1. [B&V, problem 2.8] Which of the following sets are polyhedra? Explain.
   (a) $S = \{x \in \mathbb{R}^n \mid x \succeq 0,\ \mathbb{1}^T x = 1,\ \sum_{i=1}^n x_i a_i = b_1,\ \sum_{i=1}^n x_i a_i^2 = b_2\}$, where $a_1, \ldots, a_n \in \mathbb{R}$ and $b_1, b_2 \in \mathbb{R}$.
   (b) $S = \{x \in \mathbb{R}^n \mid x \succeq 0,\ x^T y \leq 1$ for all y with $\|y\|_2 = 1\}$.
   (c) (Extra problem; will not be graded) $S = \{x \in \mathbb{R}^n \mid x \succeq 0,\ x^T y \leq 1$ for all y with $\sum_{i=1}^n |y_i| = 1\}$.

2. [B&V, problem 2.10] Let $C \subseteq \mathbb{R}^n$ be the solution set of a quadratic inequality,

$$C = \{x \in \mathbb{R}^n \mid x^T A\, x + b^T x + c \leq 0\},$$

   with $A \in \mathbf{S}^n$, $b \in \mathbb{R}^n$, and $c \in \mathbb{R}$. Show that C is convex if $A \succeq 0$.

3. [B&V, problem 2.12] Which of the following sets are convex?
   (a) A *slab*, i.e., a set of the form $\{x \in \mathbb{R}^n \mid \alpha \leq a^T x \leq \beta\}$.
   (b) A *wedge*, i.e., a set of the form $\{x \in \mathbb{R}^n \mid a_1^T x \leq b_1,\ a_2^T x \leq b_2\}$. What conditions on a_i and/or b_i would guarantee that this set is also a cone?
   (c) The set of points closer to a given point than a given set, i.e., $\{x \in \mathbb{R}^n \mid \|x - x_0\|_2 \leq \|x - y\|_2$ for all $y \in S\}$ where $S \subseteq \mathbb{R}^n$.
   (d) The set of points whose distance to a does not exceed a fixed fraction θ of the distance to b, i.e., $\{x \in \mathbb{R}^n \mid \|x - a\|_2 \leq \theta \|x - b\|_2\}$. Assume that $a \neq b$ and $0 \leq \theta \leq 1$.

4. Let $\mathcal{B}(x_c, r)$ denote a norm ball in $\mathbb{R}^n$ with center x_c and radius $r > 0$, i.e.,

$$\mathcal{B}(x_c, r) = \{x \in \mathbb{R}^n \mid \|x - x_c\| \leq r\},$$

   where $\|\cdot\|$ is *any* norm on $\mathbb{R}^n$. Use the properties of norms given in section A.1.2 of the appendix in your textbook to prove that $\mathcal{B}(x_c, r)$ is convex.