Simple Truss Problems and Linear Algebraic Systems

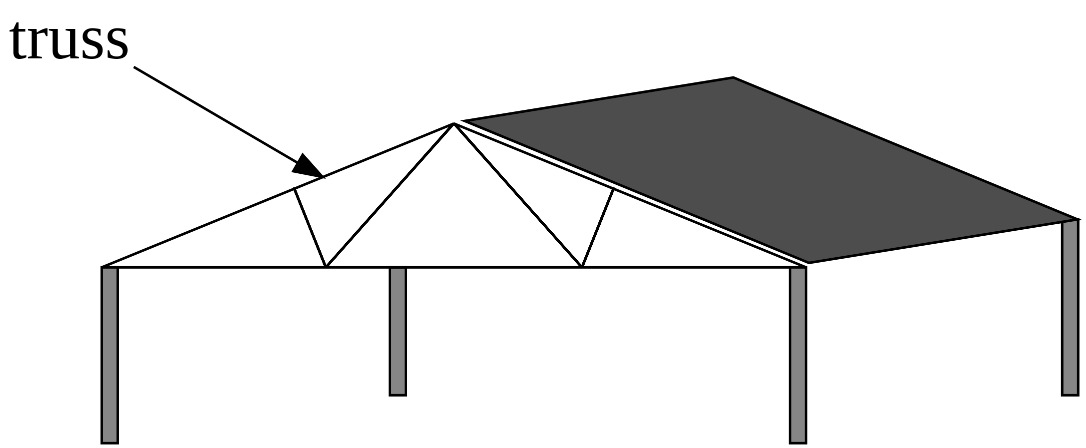
ES100

February 22, 1999

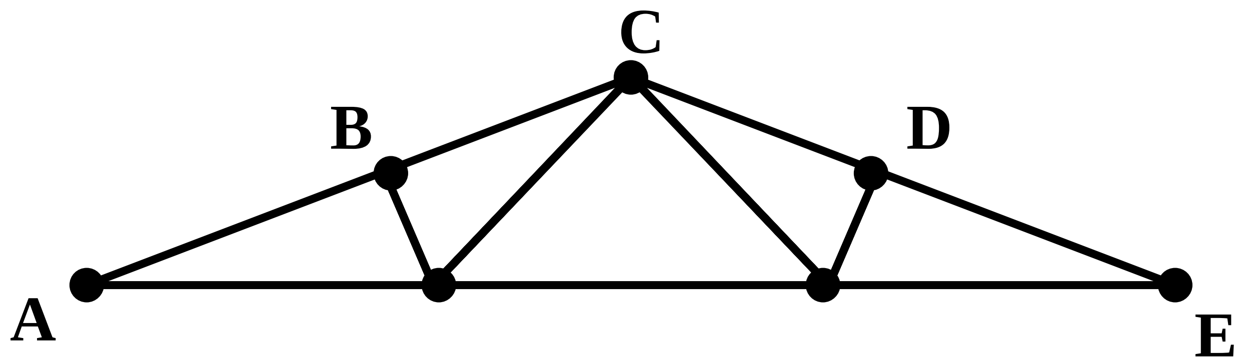
T.S. Whitten

Definition of a *truss*

- A truss is a rigid frame consisting of slender members connected at their endpoints.

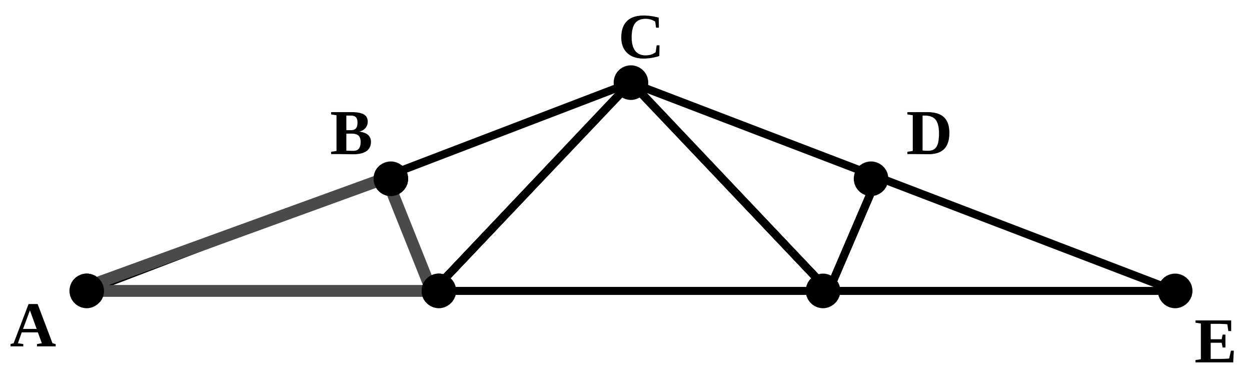


Simple Trusses



The simplest configuration for a stable truss is a triangle as shown in red above.

