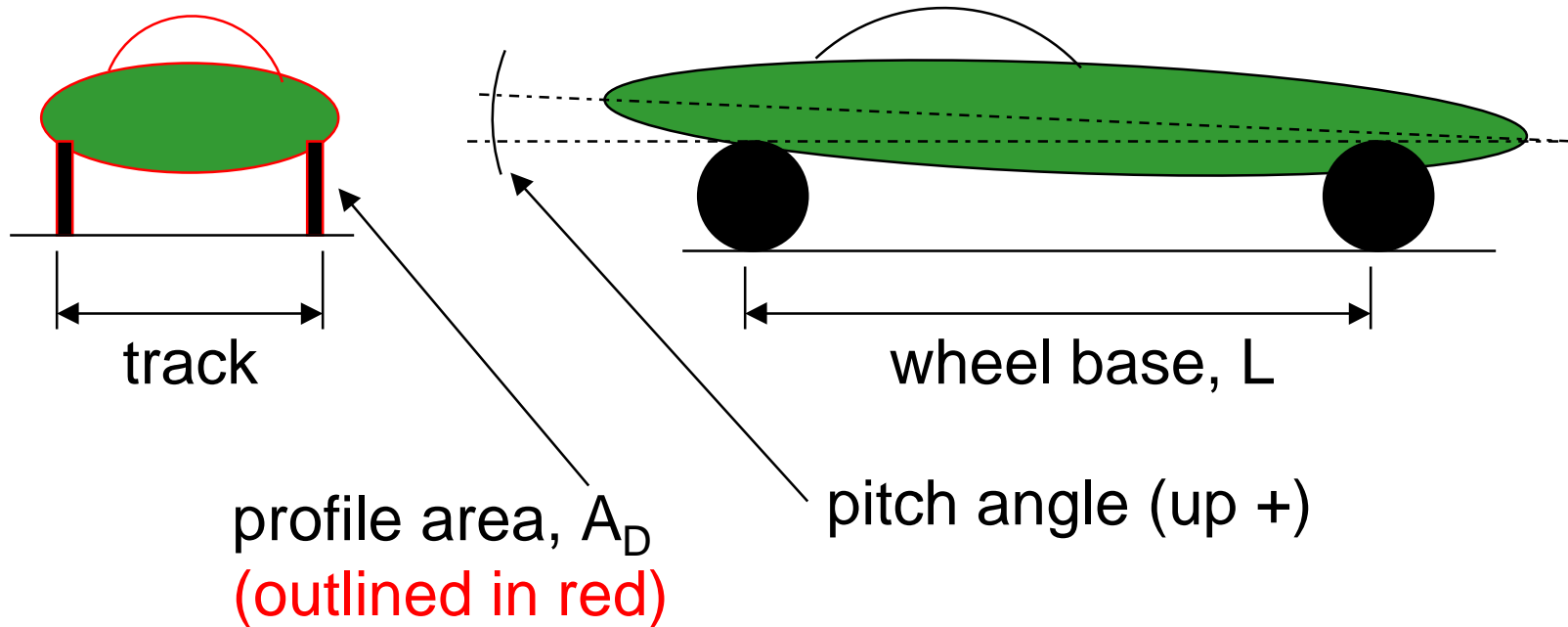
Accelerating in X-Direction

$$T - R - D - W \sin \lambda = M_e \frac{dV}{dt}$$

$$L + N - W \cos \lambda = 0$$

The effective mass, M_e , accounts for the rotational inertia of the wheels. It is about 1.03 M, typically.

