1. Consider the system

$$\dot{x} = \begin{bmatrix} -7 & -2 & 6 \\ 2 & -3 & -2 \\ -2 & -2 & 1 \end{bmatrix} x + \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} -1 & -1 & 2 \\ 1 & 1 & -1 \end{bmatrix} x.$$

   Check the controllability of this system using the controllability matrix.

2. Consider the system

$$\dot{x} = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} x + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} u$$
$$y = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} x.$$

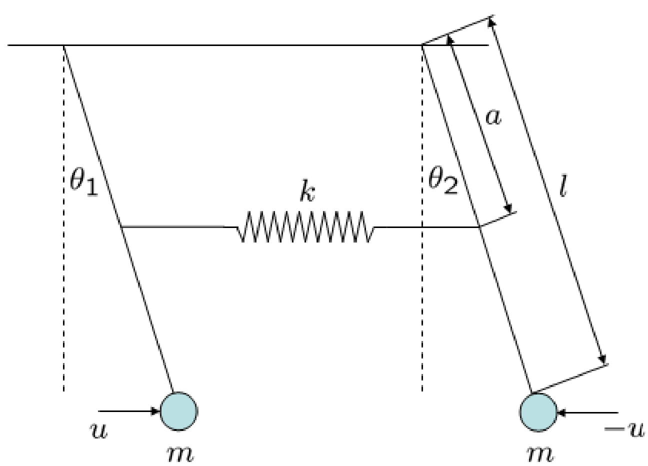
   Is this system controllable? What is the rank of the controllability matrix? What is the column space of the controllability matrix?

3. Two pendulums, coupled by a spring, are to be controlled by two equal and opposite forces u as shown in the figure below.

   The linearized equations of motion are given by

$$m\,l^2\,\ddot{\theta}_1 = -k\,a^2\,(\theta_1 - \theta_2) - m\,g\,l\,\theta_1 - u,$$
$$m\,l^2\,\ddot{\theta}_2 = -k\,a^2\,(\theta_2 - \theta_1) - m\,g\,l\,\theta_2 + u.$$

   (a) Determine the A and B matrices for a state-space representation of the system.
   (b) Is the system controllable?
   (c) Determine the column space of the controllability matrix.
   (d) If the observed output is θ_1, determine the C matrix for your state-space realization.
   (e) Assume $m = 1$, $l = 1$, $a = 0.5$, $k = 4$, $g = 10$. Is the system stable? (For this part you will have to find the eigenvalues of a 4-by-4 matrix. You can use MATLAB's `eig` command to do this.)



4. Kailath, Exercise 2.3-18: Let

$$\mathcal{C}_k = \begin{bmatrix} b & Ab & \cdots & A^{k-1}b \end{bmatrix}.$$

   Show that if $\operatorname{rank}\mathcal{C}_{k+1} = \operatorname{rank}\mathcal{C}_k$ for some k, then $\operatorname{rank}\mathcal{C}_{k+i} = \operatorname{rank}\mathcal{C}_k$ for all $i \geq 1$.

   Hint: Note that the Cayley-Hamilton theorem can not be invoked in this problem, since we do not know the size of the matrix A.