Simple Trusses



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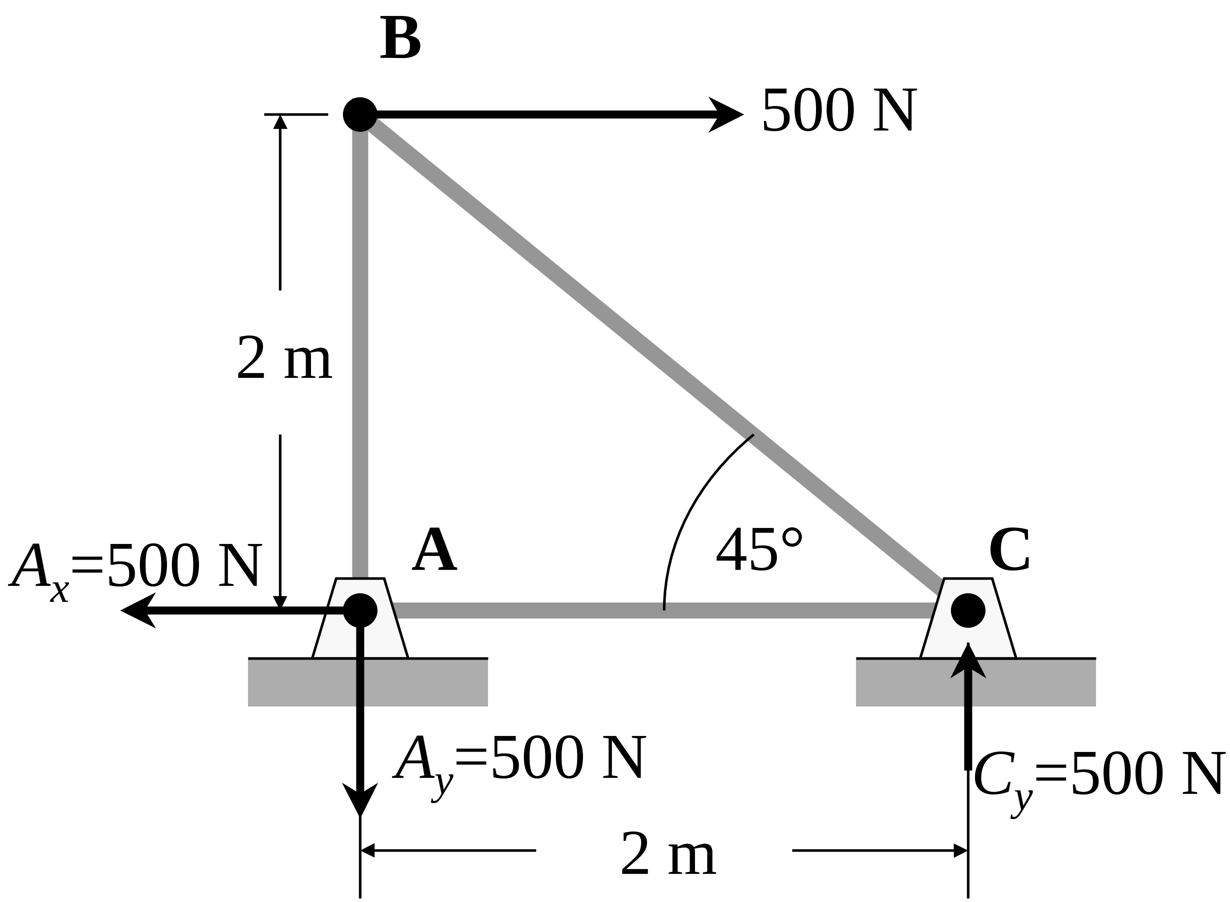
The Triangular Truss



