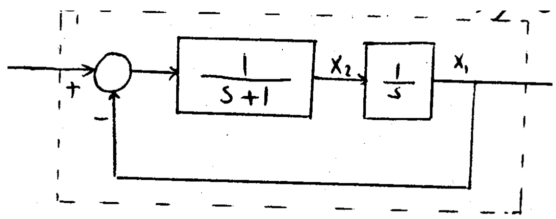
1. Write the state-space equations for the system shown in the figure below, using x_1 and x_2 as state variables. Next, design an observer for this system. Place both modes of the observer at $s = -5$.



2. In this problem we want to use a radar, that makes distance measurements, to estimate the velocity of an enemy ship. Let $d(t)$ denote the distance of the ship from shore, and let $v(t)$ denote its velocity. We assume that the ship is cruising at constant velocity. Then the equations describing the ship dynamics are $-\dot{d}(t) = v(t)$ and $\dot{v}(t) = 0$. Equivalently, in state-space form we have

$$\begin{bmatrix} \dot{d}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} d(t) \\ v(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} d(t) \\ v(t) \end{bmatrix},$$

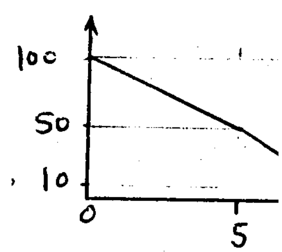
where $y(t)$ is the radar measurement of the ship's distance.

(a) For this system, design an observer whose modes are both at $s = -5$.

(b) Compute the observer transfer function (i.e., the 2-by-1 transfer function from $y(s)$ to $\begin{bmatrix} \hat{d}(s) \\ \hat{v}(s) \end{bmatrix}$).

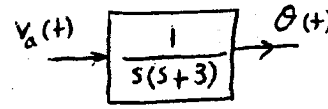
$*$(c) Assume that the ship is approaching the shore and its distance $y(t)$, as measured by the radar, is given by $y(t) = 100 - 10\,t$; see figure below. Furthermore, assume that when the observer is turned on at time $t = 0$ it has initial conditions $\hat{d}(0) = 100$ and $\hat{v}(0) = 0$. Using MATLAB, plot the values of the observer states $\hat{d}(t)$ and $\hat{v}(t)$ for $0 \leq t \leq 5$.

$*$(d) Comment on the difference between the plots of the estimated states $\hat{d}(t), \hat{v}(t)$ versus the plots of the actual states $d(t), v(t)$.



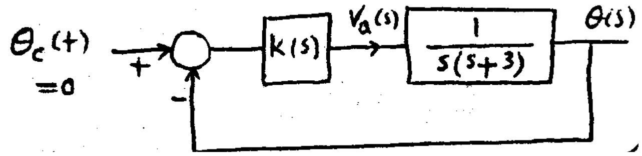
Hint: For Part (a), note that the input u is zero and therefore the observer is characterized by a state-space description that relates y to $\begin{bmatrix} \hat{d} \\ \hat{v} \end{bmatrix}$. Taking a Laplace transform gives the transfer function of the observer. For Part (c), you can use the MATLAB command `lsim` to simulate the observer; `lsim(A,B,C,0,u,t,x0)` simulates an LTI system with state-space matrices A, B, C (assuming $D = 0$), input signal u, and initial condition x_0. Type `help lsim` in the MATLAB command window for more information. Keep in mind that the input to the observer is the measurement $y(t) = 100 - 10\,t$. For Part (d), note that from the state-space equations it follows that $y(t) = d(t)$, and therefore the actual distance $d(t)$ is $100 - 10\,t$. Furthermore, since $v(t) = -\dot{d}(t)$, the actual velocity $v(t)$ is 10. Now compare these actual values to those estimated by the observer and comment on their difference.

3. In this problem we want to create a servomechanism with an electric motor. Consider a DC motor where v_a is the voltage across the rotor coil and θ is the angle of the motor shaft.



(a) Using the angle θ and the angular velocity $\omega = \dot{\theta}$ as the state variables, write the state-space equations describing the motor.

(b) Consider the closed-loop system of the figure below, and let the controller $K(s)$ be a constant gain k. Draw the root-locus plot for $k > 0$.



(c) Using the state feedback $v_a(t) = -k_1\theta(t) - k_2 w(t)$, place the closed-loop poles at $-5 \pm 5j$.

(d) Now design an observer which, using the measurement θ, estimates θ and ω. Choose the observer gain such that the poles of the observer error dynamics are located at $-20 \pm 20j$.

(e) Combine the observer and state feedback to find the observer-based controller $K(s) = k^T(sI - A + bk^T + lc^T)^{-1}l$.

$*$(f) Find the transfer function from θ_c to θ. Is the output angle $\theta(t)$ capable of tracking nonzero step commands given by θ_c? Explain.