Table 4.1 Properties of the Fourier Transform (Fourier's Song)
Dr Time and Brother Frequency
Listen to a performance by Brother Frequency (mp3) (3.5MB)

Integrate your function times a complex exponential
It's really not so hard you can do it with your pencil

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-i\omega t}\, dt$$

And when you're done with this calculation
You've got a brand new function - the Fourier Transformation

What a prism does to sunlight, what the ear does to sound
Fourier does to signals, it's the coolest trick around

Now filtering is easy, you don't need to convolve
All you do is multiply in order to solve.

$$(f * g)(t) \xleftrightarrow{\mathcal{F}} F(\omega)G(\omega)$$

From time into frequency - from frequency to time

Every operation in the time domain

Has a Fourier analog - that's what I claim

Think of a delay, a simple shift in time
It becomes a phase rotation - now that's truly sublime!

$$f(t - t_0) \xleftrightarrow{\mathcal{F}} e^{-i\omega t_0} F(\omega)$$

And to differentiate, here's a simple trick
Just multiply by J omega, ain't that slick?

$$\frac{df(t)}{dt} \xleftrightarrow{\mathcal{F}} i\omega F(\omega)$$

Integration is the inverse, what you gonna do?
Divide instead of multiply - you can do it too.

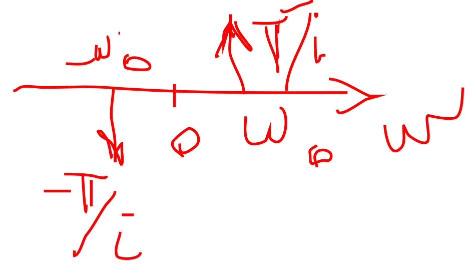
$$\int_{-\infty}^{t} f(\tau)\, d\tau \xleftrightarrow{\mathcal{F}} \frac{1}{i\omega} F(\omega)$$

From time into frequency - from frequency to time

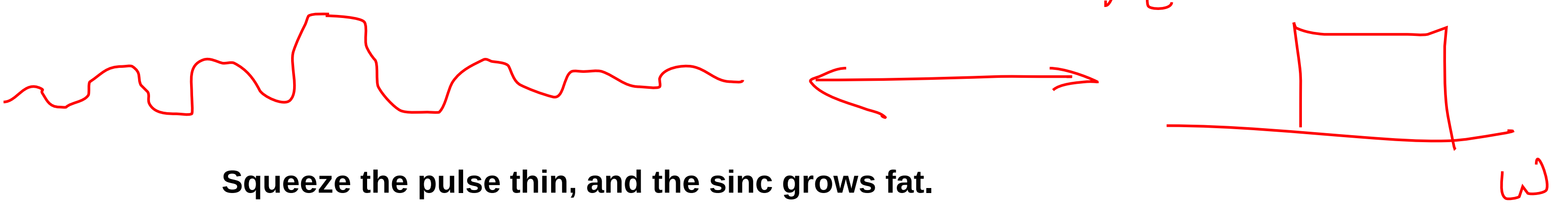
Let's do some examples... consider a sine
It's mapped to a delta, in frequency - not time

$$\sin(\omega_0 t) \xleftrightarrow{\mathcal{F}} \frac{\pi}{i}\left[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\right]$$

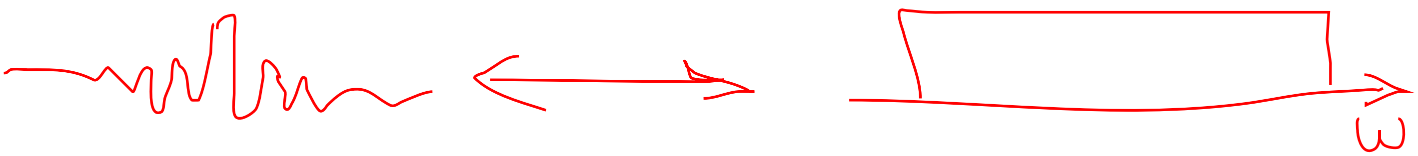
Now take that same delta as a function of time
Mapped into frequency - of course - it's a sine!



Sine x on x is handy, let's call it a sinc.
Its Fourier Transform is simpler than you think.
You get a pulse that's shaped just like a top hat...



Squeeze the pulse thin, and the sinc grows fat.



Or make the pulse wide, and the sinc grows dense,

The uncertainty principle is just common sense.