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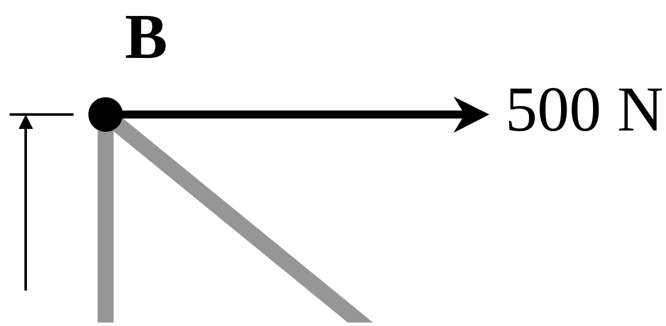

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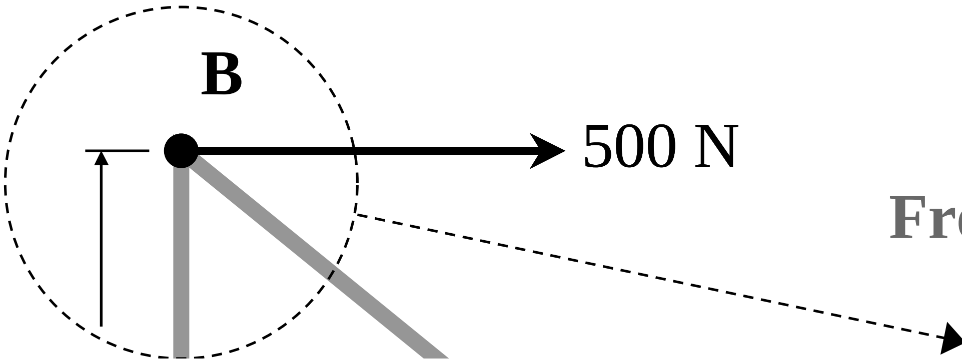
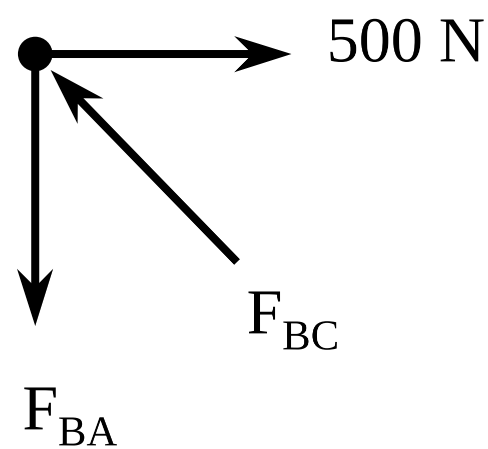
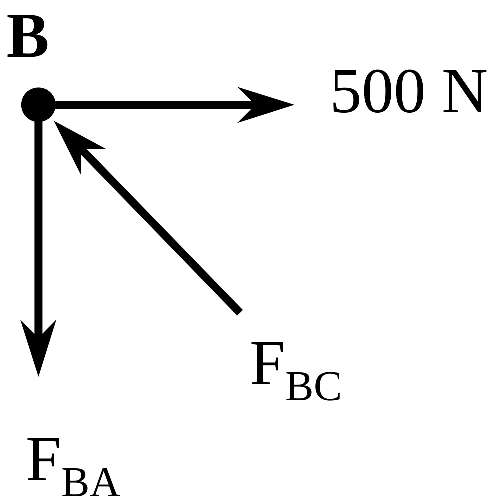


Free Body Diagram (FBD)



Static Equilibrium



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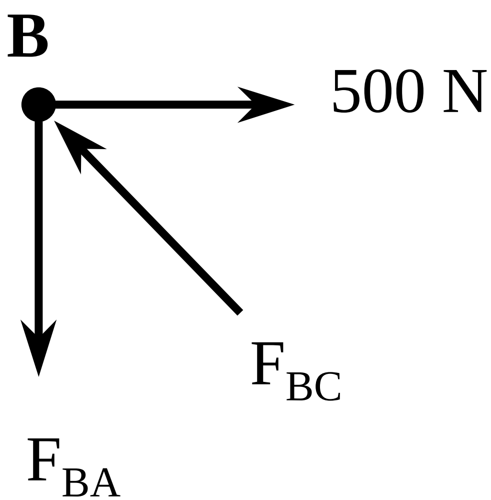

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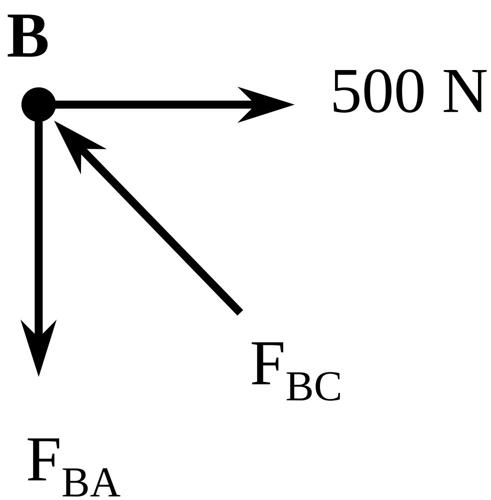
Static Equilibrium



