1. Which of the following matrices are guaranteed to equal $(A+B)^2$?

$$A^2+2AB+B^2, \quad A(A+B)+B(A+B), \quad (A+B)(B+A), \quad A^2+AB+BA+B^2.$$

2. Show that $A^2=0$ is possible for a nonzero matrix A, but that $A^TA=0$ is only possible if $A=0$.

3. In MATLAB notation, write the commands that define the matrix A and the column vectors x and b.

$$A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}, \quad x=\begin{bmatrix}5\\ -2\end{bmatrix}, \quad b=\begin{bmatrix}1\\ 7\end{bmatrix}.$$

What command would test whether or not $Ax=b$?

4. Suppose you solve $Ax=b$ for three special right-hand sides b,

$$Ax_1=\begin{bmatrix}1\\ 0\\ 0\end{bmatrix}, \quad Ax_2=\begin{bmatrix}0\\ 1\\ 0\end{bmatrix}, \quad Ax_3=\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}.$$

   (a) If the solutions x_1, x_2, x_3 are the columns of a matrix X, what is AX?

   (b) If the three solutions in Part (a) are

$$x_1=\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}, \quad x_2=\begin{bmatrix}0\\ 1\\ 1\end{bmatrix}, \quad x_3=\begin{bmatrix}0\\ 0\\ 1\end{bmatrix},$$

   solve $Ax=b$ when $b=[3,5,8]^T$.

   (c) Given the information in Parts (a) and (b), what is A?

5. Which of the following descriptions are correct? The solutions x of

$$Ax=\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

form (a) a plane; (b) a line; (c) a point; (d) the null space of A; (e) the column space of A.

6. For which vectors $[b_1,b_2,b_3]^T$ do these systems have a solution?

$$\begin{bmatrix}1 & 1 & 1\\ 0 & 1 & 1\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}b_1\\ b_2\\ b_3\end{bmatrix}, \qquad \begin{bmatrix}1 & 1 & 1\\ 0 & 1 & 1\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}b_1\\ b_2\\ b_3\end{bmatrix}.$$

7. Let w_1, w_2 w_3 be linearly independent vectors. Show that the differences $v_1=w_2-w_3$, $v_2=w_3-w_1$, $v_3=w_1-w_2$ are linearly dependent by finding a combination of the v_i's that gives zero.