0. Visit (direct link also provided on our class website)

   `http://www.engin.umich.edu/class/ctms/examples/examples.htm`

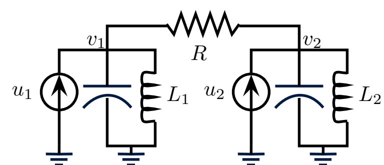
   to learn more about the state-space modeling of physical systems. Click on the pictures of different systems at the very top of the page and study the derivation of both the transfer function and the state-space representation of every system. For most systems, a numerical example with specific parameter values is given and the corresponding MATLAB code is provided.

   You do not have to hand in anything for this problem. However, this problem counts as an additional lecture on state-space modeling. Furthermore, if this is your first exposure to MATLAB, use these examples and the tutorial link on our class website to familiarize yourself with this software.

1. Consider the circuit in the figure below, in which both capacitors have unit capacitance. The current sources $u_1(t)$ and $u_2(t)$ are the inputs to the system and the node voltages $v_1(t)$ and $v_2(t)$ are the outputs. Write the state-space equations that describe the system and find the matrices A, B, C, and D. Choose

$$x(t) = \begin{bmatrix} \int_0^t v_1(\tau)\,d\tau \\ \int_0^t v_2(\tau)\,d\tau \\ v_1(t) \\ v_2(t) \end{bmatrix}$$

   as the system's state vector.



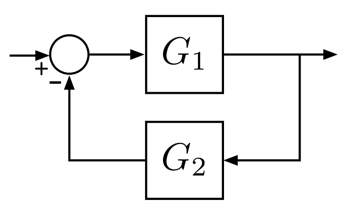
   Hint: Do not be overwhelmed by the integrals in the state vector $x(t)$! Begin by writing current laws for the nodes denoted by v_1 and v_2, as you would in a circuits course. Then identify the $\int_0^t v_i(\tau)\,d\tau$ and $v_i(t)$ terms, and remember that the latter is the derivative of the former. The rest is just algebra.

2. Using the controller canonical form, write a state-space representation for a system whose transfer function is

$$G(s) = \frac{2s^3 + s^2 + s + 3}{s^3 - s^2 + 2s + 5}.$$

   Hint: Note that the orders of the denominator and numerator polynomials are the same. Begin by pulling out a constant term from the fraction, so that the order of the numerator becomes less than that of the denominator.

3. Find a state-space representation for the negative feedback interconnection of two systems G_1 and G_2 (see picture) in terms of the state-space representations of G_1 and G_2. You can assume that D_1 and D_2 are both zero.



4. Kailath, Exercise 2.2-10.b: Realizations can have different numbers of states. Show that the realizations

$$\begin{bmatrix} A & A_1 \\ 0 & A_2 \end{bmatrix}, \quad \begin{bmatrix} b \\ 0 \end{bmatrix}, \quad \begin{bmatrix} c & q \end{bmatrix}$$
$$\begin{bmatrix} A & 0 \\ A_1 & A_2 \end{bmatrix}, \quad \begin{bmatrix} b \\ q \end{bmatrix}, \quad \begin{bmatrix} c & 0 \end{bmatrix}$$

and the realization $\{A, b, c^T\}$ all have the same transfer function for all values and (compatible) dimensions of A_1, A_2, q.

(Make sure to treat A, A_1, A_2 as matrices and *not* as scalars. For this problem, you will need to use the two matrix identities given below

$$\begin{bmatrix} U_1 & U_0 \\ 0 & U_2 \end{bmatrix}^{-1} = \begin{bmatrix} U_1^{-1} & -U_1^{-1}U_0U_2^{-1} \\ 0 & U_2^{-1} \end{bmatrix},$$
$$\begin{bmatrix} U_1 & 0 \\ U_0 & U_2 \end{bmatrix}^{-1} = \begin{bmatrix} U_1^{-1} & 0 \\ -U_2^{-1}U_0U_1^{-1} & U_2^{-1} \end{bmatrix}.)$$