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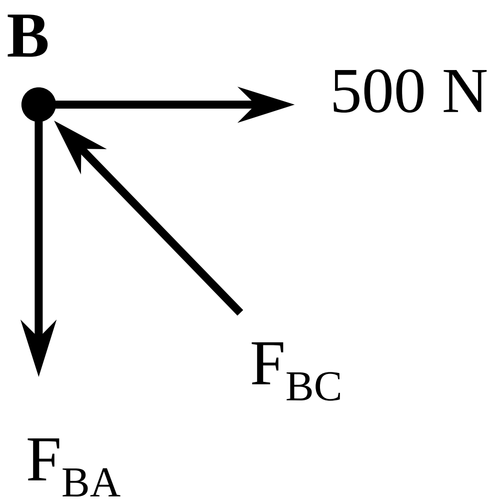
Static Equilibrium



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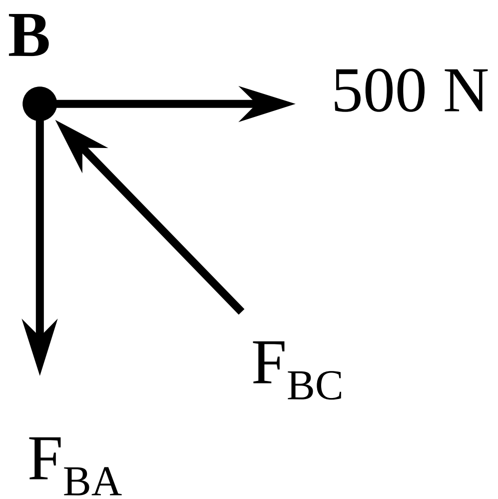
- The same is true for forces acting in the y-direction

Solving For Forces



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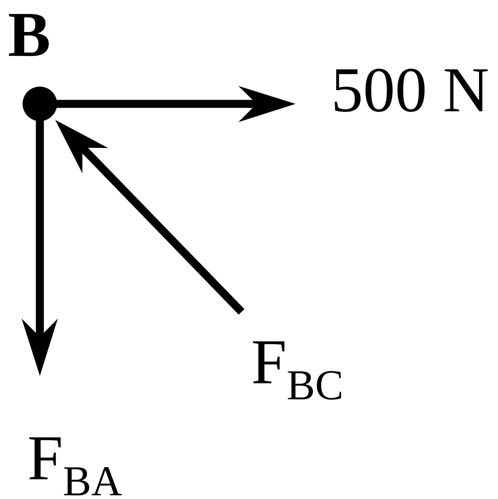

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$$F_{BC} = 707.1\,\mathrm{N}$$

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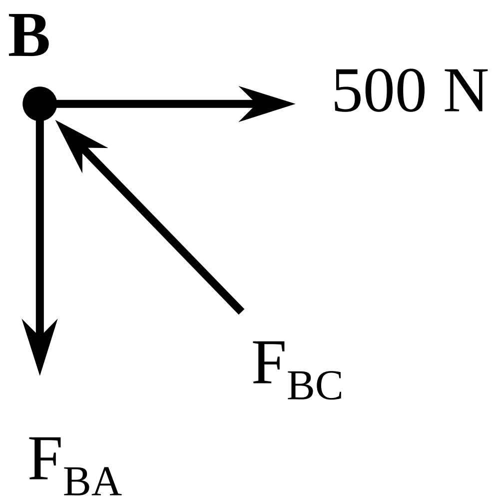


