The Ideal Gas Model and van der Waals Equation

ES100

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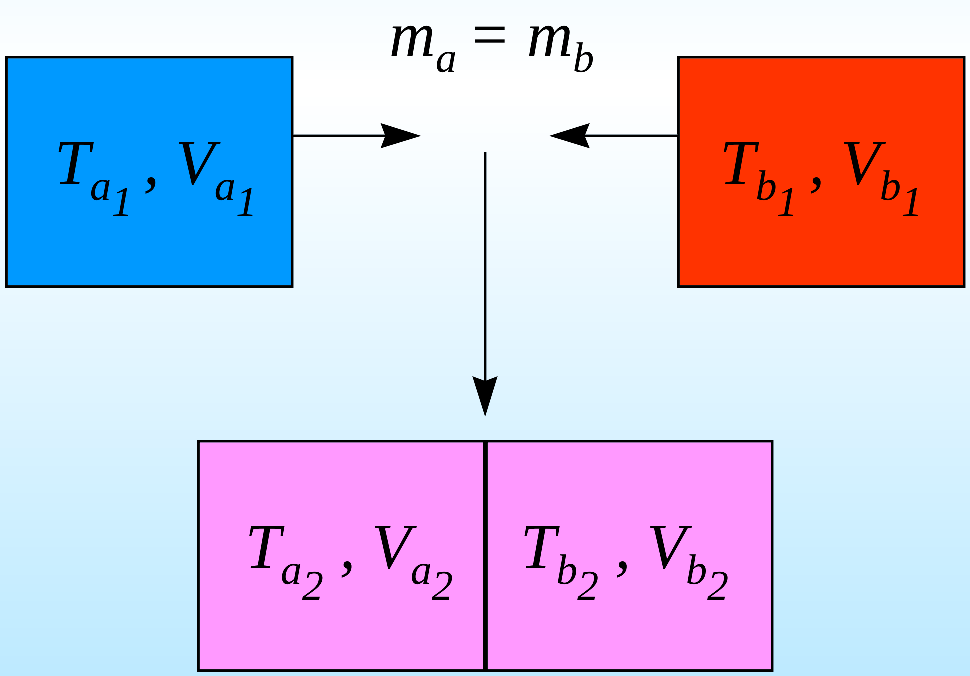
Topics To Be Introduced

- Thermodynamic equilibrium
- *p-v-T* surface
- Equations of State
- Universal Gas Constant
- Ideal Gas Model
- van der Waals Equation
- File types
- load command
- MATLAB *script* files
- MATLAB *function* files

Thermodynamic Process

Two copper blocks with different temperatures come into contact:



$$T_{a_1} < T_{b_1}$$

$$V_{a_1} < V_{b_1}$$

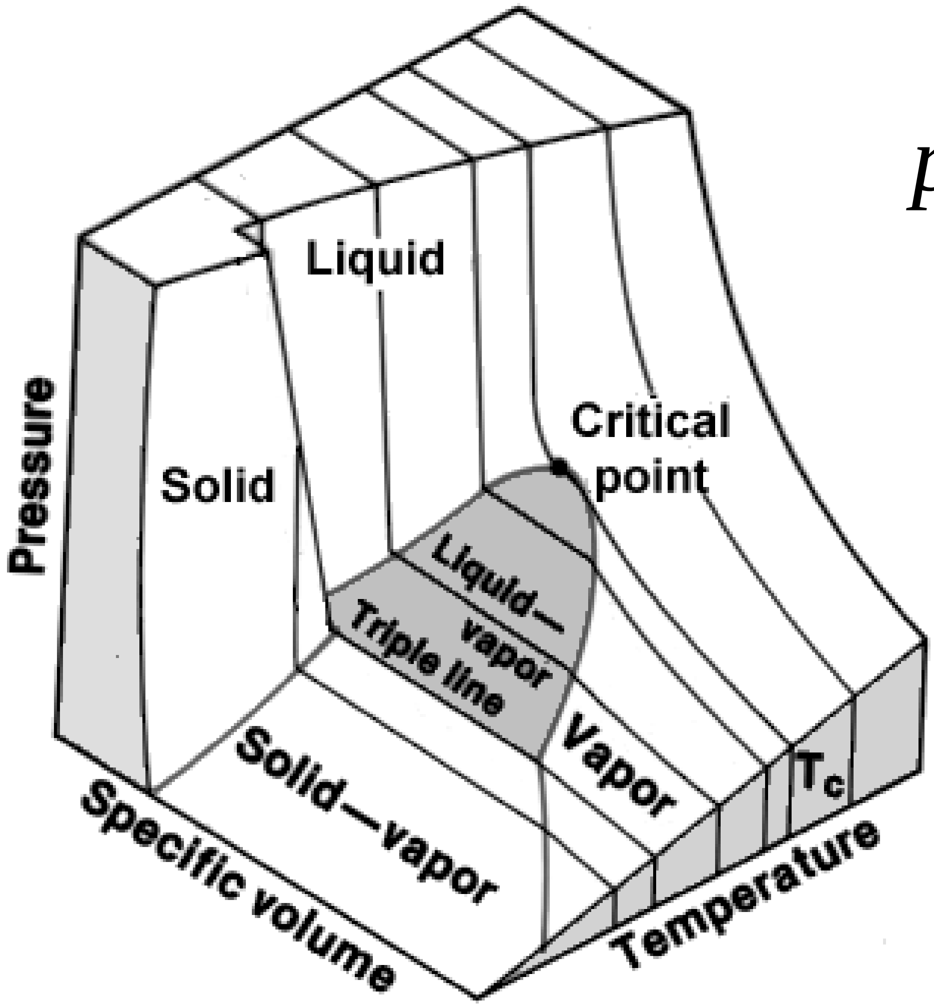
$$T_{a_2} = T_{b_2}$$

$$V_{a_2} = V_{b_2}$$

Thermodynamic Equilibrium

- *Thermodynamic equilibrium* is achieved when changes in thermodynamic properties cease. (i.e. pressure, volume, and temperature)
- In the example, when equilibrium is achieved there are also no further time dependent changes. Thus, the system is also at *steady state*.
- Steady state and thermodynamic equilibrium are *not* always mutual conditions.
- *Equations of state* are reserved for systems in thermodynamic equilibrium.