Some Vehicle Geometry



“drag area” is $c_D A_D$, [m²]

Lift Coefficient of Car

$$c_L = \frac{L}{A_D \frac{1}{2} \rho V^2}$$

$$\frac{N}{(m^2) \left(\frac{N}{m^2} \right)} \Rightarrow c_L \text{ is dimensionless}$$

Lift Coefficient of Car

lift coefficient

$$c_L = \frac{L}{A_D \frac{1}{2} \rho V^2}$$

$$\frac{N}{(m^2) \left(\frac{N}{m^2} \right)} \Rightarrow c_L \text{ is dimensionless}$$

Lift Coefficient of Car

lift coefficient \rightarrow

$$c_L = \frac{L}{A_D \frac{1}{2} \rho V^2}$$

\leftarrow lift force

$$\frac{N}{(m^2) \left(\frac{N}{m^2} \right)} \Rightarrow c_L \text{ is dimensionless}$$

Lift Coefficient of Car

lift coefficient $\rightarrow c_L = \frac{L}{A_D \frac{1}{2} \rho V^2}$ lift force

profile area \rightarrow

$$\frac{N}{(m^2) \left(\frac{N}{m^2} \right)} \Rightarrow c_L \text{ is dimensionless}$$

Lift Coefficient of Car

lift coefficient $\rightarrow c_L = \frac{L}{A_D \frac{1}{2} \rho V^2}$ lift force

profile area \rightarrow air density \rightarrow

$$\frac{N}{(m^2) \left(\frac{N}{m^2} \right)} \Rightarrow c_L \text{ is dimensionless}$$

Lift Coefficient of Car

lift coefficient $\rightarrow c_L = \frac{L}{A_D \frac{1}{2} \rho V^2}$ lift force

profile area \rightarrow air density \rightarrow relative air speed \rightarrow

$$\frac{N}{(m^2) \left(\frac{N}{m^2} \right)} \Rightarrow c_L \text{ is dimensionless}$$