$$\sum F_x = 0; \quad 500 - F_{BC} \sin 45° = 0$$

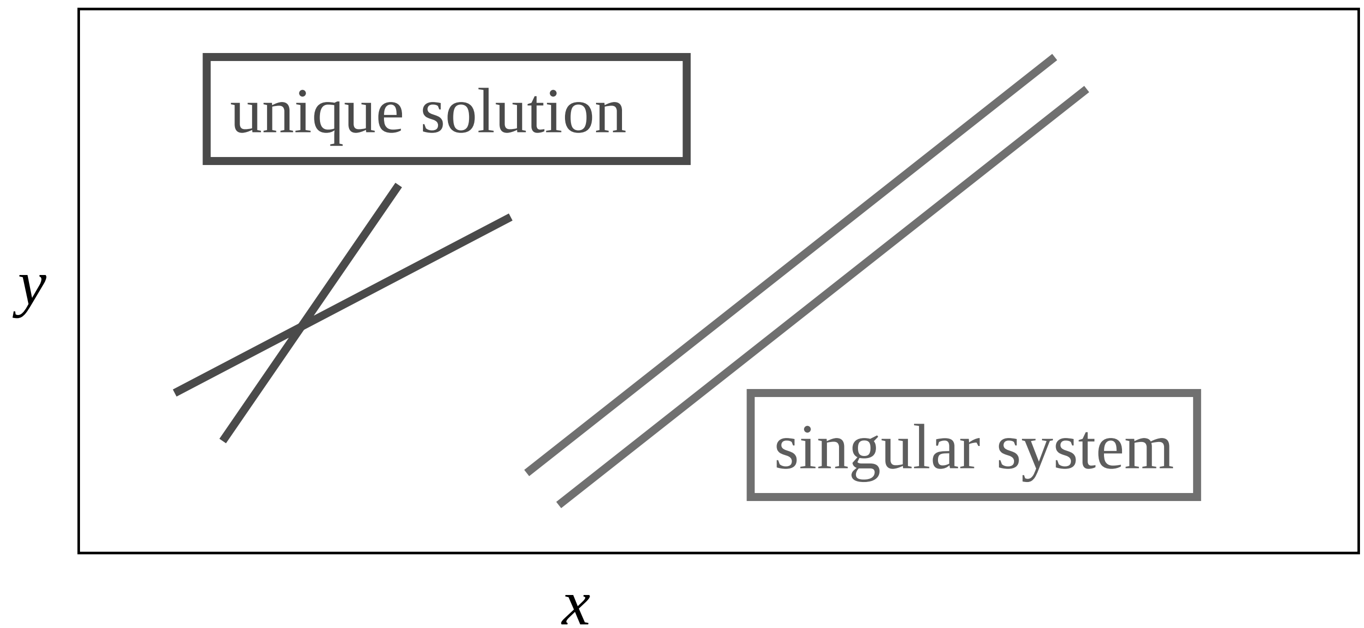
$$F_{BC} = 707.1\ \text{N}$$

$$\sum F_y = 0; \quad F_{BC} \cos 45° - F_{BA} = 0$$

$$F_{BA} = 500\ \text{N}$$

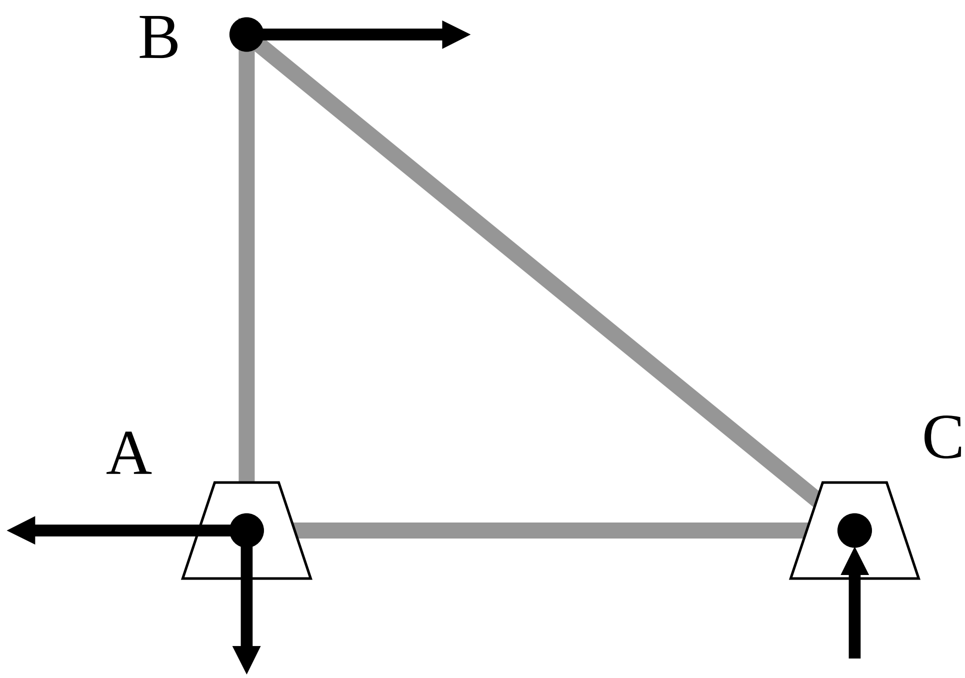
Unique vs. Singular Systems

- Some systems of equations do not have unique solutions.



