4.7-1 (i) For $m(t) = \cos 1000t$

$$\varphi_{\text{DSB-SC}}(t) = m(t)\cos 10,000t = \cos 1000t \cos 10,000t$$
$$= \frac{1}{2}[\underbrace{\cos 9000t}_{\text{LSB}} + \underbrace{\cos 11,000t}_{\text{USB}}]$$

(ii) For $m(t) = 2\cos 1000t + \cos 2000t$

$$\varphi_{\text{DSB-SC}}(t) = m(t)\cos 10,000t = [2\cos 1000t + \cos 2000t]\cos 10,000t$$
$$= \cos 9000t + \cos 11,000t + \frac{1}{2}[\cos 8000t + \cos 12,000t]$$
$$= \underbrace{[\cos 9000t + \frac{1}{2}\cos 8000t]}_{\text{LSB}} + \underbrace{[\cos 11,000t + \frac{1}{2}\cos 12,000t]}_{\text{USB}}$$

(iii) For $m(t) = \cos 1000t \cos 3000t$

$$\varphi_{\text{DSB-SC}}(t) = m(t)\cos 10,000t = \frac{1}{2}[\cos 2000t + \cos 4000t]\cos 10,000t$$
$$= \frac{1}{2}[\cos 8000t + \cos 12,000t] + \frac{1}{2}[\cos 6000t + \cos 14,000t]$$
$$= \frac{1}{2}\underbrace{[\cos 8000t + \cos 6000t]}_{\text{LSB}} + \frac{1}{2}\underbrace{[\cos 12,000t + \cos 14,000t]}_{\text{USB}}$$

This information is summarized in a table below. Figure S4.7-1 shows various spectra.

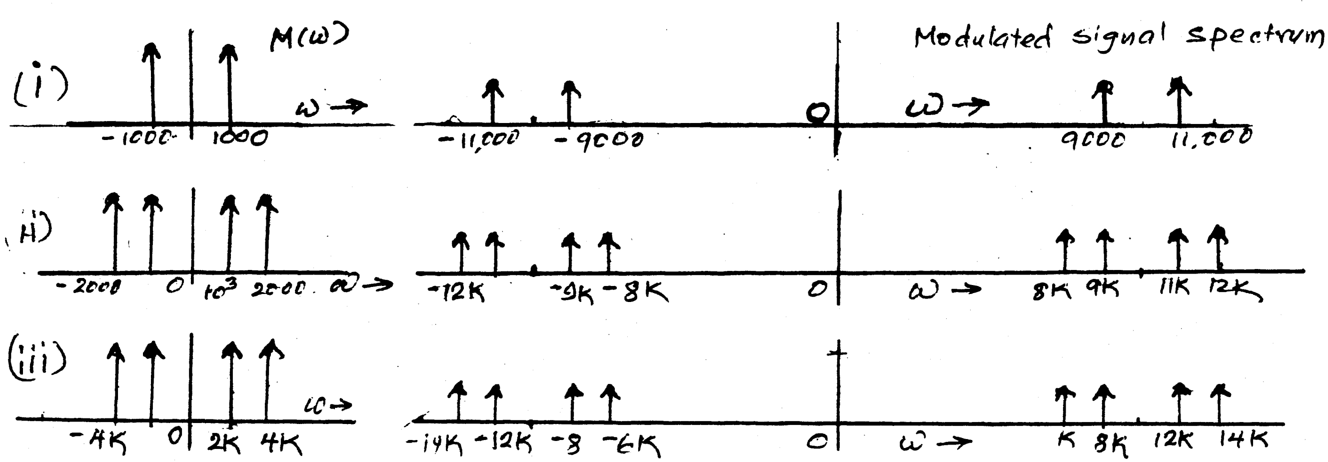

Fig. S4.7-1

case	Baseband frequency	DSB frequency	LSB frequency	USB frequency
i	1000	9000 and 11,000	9000	11,000
ii	1000	9000 and 11,000	9000	11,000
	2000	8000 and 12,000	8000	12,000
iii	2000	8000 and 12,000	8000	12,000
	4000	6000 and 14,000	6000	14,000

4.7-2 (a) The signal at point b is

$$\begin{aligned} f_a(t) &= m(t)\cos^3 \omega_c t \\ &= m(t)\left[\frac{3}{4}\cos\omega_c t + \frac{1}{4}\cos 3\omega_c t\right] \end{aligned}$$

The term $\frac{3}{4}m(t)\cos\omega_c t$ is the desired modulated signal, whose spectrum is centered at $\pm\omega_c$. The remaining term $\frac{1}{4}m(t)\cos 3\omega_c t$ is the unwanted term, which represents the modulated signal with carrier frequency $3\omega_c$ with spectrum centered at $\pm 3\omega_c$ as shown in Fig. S4.7-2. The bandpass filter centered at $\pm\omega_c$ allows to pass the desired term $\frac{3}{4}m(t)\cos\omega_c t$, but suppresses the unwanted term $\frac{1}{4}m(t)\cos 3\omega_c t$. Hence, this system works as desired with the output $\frac{3}{4}m(t)\cos\omega_c t$.

