

See attached page (from Norman Nise, *Control Systems Engineering*, Seventh Edition, Wiley, 2015, Chapter 2)

1. Chapter 2, Problem 52.

2. Chapter 2, Problem 53, Part a.

The problem statement in the book is incomplete. It is desired to linearize the differential equation for small excursions *around the nominal point*  $x_0 = 0$ .

3. Chapter 2, Problem 55.

4. Chapter 2, Problem 56.

The problem statement in the book could use some clarification. The differential equation describing the system is

$$2\frac{d^2x(t)}{dt^2} + 2\frac{dx(t)}{dt} - \ln(1 - x(t)) = f(t).$$

The nominal force is  $f_0 = 1$ , which, using the equations of motion in steady-state and with all derivatives set to zero, will give the nominal displacement  $x_0$ . Now, small excursions  $\delta f$  in the input force result in small excursions  $\delta x$  in the output displacement, i.e.,  $f = f_0 + \delta f$  renders  $x = x_0 + \delta x$ . It is desired to linearize the equations around the nominal solution  $(f_0, x_0)$ , and to find  $\delta X(s)/\delta F(s)$ .

50. Find the series and parallel analogs for the translational mechanical system shown in Figure 2.20 in the text. [Section: 2.9]
51. Find the series and parallel analogs for the rotational mechanical systems shown in Figure P2.17(b) in the problems. [Section: 2.9]
52. A system's output,  $c$ , is related to the system's input,  $r$ , by the straight-line relationship,  $c = 5r + 7$ . Is the system linear? [Section: 2.10]
53. Consider the differential equation

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = f(x)$$

where  $f(x)$  is the input and is a function of the output,  $x$ . If  $f(x) = \sin x$ , linearize the differential equation for small excursions. [Section: 2.10]

- a.  $x = 0$   
b.  $x = \pi$

54. Consider the differential equation

$$\frac{d^3x}{dt^3} + 10\frac{d^2x}{dt^2} + 20\frac{dx}{dt} + 15x = f(x)$$

where  $f(x)$  is the input and is a function of the output,  $x$ . If  $f(x) = 3e^{-5x}$ , linearize the differential equation for  $x$  near 0. [Section: 2.10]

55. Many systems are *piecewise* linear. That is, over a *large* range of variable values, the system can be described linearly. A system with amplifier saturation is one such example. Given the differential equation

$$\frac{d^2x}{dt^2} + 17\frac{dx}{dt} + 50x = f(x)$$

assume that  $f(x)$  is as shown in Figure P2.34. Write the differential equation for each of the following ranges of  $x$ : [Section: 2.10]

- a.  $-\infty < x < -3$   
b.  $-3 < x < 3$   
c.  $3 < x < \infty$

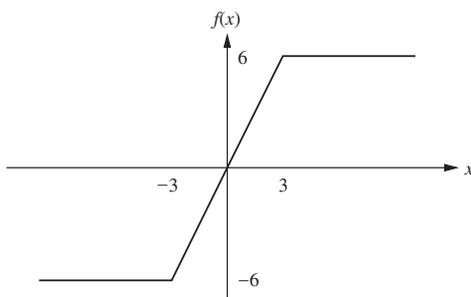


FIGURE P2.34

56. For the translational mechanical system with a nonlinear spring shown in Figure P2.35, find the transfer function,  $G(s) = X(s)/F(s)$ , for small excursions around  $f(t) = 1$ . The spring is defined by  $x_s(t) = 1 - e^{-f_s(t)}$ , where  $x_s(t)$  is the spring displacement and  $f_s(t)$  is the spring force. [Section: 2.10]

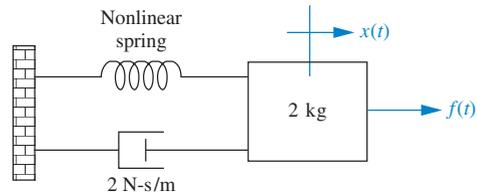


FIGURE P2.35

57. Enzymes are large proteins that biological systems use to increase the rate at which reactions occur. For example, food is usually composed of large molecules that are hard to digest; enzymes break down the large molecules into small nutrients as part of the digestive process. One such enzyme is amylase, contained in human saliva. It is commonly known that if you place a piece of uncooked pasta in your mouth its taste will change from paper-like to sweet as amylase breaks down the carbohydrates into sugars. Enzyme breakdown is often expressed by the following relation:



In this expression a substrate ( $S$ ) interacts with an enzyme ( $E$ ) to form a combined product ( $C$ ) at a rate  $k_1$ . The intermediate compound is reversible and gets disassociated at a rate  $k_{-1}$ . Simultaneously some of the compound is transformed into the final product ( $P$ ) at a rate  $k_2$ . The kinetics describing this reaction are known as the Michaelis-Menten equations and consist of four nonlinear differential equations. However, under some conditions these equations can be simplified. Let  $E_0$  and  $S_0$  be the initial concentrations of enzyme and substrate, respectively. It is generally accepted that under some energetic conditions or when the enzyme concentration is very big ( $E_0 \gg S_0$ ), the kinetics for this reaction are given by

$$\frac{dS}{dt} = k_{\psi}(\tilde{K}_s C - S)$$

$$\frac{dC}{dt} = k_{\psi}(S - \tilde{K}_M C)$$

$$\frac{dP}{dt} = k_2 C$$

where the following constant terms are used (*Schnell, 2004*):

$$k_{\psi} = k_1 E_0$$

$$\tilde{K}_s = \frac{k - 1}{k_{\psi}}$$