Statically Indeterminate Example

- Previously, we showed a system of two equations that had two unknowns.   Now, adding member BD and CD:



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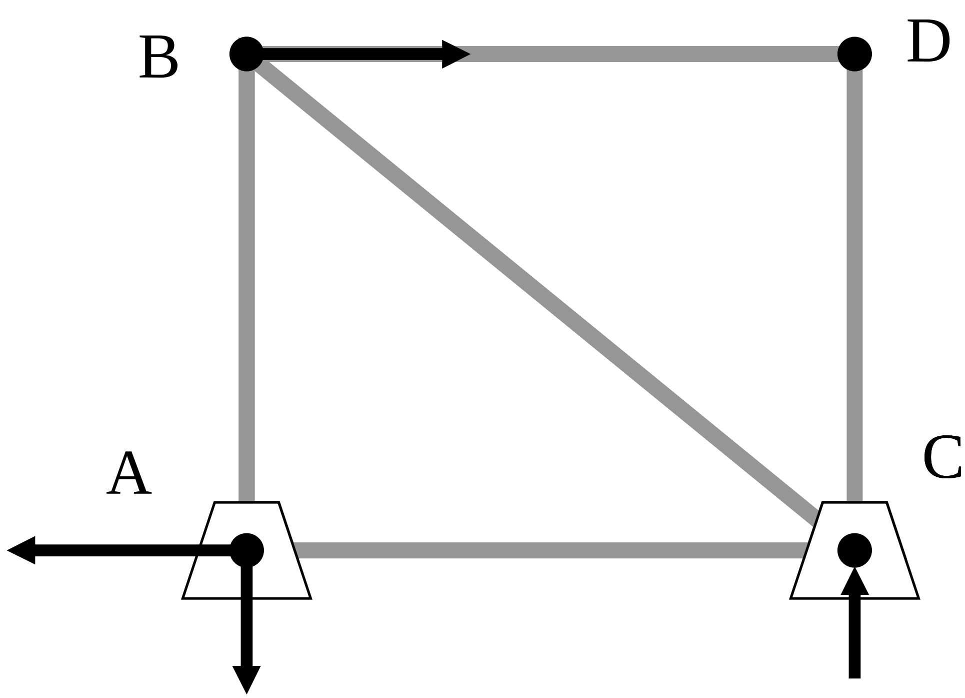

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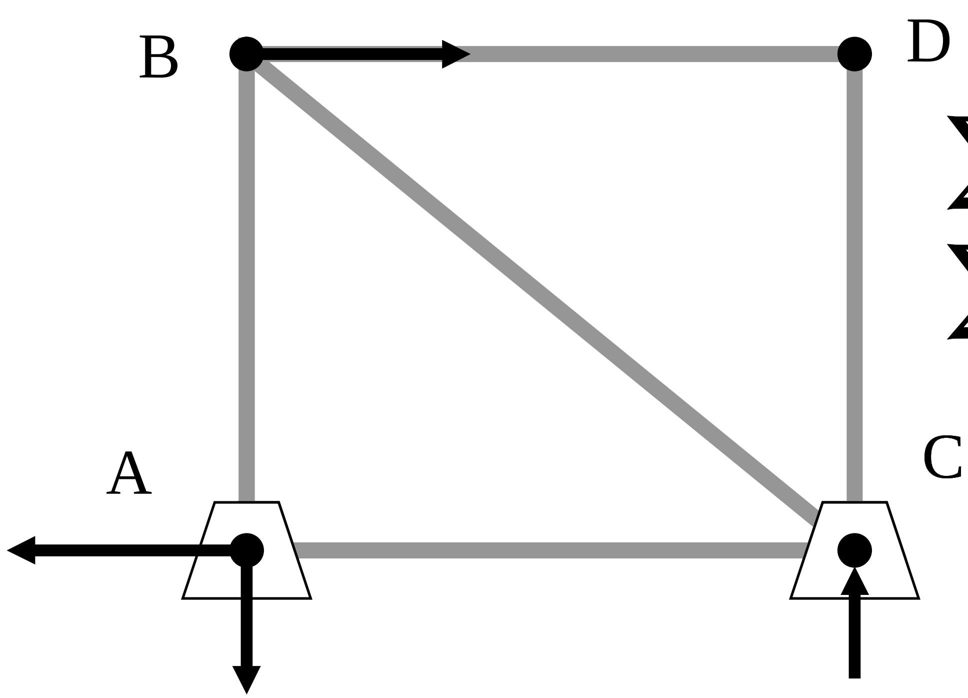

