Sol to Hw 3

52.

An input r_1 yields $c_1 = 5r_1+7$. An input r_2 yields $c_2 = 5r_2 +7$. An input $r_1 +r_2$ yields, $5(r_1+r_2)+7 = 5r_1+7+5r_2 = c_1+c_2-7$. Therefore, not additive. What about homogeneity? An input of Kr_1 yields $c = 5Kr_1+7 \neq Kc_1$. Therefore, not homogeneous. The system is not linear.

53.

a. Let $x = \delta x+0$. Therefore,

$$\ddot{\delta x} + 3\dot{\delta x} + 2\delta x = \sin(0+\delta x)$$

But, $\sin(0+\delta x) = \sin 0 + \left.\frac{d \sin x}{dx}\right|_{x=0} \delta x = 0 + \cos x \Big|_{x=0} \delta x = \delta x$

Therefore, $\ddot{\delta x} + 3\dot{\delta x} + 2\delta x = \delta x$. Collecting terms, $\ddot{\delta x} + 3\dot{\delta x} + \delta x = 0$

55.

The given curve can be described as follows:

$f(x) = -6$; $-\infty<x<-3$;

$f(x) = 2x$; $-3<x<3$;

$f(x) = 6$; $3<x<+\infty$

Thus,

a. $\ddot{x}+17\dot{x}+50x=-6$

b. $\ddot{x}+17\dot{x}+50x=2x$ or $\ddot{x}+17\dot{x}+48x=0$

c. $\ddot{x}+17\dot{x}+50x=6$

56.

The relationship between the nonlinear spring's displacement, $x_s(t)$ and its force, $f_s(t)$ is

$$x_s(t)=1-e^{-f_s(t)}$$

Solving for the force, $f_s(t)=-\ln(1-x_s(t))$ (1)

Writing the differential equation for the system by summing forces,

$$2\frac{d^2x(t)}{dt^2}+2\frac{dx(t)}{dt}-\ln(1-x(t))=f(t) \quad (2)$$

Letting $x(t) = x_0 + \delta x$ and $f(t) = 1 + \delta f$, linearize $\ln(1 - x(t))$.

$$\ln(1-x)-\ln(1-x_0)=\left.\frac{d\ln(1-x)}{dx}\right|_{x=x_0}\delta x$$

Solving for $\ln(1-x)$,

$$\ln(1-x)=\ln(1-x_0)-\left.\frac{1}{1-x}\right|_{x=x_0}\delta x=\ln(1-x_0)-\frac{1}{1-x_0}\delta x \qquad (3)$$

When f = 1, δx = 0. Thus from Eq. (1), $1 = -\ln(1-x_0)$.

Solving for x_0, $1 - x_0 = e^{-1}$, or $x_0 = 0.6321$.

Substituting $x_0 = 0.6321$ into Eq. (3),

$$\ln(1-x)=\ln(1-0.6321)-\frac{1}{1-0.6321}\delta x=-1-2.718\delta x$$

Placing this value into Eq. (2) along with $x(t) = x_0 + \delta x$ and $f(t) = 1 + \delta f$, yields the linearized differential equation, $2\frac{d^2\delta x}{dt^2}+2\frac{d\delta x}{dt}+1+2.718\delta x=1+\delta f$

or

$$2\frac{d^2\delta x}{dt^2}+2\frac{d\delta x}{dt}+2.718\delta x=\delta f$$

Taking the Laplace transform and rearranging yields the transfer function,

$$\frac{\delta x(s)}{\delta f(s)}=\frac{1}{2s^2+2s+2.718}$$