Review

- **ldgvdw.m**
  - » Inputs: *vol_temp.txt*, molecular weight, two van der Walls constants
  - » Outputs: plot of pressure vs. temperature for ideal gas & a plot of "*Z*" vs. pressure for a van der Waals gas

$$Z = \frac{p\bar{v}}{\bar{R}T}$$

Statistics

E. F. Thacher

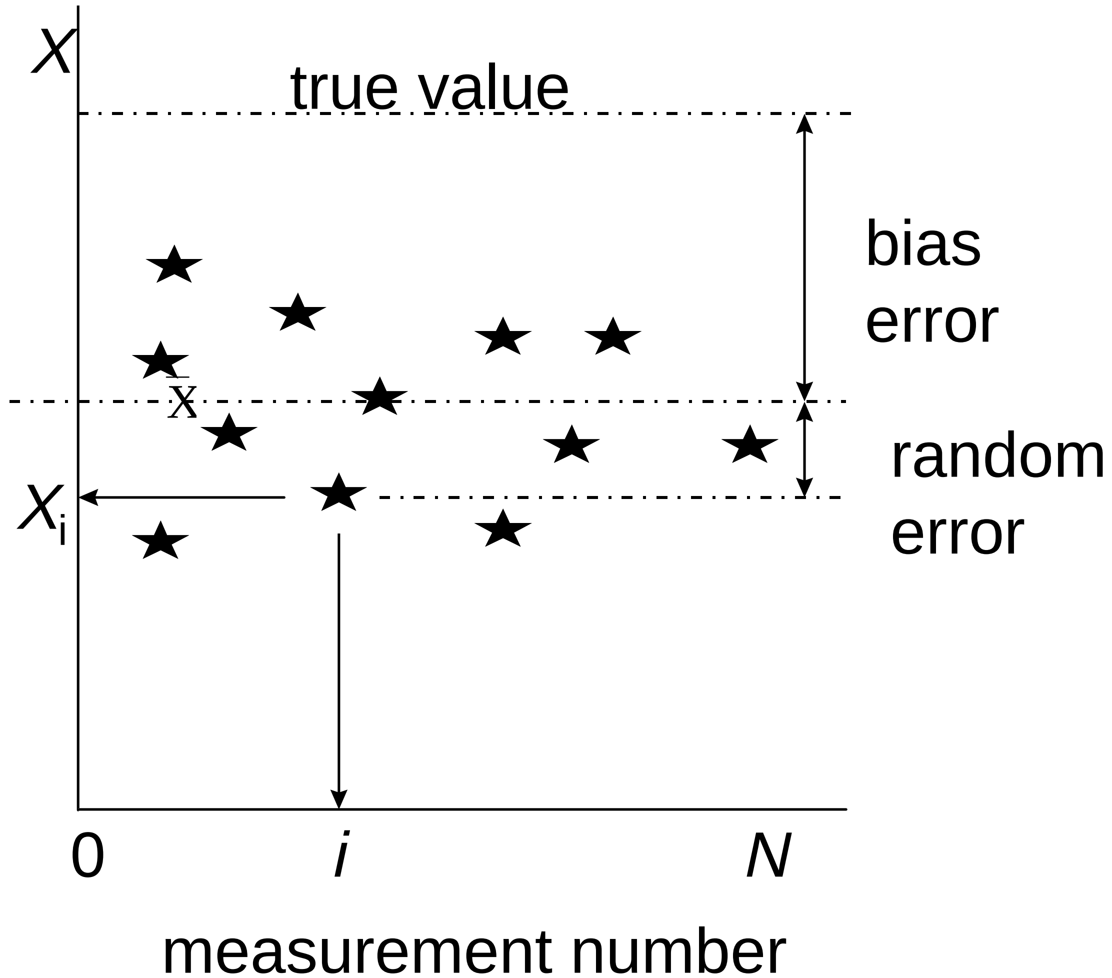
Launcher Design

- How precise must the launch angle be so that the range lies within a specified error?