Drag Coefficient of Car

drag coefficient

$$c_D = \frac{D}{A_D \frac{1}{2} \rho V^2}$$

drag force

profile area

air density

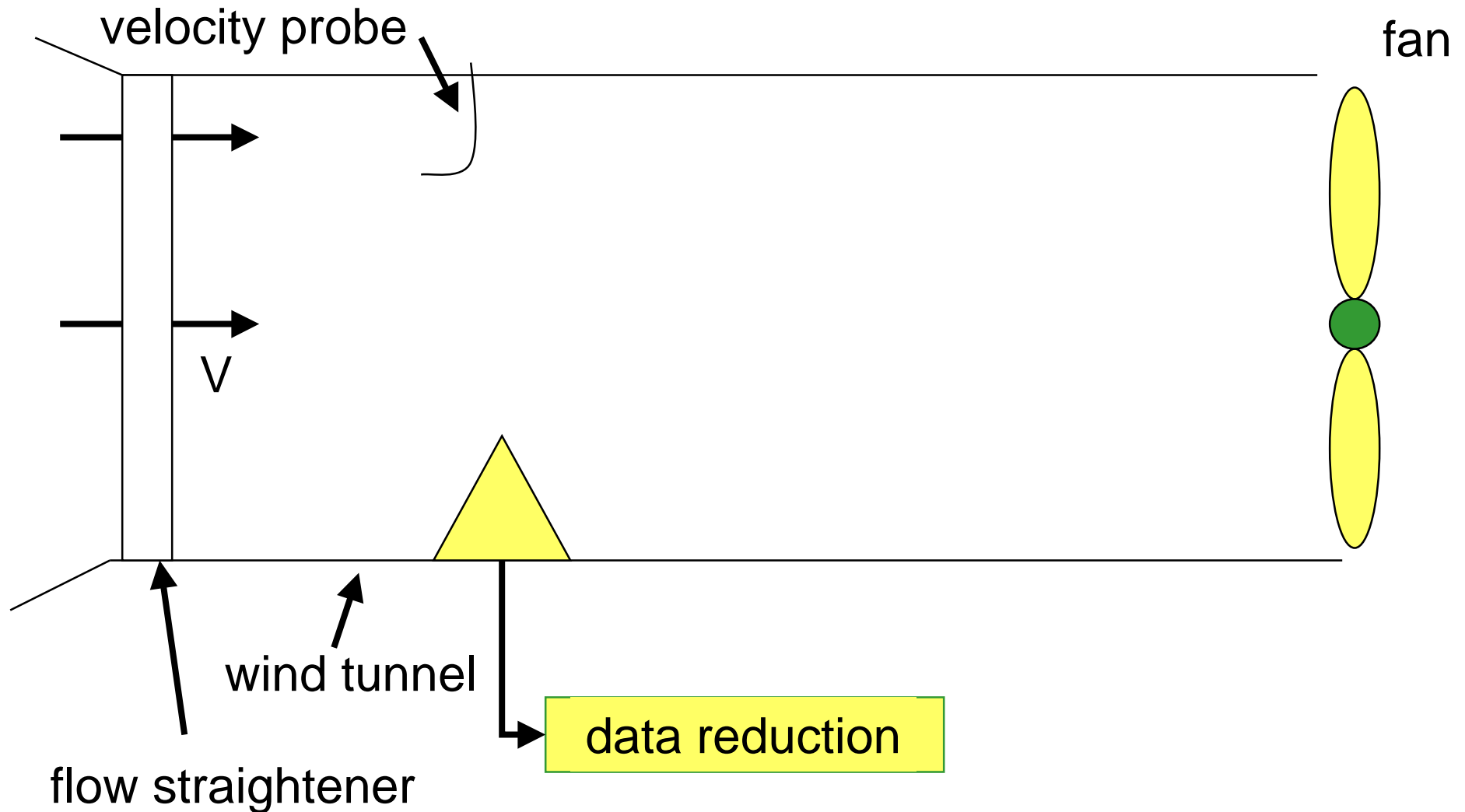
relative air speed

$$\frac{N}{(m^2) \left(\frac{N}{m^2} \right)} \Rightarrow c_D \text{ is dimensionless}$$