At pin B:

$$\sum F_x = 500 - F_{BC_x} + F_{BD} = 0$$

$$\sum F_y = F_{BC_y} - F_{BA_y} = 0$$

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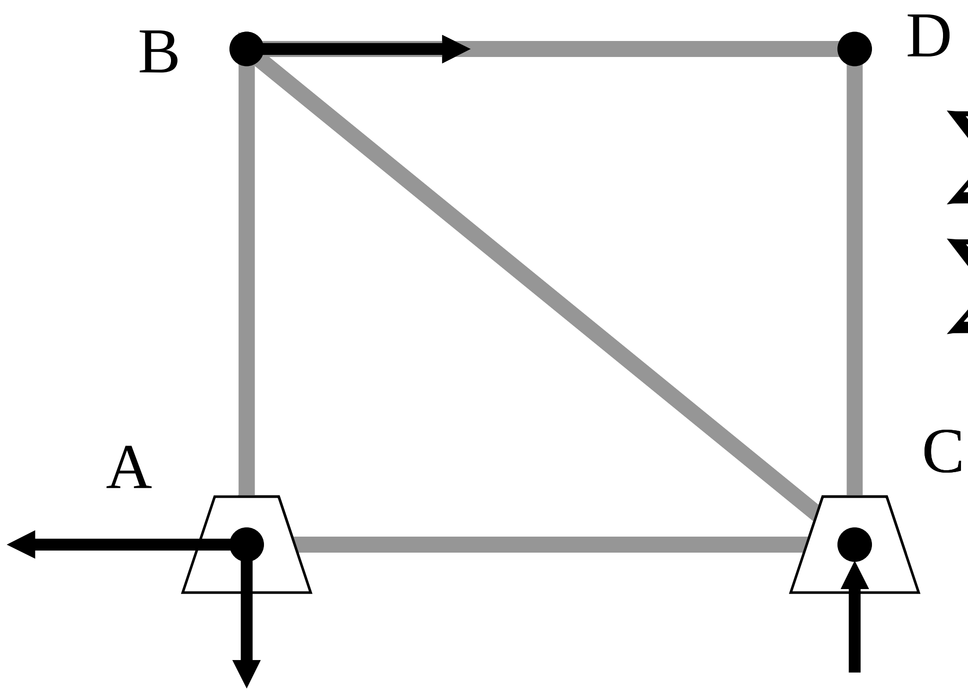


At pin B:

$$\sum F_x = 500 - F_{BC\,x} + F_{BD} = 0$$

$$\sum F_y = F_{BC\,y} - F_{BA_y} = 0$$

Hence, we have only two equations for three unknowns.

Expressing Sets of Linear Algebraic Equations in Matrix Form

- Summation of the forces in the x and y directions can be written as:

$$-F_{BC}\sin 45^\circ + (0)F_{BA} = -500$$

$$F_{BC}\cos 45^\circ - (1)F_{BA} = 0$$

- These two equations can be equivalently expressed in matrix form as...

Multiplication of Two Matrices

$$\underbrace{\begin{bmatrix} -\sin 45^\circ & 0 \\ \cos 45^\circ & 1 \end{bmatrix}}_{coefficients} \cdot \underbrace{\begin{bmatrix} F_{BC} \\ F_{BA} \end{bmatrix}}_{variables} = \begin{bmatrix} -500 \\ 0 \end{bmatrix}$$

Generalized System of Linear Equations

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\
\vdots \qquad\qquad \vdots \qquad\qquad\quad \vdots \qquad & \quad \vdots \\
a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &= b_m
\end{aligned}
$$