p-v-T Relationship of Water

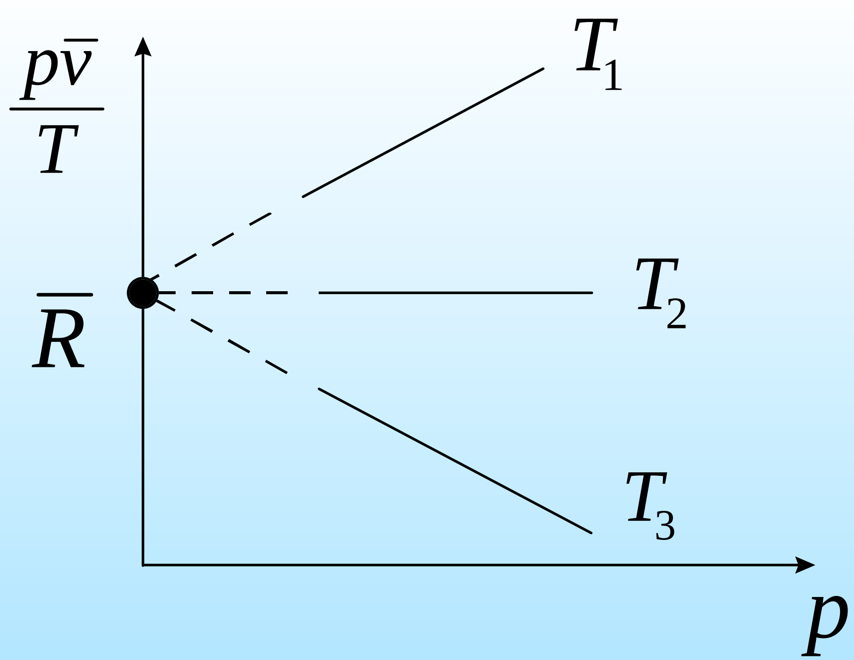


$$p = p(v, T)$$

Universal Gas Constant

Let $p = p(\bar{v}, T)$ where $\bar{v}$ is volume per mole.

If experimental values of $p\bar{v}/T$ are plotted vs. pressure for various temperatures and extrapolated to zero pressure, they converge at a point.



$$\lim_{p \to 0} \frac{p\bar{v}}{T} = \bar{R}$$

Values for the Gas Constant

$$\bar{R} = \begin{cases} 8.314\ \text{kJ} / \text{kmol} \cdot \text{K} \\ 1.986\ \text{Btu} / \text{lbmol} \cdot {}^\circ\text{R} \\ 1545\ \text{ft} \cdot \text{lbf} / \text{lbmol} \cdot {}^\circ\text{R} \end{cases}$$

Ideal Gas Model

The ratio $Z = p\bar{v}/\bar{R}T$ is a dimensionless term for which a special case arises when $Z = 1$. Under this condition, we can write $p = p(\bar{v}, T)$ as

$$\frac{p\bar{v}}{T} = \bar{R} \qquad (1)$$

The above equation is the *Ideal Gas Model*

Other Expressions of the Ideal Gas Model

$$\frac{p\bar{v}}{T} = \bar{R} \qquad (1)$$

specific volume, *v*, is equal to,

$$v = \frac{\bar{v}}{M}$$ where *M* is molecular weight

thus, dividing (1) by *M* yields

$$pv = \frac{\bar{R}}{M}T \qquad (2)$$

Other Expressions cont.(1)

$$\frac{\bar{R}}{M} = R \qquad (3)$$

substituting (3) into (2) gives,

$$pv = RT \qquad (4)$$

Specific volume can also be represented in terms of whole volume and mass as, $v = V/m$

Multiplying (4) by m

$$pV = mRT \qquad (5)$$

Other Expressions cont.(2)

Using (3),

$$pV = \frac{m}{M}\bar{R}T$$

where *m/M* is equal to *n*, the number of moles

Finally,

$$pV = n\bar{R}T \qquad (6)$$

van der Waals Equation

The ideal gas equation of state is an accurate model to use for a number of gases over a wide range of conditions but it is still an approximation. Another equation of state, used in an attempt to more accurately model the thermodynamic state of a gas, was proposed by van der Waals (1873).

$$p = \frac{\overline{R}T}{\overline{v} - b} - \frac{a}{\overline{v}^2} \qquad (7)$$

Physically speaking, the two constants *a* and *b* are used to correct for the volume occupied by molecules themselves and the forces of attraction between them.

File Types used by MATLAB

- *.m files - *store a series of commands*
- *.mat files - *store variable names and values*
- ascii data files
  - text (i.e. *diary* files)
  - external data

Use the `path` and `addpath` commands to assisst MATLAB in locating and accessing your MATLAB files

MATLAB *.m-files

- Script files
  - Stores a set of executable commands
- Function files
  - Similar to script files but variables are local inside the function. This feature allows the user to include *calls* to a function file, from within a script file, without affecting the vales of variable used in the script file.