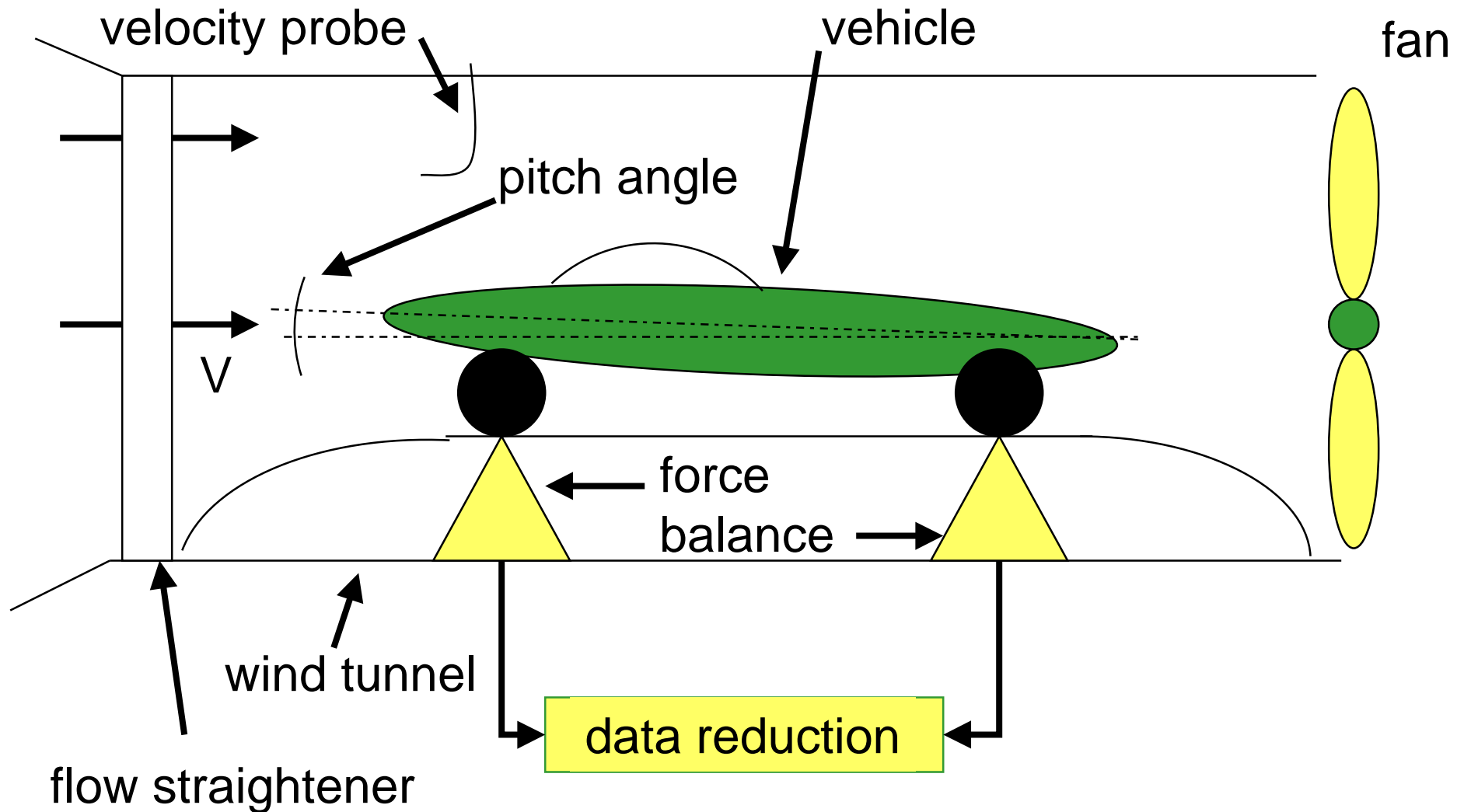
Lift, Drag in Straight Ahead Flow

- Lift and drag depend on
 - The **relative airspeed**
 - The **viscosity** and **density** of the air
 - The **shape** and **smoothness** of the vehicle
 - Including wheels and wheel wells, and the
 - Ventilation system
 - The **pitch angle** of the vehicle
 - The **proximity** of the **ground**

Measuring c_L and c_D in Tunnel



Measuring c_L and c_D in Tunnel



Utility of Model Tests

- Use c_L and c_D *for full scale car*
 - Geometrically similar car
 - Same Reynolds number
- However, drag of actual car usually greater than model
 - Model usually simplified
 - Some full-scale test facilities exist



Reynolds Number

Proportional to the
ratio of dynamic to viscous forces
acting on a fluid element

$$\text{Re} = \rho V L / \mu$$

$$\frac{\left(\frac{\text{kg}}{\text{m}^3} \right) \left(\frac{\text{m}}{\text{sec}} \right) (m)}{\frac{\text{N sec}}{\text{m}^2}} = \frac{\text{N}}{\frac{\text{m}^2}{\text{N}}} \Rightarrow n.d.$$

L is wheel base

Rolling Resistance

- Contact patch force
 - Proportional to N
- Bearing friction (small)
 - Proportional to V and W
- Wheel rotational drag (smallest)
 - Proportional to V

$$R = \mu W = (\mu_1 \cos \lambda + \mu_2 V) W$$

Coast-Down Test to Measure

$$c_D A_D, \mu_1, \mu_2$$

- Measure W , p (pressure), and T (temp. Celcius)
- No wind, horizontal road (ideally)
- Initial speed greater than about 40 mph
- Coast down in straight line and record V at intervals
- [Note: μ means “rolling resistance coefficient,” in this case -- not viscosity]

Reducing Coast Down Data

- Fit $V(t)$, find $\dot{V}(t)$, fit a quadratic in V to it:

$$\dot{V}(t) = a_2 V^2 + a_1 V + a_0$$

- Where

$$a_2 = \frac{c_D A_D \rho}{2M_e}, \quad a_1 = \frac{\mu_2 W}{M_e}, \quad a_0 = \frac{\mu_1 W}{M_e}$$

Making Decisions II

- If the user selects numeric derivative
 - ...then do that method and go on
- Else she wants to smooth the data first
 - ...then do that and go on
- End of decision

First Derivative Approximations

Forward difference

$$\frac{dV}{dt} \approx \frac{V_{t+\Delta t} - V_t}{\Delta t} = \frac{V_{i+1} - V_i}{\Delta t}$$