(b) Figure S4.7-2 shows the spectra at points b and c.

(c) The minimum usable value of ω_c is $2\pi B$ in order to avoid spectral folding at dc.

(d)

$$\begin{aligned} m(t)\cos^2\omega_c t &= \frac{m(t)}{2}\left[1 + \cos 2\omega_c t\right] \\ &= \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos 2\omega_c t \end{aligned}$$

The signal at point b consists of the baseband signal $\frac{1}{2}m(t)$ and a modulated signal $\frac{1}{2}m(t)\cos 2\omega_c t$, which has a carrier frequency $2\omega_c t$, not the desired value $\omega_c t$. Both the components will be suppressed by the filter, whose center center frequency is ω_c. Hence, this system will not do the desired job.

(e) The reader may verify that the identity for $\cos n\omega_c t$ contains a term $\cos\omega_c t$ when n is odd. This is not true when n is even. Hence, the system works for a carrier $\cos^n \omega_c t$ only when n is odd.

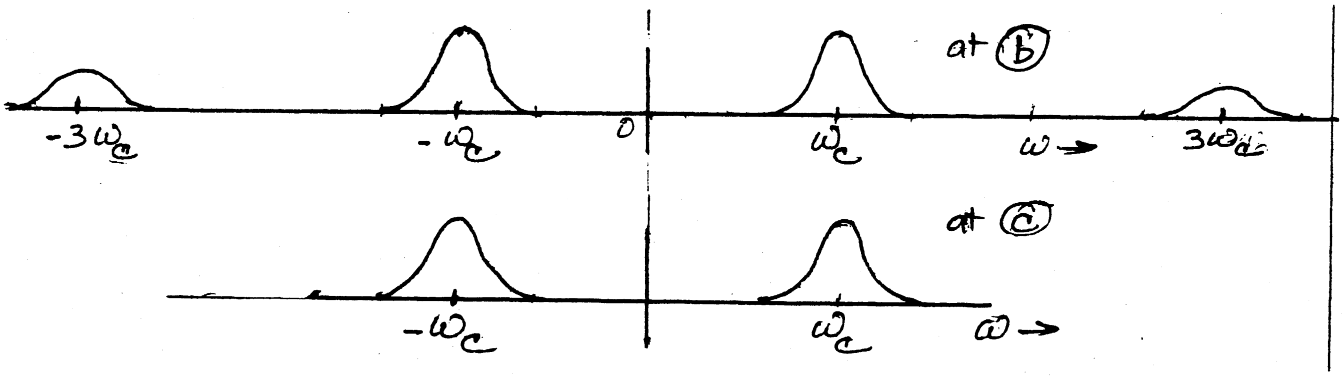

Fig. S4.7-2

4.7-3 This signal is identical to that in Fig. 3.8a with period T_0 (instead of 2π). We find the Fourier series for this signal as

$$x(t) = \frac{1}{2} + \frac{2}{\pi}\left[\cos\,\omega_c t - \frac{1}{3}\cos\,3\omega_c t + \frac{1}{5}\cos\,5\omega_c t + \cdots\right]$$

Hence, $y(t)$, the output of the multiplier is

$$y(t) = m(t)x(t) = m(t)\left[\frac{1}{2} + \frac{2}{\pi}\left(\cos\,\omega_c t - \frac{1}{3}\cos\,3\omega_c t + \frac{1}{5}\cos\,5\omega_c t + \cdots\right)\right]$$

The bandpass filter suppresses the signals $m(t)$ and $m(t)\cos\,n\omega_c t$ for all $n \neq 1$. Hence, the bandpass filter output is

$$km(t)\cos\,\omega_c t = \frac{2}{\pi}m(t)\cos\,\omega_c t$$

4.7-4 $f_a(t) = [A + m(t)] \cos \omega_c t$. Hence,

$$f_b(t) = [A + m(t)] \cos^2 \omega_c t$$
$$= \frac{1}{2}[A + m(t)] + \frac{1}{2}[A + m(t)] \cos 2\omega_c t$$

The first term is a lowpass signal because its spectrum is centered at $\omega = 0$. The lowpass filter allows this term to pass, but suppresses the second term, whose spectrum is centered at $\pm 2\omega_c$. Hence the output of the lowpass filter is

$$y(t) = A + m(t)$$

When this signal is passed through a dc block, the dc term A is suppressed yielding the output $m(t)$. This shows that the system can demodulate AM signal regardless of the value of A. This is a synchronous or coherent demodulation.