Generalized Matrix Representation of Linear System

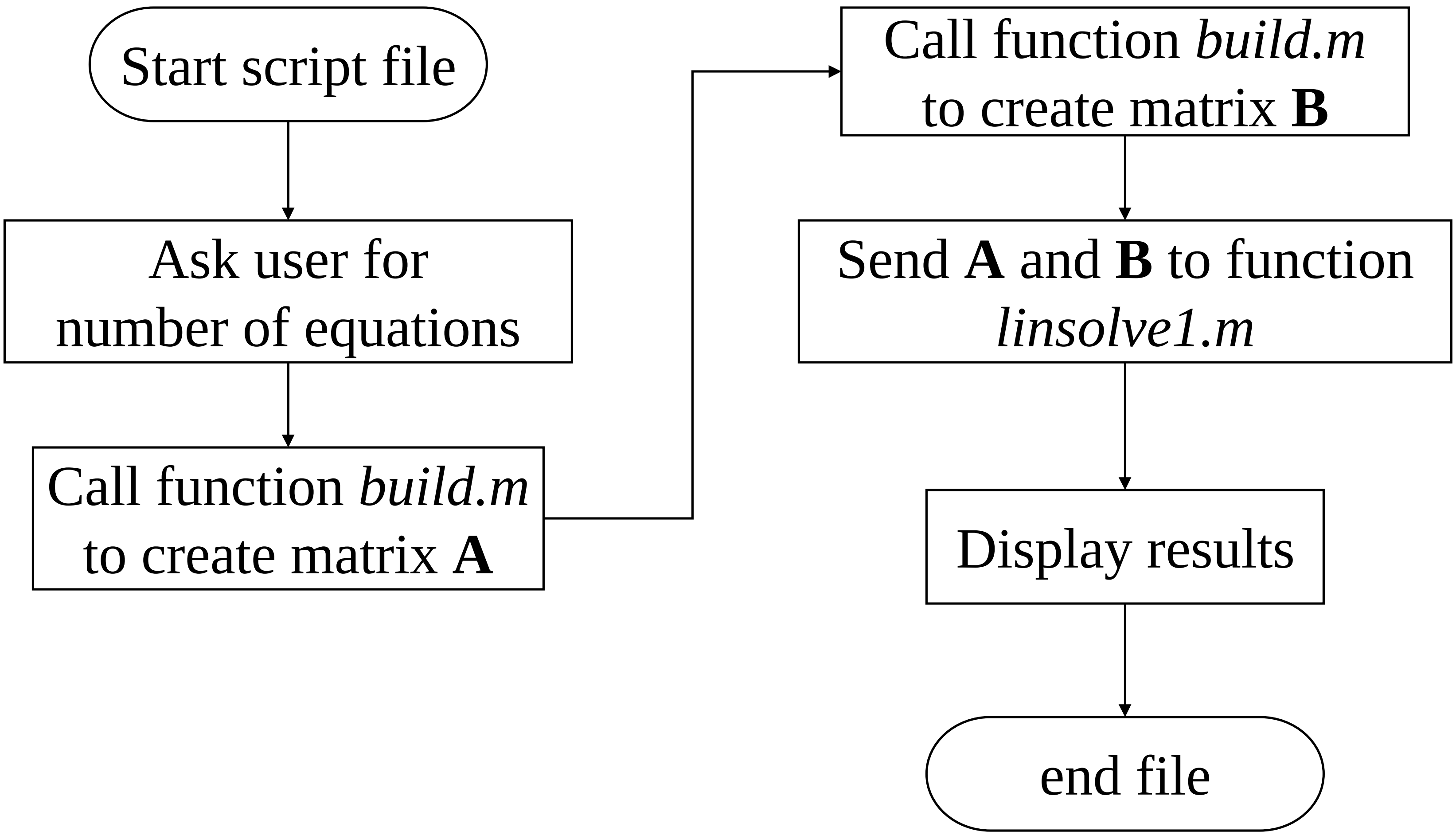
$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

In matrix **A**, m is the index that identifies the row and n is the index that identifies the column. Thus, the requirement that the number of unknowns must equal the number of equations in order for a unique solution to exist, is at the root of matrix multiplication. i.e. m must be equal to n

Solution Techniques

- One method of solving involves successive elimination of variables until only one equation and one unknown variable remains. *Gauss Elimination*
- *Cramer's Method* is based on finding matrix determinants for the system
- Another technique particularly suited to MATLAB is based on the matrix inverse method

Solution of Linear System Using MATLAB



Script matalg.m

- Calls the function build.m *twice*
  - *build.m performs a dedicated task to inout data*

- Calls a separate function, linsolve1.m to do the dedicated task of computing the solution

- Displays the answer

Function build.m

- Function uses a for loop to iterate through matrix position.

Function linsolve1.m

- This function introduces use of the MATLAB backslash(\ ), matrix operator to solve linear systems of the general form:

$$\mathbf{Ax} = \mathbf{b}$$

MATLAB Demo

run matalg.m