“i” \implies t

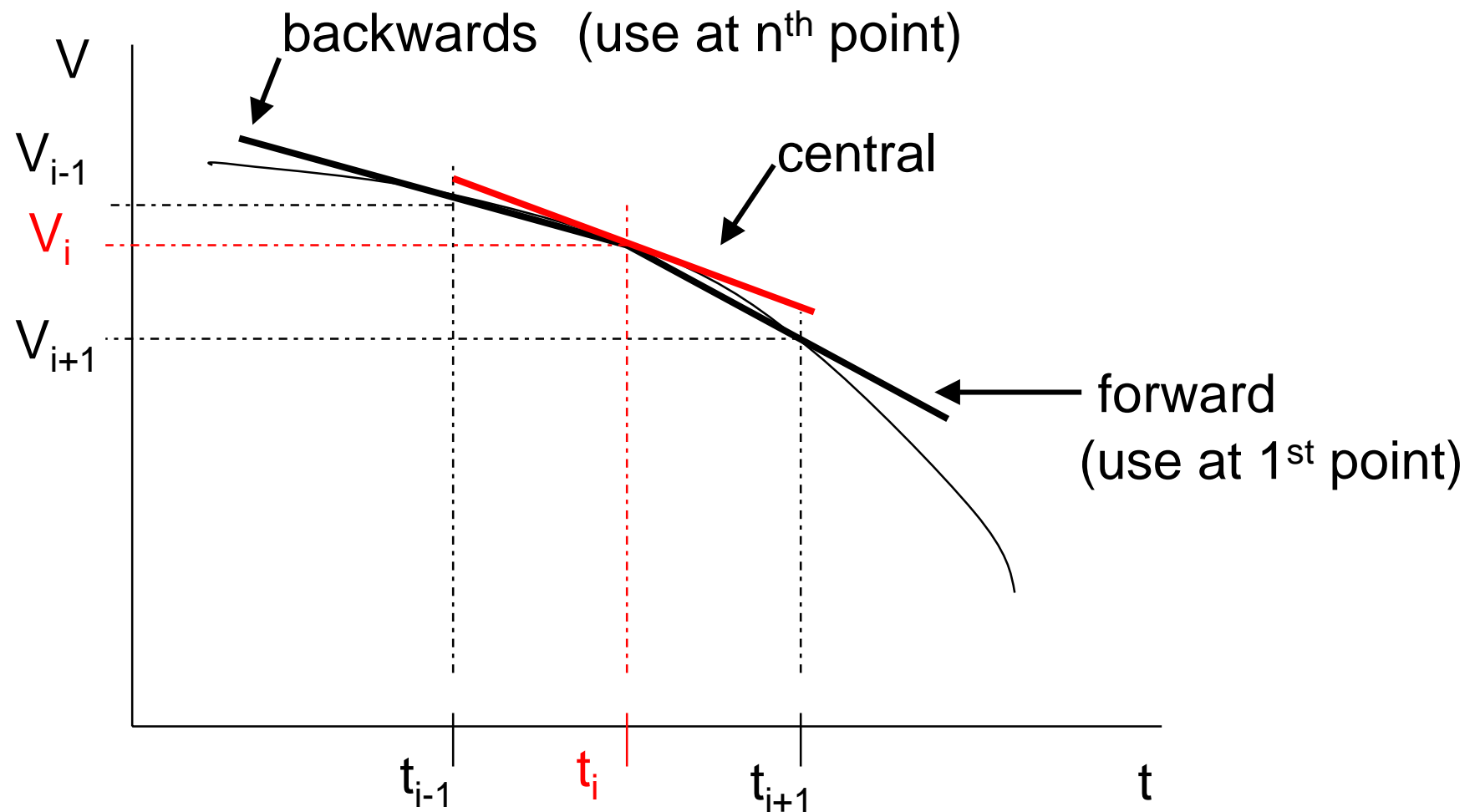
Central difference

$$\frac{dV}{dt} \approx \frac{V_{i+1} - V_{i-1}}{2\Delta t}$$

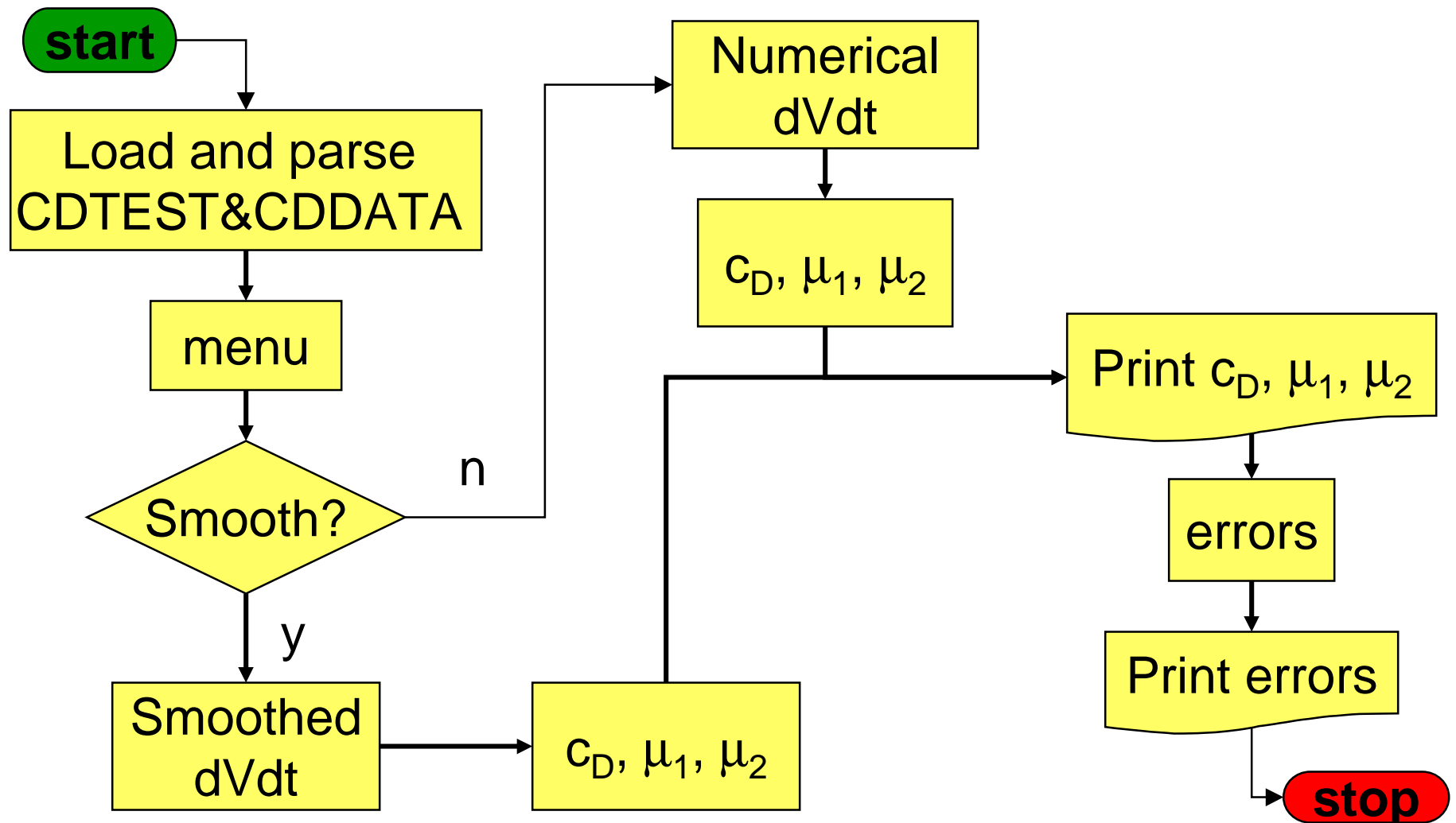
Backward difference