Error Classification

- Random error (scatter)
  - » Random fluctuations in measurement conditions
  - » Noise introduced by signal processing
- Bias error (constant offset)
  - » Poor calibration, laboratory conditions, etc.
  - » Built into model
- Both propagate through model

Bias and Random Error



Some Relevant Statistics

- Mean value

$$\bar{\theta}_0 = \frac{1}{N}\sum_{i=1}^{N}\theta_{0i}$$

- Standard deviation

$$s_{\theta_0} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(\theta_{0_i} - \bar{\theta}_0)^2}$$

Applications

- Probabilistic design
- Statistical process control
- Turbulence research

MATLAB: Pressure Statistics

Measurement	p (psi)
1	4.95
2	4.87
3	5.1
4	4.99
5	5.06
6	5.09
7	4.95
8	5.01
9	4.98
10	4.99
11	5.0
12	5.05
13	5.0
14	5.02
15	5.01
16	5.06
17	4.99
18	5.01
19	4.9

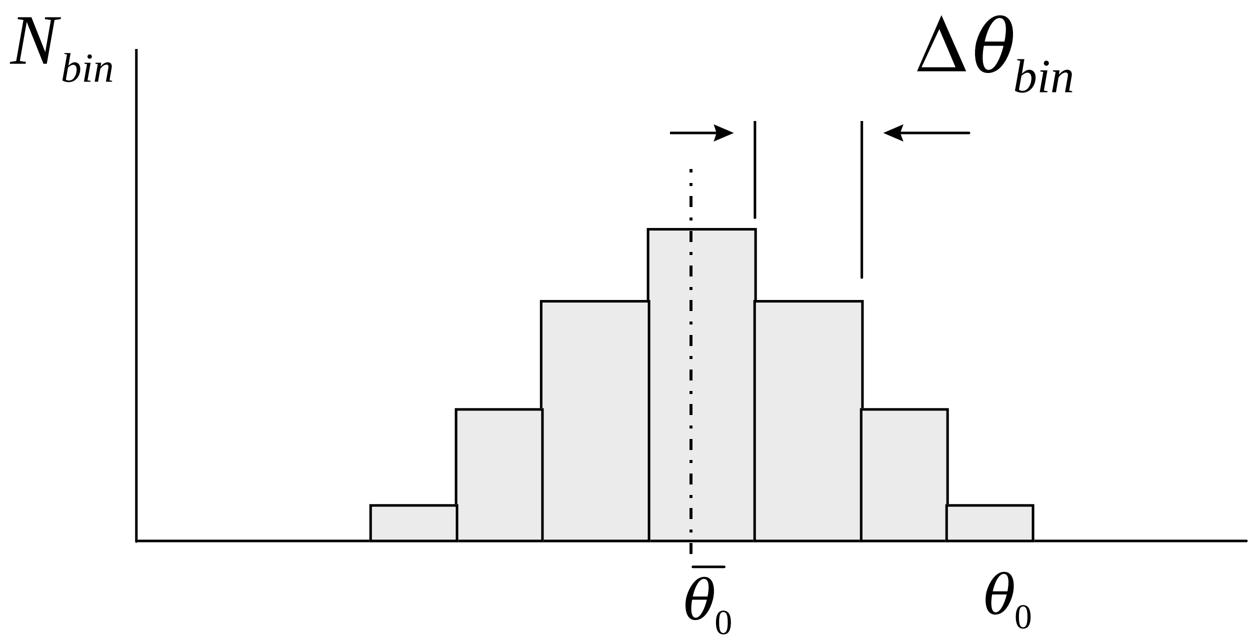
Use stats.m

MATLAB:
Mean and Sigma

- **stats.m**
  - » Inputs: vector of data
  - » Outputs: mean and standard deviation of the data

