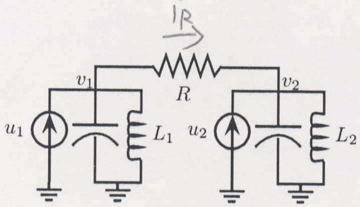
Solutions to Homework 4

①



$C=1$.

$$\dot{V}_1 + i_{L_1} + i_R = u_1$$

$$\begin{cases} \dot{V}_2 + i_{L_2} - i_R = u_2 \\ i_R = \dfrac{V_1 - V_2}{R} \\ L_1 \dot{i}_{L_1} = V_1 \\ L_2 \dot{i}_{L_2} = V_2 . \end{cases}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \int_0^t V_1(\tau)\,d\tau \\ \int_0^t V_2(\tau)\,d\tau \\ V_1(t) \\ V_2(t) \end{bmatrix}$$

$$\Rightarrow \begin{cases} \dot{x}_1 = x_3 \\ \dot{x}_2 = x_4 \\ \dot{x}_3 = u_1 - \dfrac{x_3 - x_4}{R} - \dfrac{x_1}{L_1} \\ \dot{x}_4 = u_2 + \dfrac{x_3 - x_4}{R} - \dfrac{x_2}{L_2} \end{cases}$$

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{1}{L_1} & 0 & -\frac{1}{R} & \frac{1}{R} \\ 0 & -\frac{1}{L_2} & \frac{1}{R} & -\frac{1}{R} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} .$$

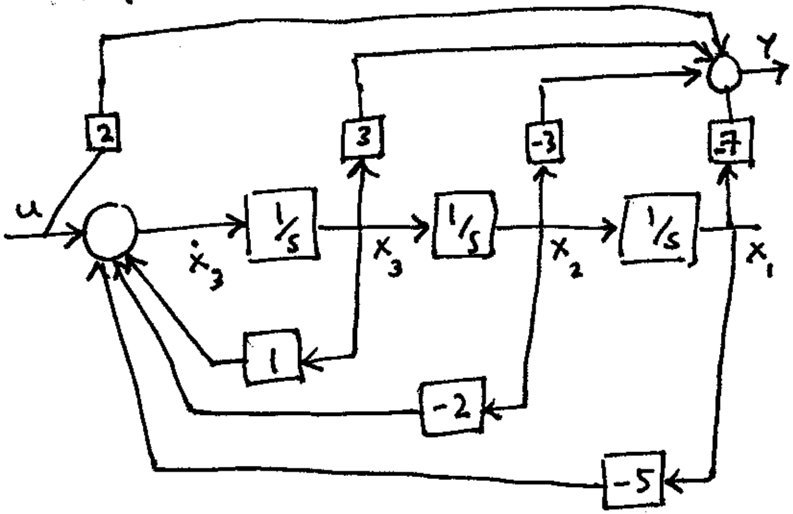
$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + 0 \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}.$$

$$\therefore A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{1}{L_1} & 0 & -\frac{1}{R} & \frac{1}{R} \\ 0 & -\frac{1}{L_2} & \frac{1}{R} & -\frac{1}{R} \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad D = 0.$$

(2)

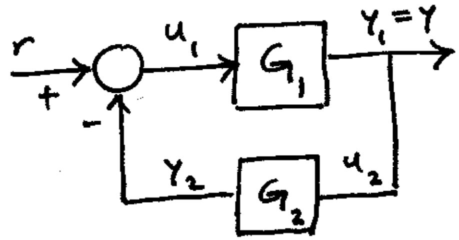
$$G(s) = \frac{2s^3 + s^2 + s + 3}{s^3 - s^2 + 2s + 5} = 2 + \frac{3s^2 - 3s - 7}{s^3 - s^2 + 2s + 5}$$



$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -2 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} -7 & -3 & 3 \end{bmatrix} x + 2u$$

③



$$\begin{aligned}\dot{x}_1 &= A_1 x_1 + B_1 u_1 \\ &= A_1 x_1 + B_1 (r - Y_2) \\ &= A_1 x_1 + B_1 r - B_1 C_2 x_2 \qquad (*)\end{aligned}$$

$$Y_1 = C_1 x_1$$

$$\begin{aligned}\dot{x}_2 &= A_2 x_2 + B_2 u_2 \\ &= A_2 x_2 + B_2 Y_1 \\ &= A_2 x_2 + B_2 C_1 x_1 \qquad (**)\end{aligned}$$

$$Y_2 = C_2 x_2$$

$(*)$ & $(**)$ $\Rightarrow$ $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_1 & -B_1 C_2 \\ B_2 C_1 & A_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} r$$

$$Y = \begin{bmatrix} C_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

④

$$G(s) = [c \quad q]\left(sI - \begin{bmatrix} A & A_1 \\ 0 & A_2 \end{bmatrix}\right)^{-1}\begin{bmatrix} b \\ 0 \end{bmatrix}$$

$$= [c \quad q]\left(\begin{bmatrix} sI & 0 \\ 0 & sI \end{bmatrix} - \begin{bmatrix} A & A_1 \\ 0 & A_2 \end{bmatrix}\right)^{-1}\begin{bmatrix} b \\ 0 \end{bmatrix}$$

$$= [c \quad q]\begin{bmatrix} sI-A & -A_1 \\ 0 & sI-A_2 \end{bmatrix}^{-1}\begin{bmatrix} b \\ 0 \end{bmatrix}$$

$$= [c \quad q]\begin{bmatrix} (sI-A)^{-1} & (sI-A)^{-1}A_1(sI-A_2)^{-1} \\ 0 & (sI-A_2)^{-1} \end{bmatrix}\begin{bmatrix} b \\ 0 \end{bmatrix}$$

$$= [c \quad q]\begin{bmatrix} (sI-A)^{-1}b \\ 0 \end{bmatrix} = c(sI-A)^{-1}b$$

$$G(s) = [c \quad 0]\left(sI - \begin{bmatrix} A & 0 \\ A_1 & A_2 \end{bmatrix}\right)^{-1}\begin{bmatrix} b \\ q \end{bmatrix}$$

$$= [c \quad 0]\left(\begin{bmatrix} sI & 0 \\ 0 & sI \end{bmatrix} - \begin{bmatrix} A & 0 \\ A_1 & A_2 \end{bmatrix}\right)^{-1}\begin{bmatrix} b \\ q \end{bmatrix}$$

$$= [c \quad 0]\begin{bmatrix} sI-A & 0 \\ -A_1 & sI-A_2 \end{bmatrix}^{-1}\begin{bmatrix} b \\ q \end{bmatrix}$$

$$= [c \quad 0]\begin{bmatrix} (sI-A)^{-1} & 0 \\ (sI-A_2)^{-1}A_1(sI-A)^{-1} & (sI-A_2)^{-1} \end{bmatrix}\begin{bmatrix} b \\ q \end{bmatrix}$$

$$= [c(sI-A)^{-1} \quad 0]\begin{bmatrix} b \\ q \end{bmatrix} = c(sI-A)^{-1}b$$

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