$$\frac{dV}{dt} \approx \frac{V_i - V_{i-1}}{\Delta t}$$

Numerical Derivatives II

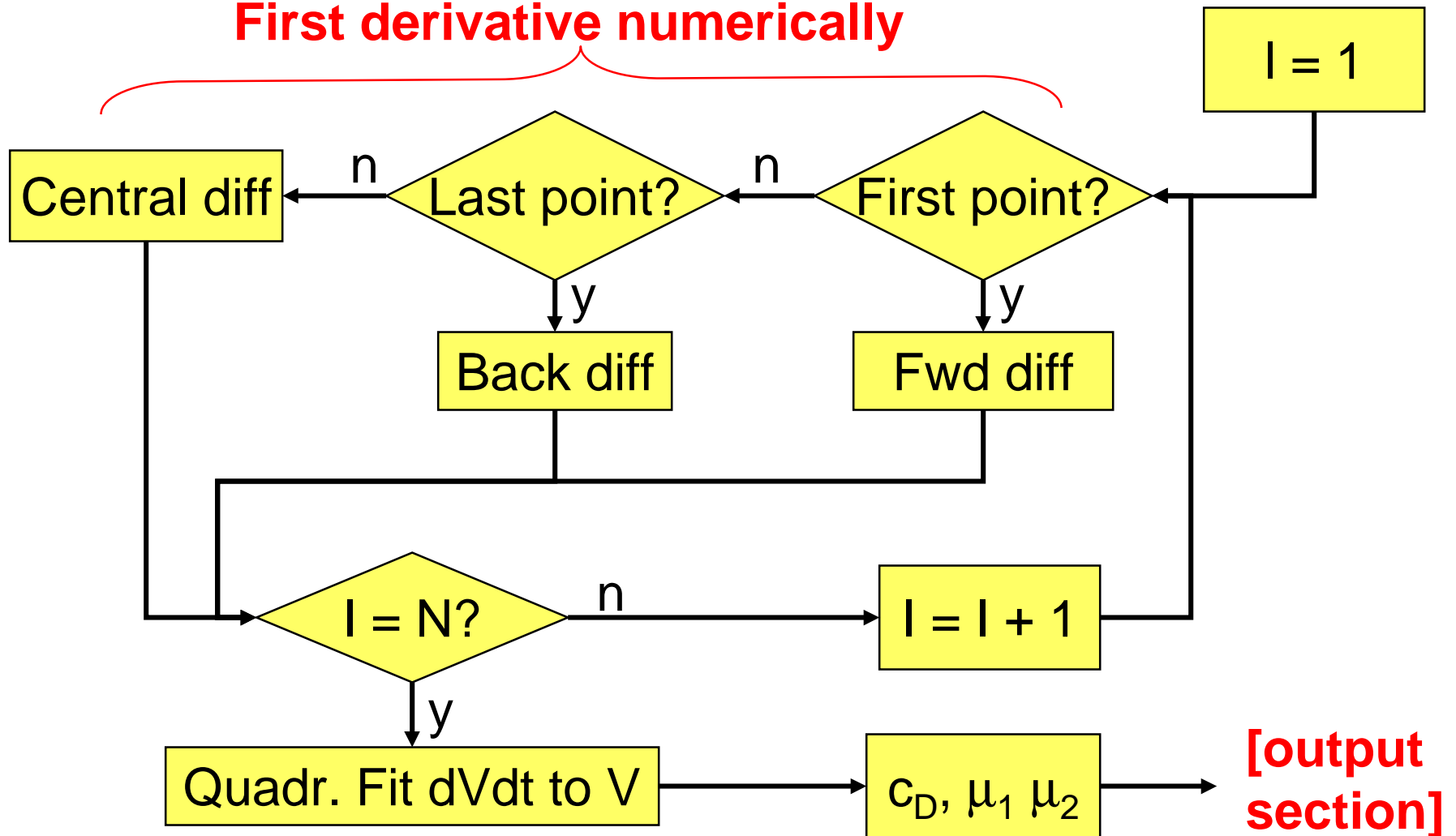


COASTD.M Flow Chart I



Numerical dV/dt

First derivative numerically



COASTD.M I

% Load and parse the data file

```
load a:\cddata -ascii;
```

```
t = ld1(:,1);           % vector of times (sec)
```

```
V = ld1(:,2);           % vector of car speeds at  
                           % times (m/s)
```