Histogram

- After N measurements



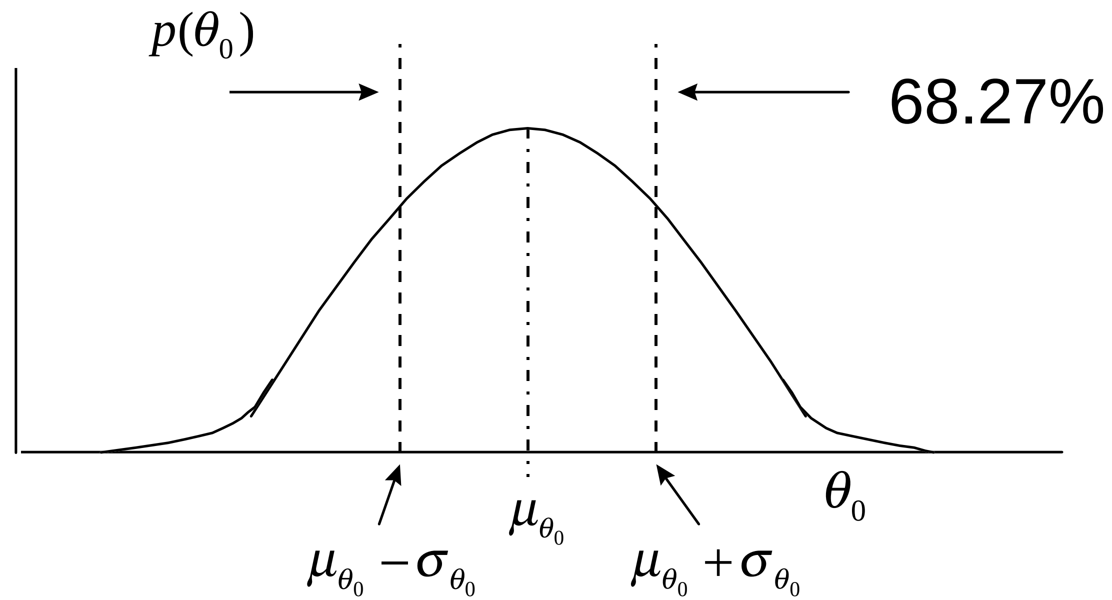
MATLAB: Histograms

- histdemo.m
  - » <u>Inputs</u>: *p_data.txt*, number of bins
  - » Outputs: two histograms

- hgramf.m
  - » <u>Inputs</u>: data vector, number of bins
  - » Outputs: two histograms

Normal Distribution

- After <u>many</u> measurements…



$$P(\bar{\theta}_0 - \sigma_{\theta_0} \le \theta_0 \le \bar{\theta}_0 + \sigma_{\theta_0}) = 68.27\%$$

Normal Distribution Function

$$p(\theta) = \frac{1}{\sigma_{\theta_0}\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\theta_0 - \mu_{\theta_0}}{\sigma} \right)^2 \right]$$

Finite Statistics Estimate Population Statistics

- Standard deviation measures spread
- Mean is most probable value
- As *N* becomes large

