1. Consider the system

$$\dot{x} = \begin{bmatrix} -1/2 & -3/2 \\ -3/2 & -1/2 \end{bmatrix} x + \begin{bmatrix} 2 \\ 2 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1/2 & 1/2 \end{bmatrix} x.$$

   (a) Diagonalize the system using an appropriate coordinate trasformation.

   (b) Is the system stable?

   (c) Using the diagonal representation you obtained, can you say which mode of the system is controllable? Which mode is observable?

   (d) Show that the transfer function of the system only depends on the mode that is both controllable and observable.

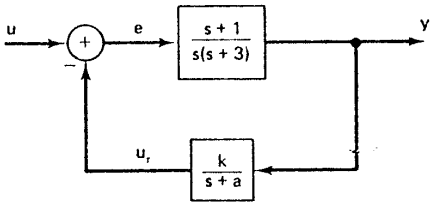
2. Consider the system illustrated in the figure.

   (a) Find a state-space representation for this system. Do this by first finding a state-space realization for system $G_1(s) = (s+1)/(s(s+3))$ in the forward path and system $G_2(s) = k/(s+a)$ in the feedback path, and then using

$$A = \begin{bmatrix} A_1 & -B_1C_2 \\ B_2C_1 & A_2 \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} C_1 & 0 \end{bmatrix}.$$

   (Recall that the above expressions for the A, B, C matrices of a feedback interconnection were derived as a homework problem in HW 2.)

   (b) Form the controllability and observability matrices, and show that one or both of these matrices lose rank for $a = 1$, i.e., when $a = 1$ the system is either uncontrollable or unobservable or both.



3. Consider the system illustrated in the figure and choose the state variables as shown.

   (a) Write the state equations for the state variables x_1 and x_2 defined in the figure.

   (b) Is this realization controllable? Is it observable?

   (c) Write the transfer function from u to y.