% Car mass and profile area

```
load a:\cdtest -ascii;
```

```
Ap = cdtest(2);         % m2
```

```
M = cdtest(5);         % kg
```

```
p = cdtest(6);
```

```
Tc = cdtest(7);
```

```
T = tk (Tc);
```

```
rho = airden (p*1000, T);
```

COASTD.M II

% Menu

```
fprintf ( '\nMENU:\n\t1. Unsmoothed data,  
         numeric derivative.\n\t2.  
         Cubic smoothing.\n\n\t');
```

```
choice = input ( 'Enter number of  
                choice: ');
```

The “choice” if-block

if choice == 1

[un-smoothed data, numeric derivative statements]

else

[cubic-polynomial fitted to data, derivative is derivative of polynomial]

end

COASTD.M III

```
% Start of "choice" if-block
```

```
if choice == 1
```

```
% Un-smoothed data, numeric derivative
```

```
n = length(V);
```

```
clear dVdt
```

```
dt = t(2)-t(1);
```

```
% Loop through all data points
```

```
[next slide]
```

[if choice == 1, continued]

```
for i = 1:n
    if i == 1
        dvdt(i) = ( v(i+1)-v(i) )/dt;
    elseif i == n
        dvdt(i) = ( v(i)-v(i-1) )/dt;
    else
        dvdt(i) = ( v(i+1)-v(i-1) )/( 2*dt);
    end
end
% End of loop through data
% Reduce the results
a = polyfit (V, dvdt', 2);
[cDm mu1m mu2m] = reduce (a, cdtest);
```

COASTD.M IV

else

% Try cubic smoothing

```
b3 = polyfit (t, V, 3);
```

```
vf3 = b3(1)*t.^3 + b3(2)*t.^2 + ...  
      b3(3)*t + b3(4);
```

```
dvdt3 = 3*b3(1)*t.^2 + 2*b3(2)*t +...  
        b3(3);
```

% Reduce the results

```
a3 = polyfit (vf3, dvdt3, 2);
```

```
[cDm mu1m mu2m] = reduce (a3, cdtest);
```

end % End of "choice" if-block

COASTD.M V

```
% Print results
```

```
fprintf ('\nResults:\n\tcD = %g\n\tmu1 = %g\n\tmu2 = %g (s/m)\n\n', ...  
        cDm, mu1m, mu2m);
```

```
% Calculate errors
```

```
errors (cDm, mu1m, mu2m, cdtest);
```

New Functions in COASTD.M I

```
function errors ( cD, mu1, mu2, cdtest )

ecD = abs( (cD/cdtest(1)) - 1 )*100;
emu1 = abs( (mu1/cdtest(3)) - 1 )*100;
emu2 = abs( (mu2/cdtest(4)) - 1 )*100;
fprintf ( '\n\tError in cD = %g %%\n
\tError in mu1 = %g %%\n
\tError in mu2 = %g %%\n\n', ...
ecD, emu1, emu2)
```

New Functions in COASTD.M II

```
function [cD, mu1, mu2] =  
    reduce(a, cdtest);  
  
% Assign convenient names  
AD = cdtest(2);  
M = cdtest(5);  
p = cdtest(6);  
Tc = cdtest(7);  
  
% Calculate T, rho, Me, and W  
T = tk (Tc);  
rho = airden (p*1000, T);  
Me = 1.03*M;  
W = M*9.807;
```

Function REDUCE.M, cont.

```
% Find cD, mu1, mu2. Recall that:  
% a(1) = -.5*cD*AD*rho/Me;  
% a(2) = -mu2*W/Me;  
% a(3) = -mu1*W/Me;  
cD = -a(1)*2*Me/(AD*rho);  
mu1 = -a(3)*Me/W;  
mu2 = -a(2)*Me/W;
```

Results from COASTD.M

- Effect of number of measurements
- Effect of standard deviation