$$s_{\theta_0} \to \sigma_{\theta_0}$$

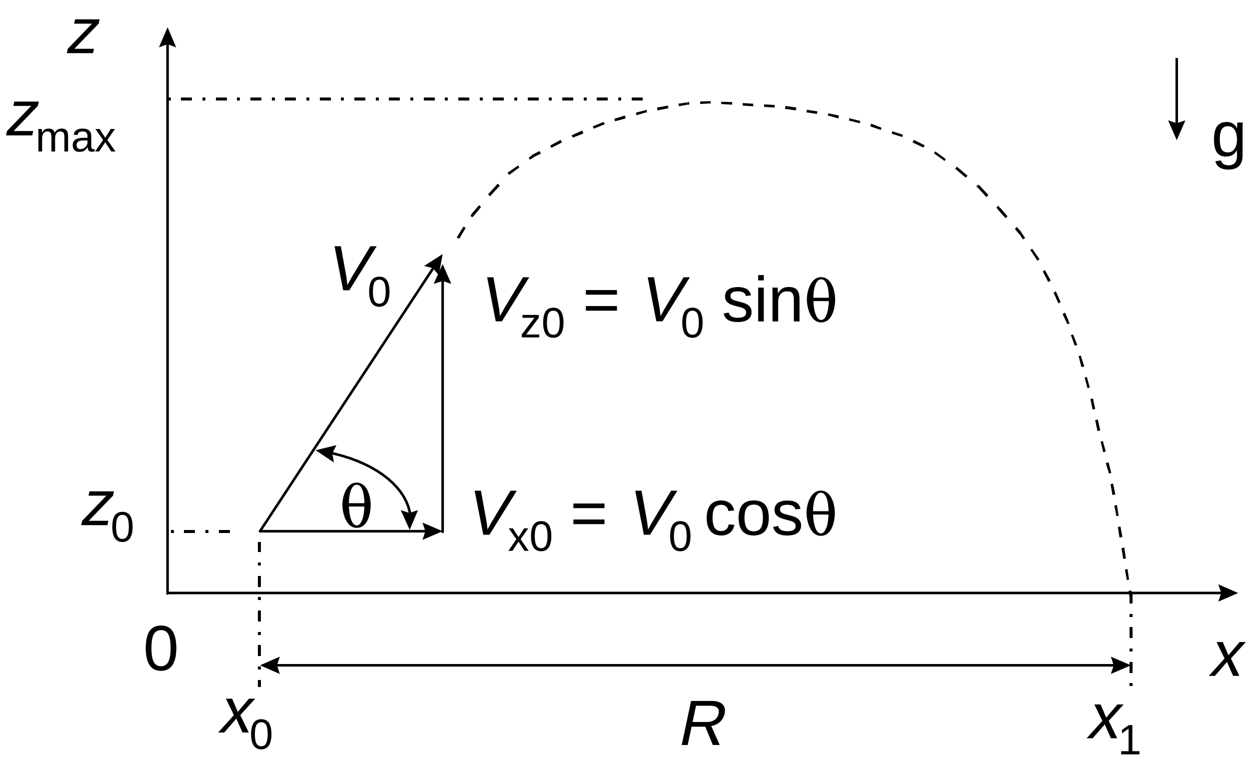
$$\overline{\theta}_0 \to \mu_{\theta_0}$$

MATLAB: Effect of Sigma

- **normplt.m**
  - » Uses **genx.m** to generate sampled values
  - » Uses overlay plotting
  - » Uses **`gtext`** command to label plots
- Inputs: vector of data
- Outputs: three overlaid plots of normal p.d.f. (one for each of three sigma values)

Launch Problem

- <u>Case</u>: vacuum



Range

When $z = 0$, $x = x_1$ and $t = t_1$. Therefore

$$-\frac{1}{2} g t_1^{\,2} + V_{z0} t_1 + z_0 = 0$$

$$t_1 = \frac{V_{z0}}{g} + \sqrt{\left(\frac{V_{z0}}{g}\right)^2 + \frac{2 z_0}{g}}$$

$$R = x_1 - x_0 \qquad R = V_{x0} t_1$$

Solved by **range0.m**

MATLAB: Range Solver

- Inputs: V_0 , θ_0 , x_0 , z_0
  - » Launch angle is a vector
- Outputs: vector ***R*** containing a range for each launch angle

MATLAB: Normal Statistics

- *r* = `randn`(1,n)
  - » Generates *n* random numbers in vector *r* from normal p.d.f. with mean of 0 and sigma of 1 $\bar{x}, \sigma$
- Transform to $x = \sigma r + \bar{x}$
- Use to do simulated pointing experiment

MATLAB: Pointing Experiment

- **genstat.m**
  - » commands used: **`clc`, `disp`, `input`, `randn`, `fprintf`**
  - » functions used: **range0.m**, **stats.m**
- Inputs: number of samples, mean and sigma desired, V_0, x_0, z_0
- Outputs: *mean_of_R*, *sigma_of_R*