On Optimal Periodic Sensor Scheduling for Field Estimation in Wireless Sensor Networks

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***Abstract*—We consider the problem of finding the optimal time-periodic sensor schedule for field estimation in wireless sensor networks. The sensors are subject to resource constraints in the form of a limit on the number of times each sensor can be activated over one period. A correspondence between active sensors and the nonzero columns of the estimator gain matrix is established. We formulate an optimization problem which seeks the optimal periodic estimator gain matrix that minimizes the error covariance over one period, subject to the sensor resource constraints. The optimization problem is combinatorial in nature, and we apply the alternating direction method of multipliers (ADMM) to find its locally optimal solutions. Numerical results are provided to illustrate the effectiveness of the proposed method.**

I. Introduction

Wireless sensor networks (WSNs), consisting of a large number of spatially distributed sensors, have been used in a wide range of applications. In this paper, we study the problem of dynamic field estimation, where in a given region of interest, the field intensities that commonly evolve as part of a linear dynamical system are monitored by local sensors. However, due to the limited communication and energy resources, it is desirable that only a subset of sensors be activated over time and space. Therefore, the problem of sensor selection/scheduling arises.

In the literature on estimation and control over WSNs, sensor selection/scheduling problems have been studied extensively [1]–[5]. In [1], the problem of sensor selection for a *single* time step is relaxed to a convex optimization problem and solved efficiently. In [2], a *multi-step* sensor selection strategy is proposed by reformulating the Kalman filter, which is able to address different performance metrics and constraints on available resources. However, if the length of the time horizon becomes large then finding an optimal non-myopic schedule becomes difficult in that the number of sensor sequences grows intractably large as the time horizon grows. Therefore, some works such as [3], [4] consider periodic sensor schedules on an infinite time horizon. For a finite horizon sensor scheduling problem, periodicity in the sensor schedule has been observed in [5]. In [3], it is further shown that the solutions to the infinite horizon problems can be approximated arbitrarily well by a periodic schedule with a finite period. We emphasize that the results in [3] are nonconstructive, in the sense that it is proved that the optimal sensor schedule is time-periodic but an algorithm for obtaining this schedule is not provided. A method for designing optimal periodic sensor schedules is proposed in [4] for the deep space problem. However, for tractability in optimization the authors assume that the process noise is negligible, which is not a practical assumption for other applications such as target tracking and environment monitoring; see [2], [6].

In this paper, we propose a novel framework for the joint design of optimal periodic sensor schedules and optimal estimator (Kalman filter) gain matrices. This is based on the fact that a sensor being off at a certain time instant is equivalent to the corresponding column of the estimator gain matrix being identically zero. By using cardinality functions that count the number of nonzero columns of a matrix, measurement frequency constraints (a.k.a. transmission bounds in [5]) are also incorporated into the optimization formulation. As our objective, we minimize the trace of the periodic covariance matrix over one period. The covariance matrix satisfies a sequence of periodic Lyapunov *recursions*. We introduce a cyclic matrix representation that effectively replaces all recursive equations with one algebraic Lyapunov *equation*. This representation lends itself more readily to numerical optimization methods. The proposed constrained optimization problem is solved by using the alternating direction method of multipliers (ADMM) [7]. The approach presented here is similar to ADMM applied to the problem of optimal sparse feedback control [8]–[10], whereas we apply it to the sensor scheduling problem in this paper.

II. Problem Formulation

Consider a discrete-time linear dynamical system evolving according to the equations

$$x_{k+1} = Ax_k + Bw_k \tag{1}$$

$$y_k = Cx_k + v_k \tag{2}$$

where x_k is the N-dimensional state vector including all the field values of interest at time k, y_k is the M-dimensional measurement vector, A is the system transition matrix which is determined by the physical model of the field (e.g., the diffusion process governed by the partial differential equation (PDE) model in [2], [11]), C is the observation matrix, and the pair (A, C) is observable. The inputs w_k and v_k are white, Gaussian, zero-mean random vectors with covariance matrices Q and R, respectively.

For the above system, we consider state estimators of the form

$$\hat{x}_{k+1} = A\hat{x}_k + L_k(y_k - C\hat{x}_k) = (A - L_kC)\hat{x}_k + L_ky_k,$$

where L_k is the time-dependent estimator gain to be determined. We define the estimation error covariance P_k as,

$$P_k = \mathcal{E}\{(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T\},$$

where $\mathcal{E}$ is the expectation operator.[1] It is easy to show that P_k satisfies the Lyapunov recursion,

$$P_{k+1} = (A - L_k C) P_k (A - L_k C)^T + BQB^T + L_k R L_k^T. \quad (3)$$

Partitioning the matrices L_k and C into their respective columns and rows, we have

$$L_k C = L_{k,1} C_1^T + L_{k,2} C_2^T + \cdots + L_{k,M} C_M^T$$

where $L_{k,m}$ denotes the mth column of L_k and C_m^T is the mth row of C. Note that each row of C characterizes the measurement of one sensor. For example, suppose that at time step k only the μth sensor reports a measurement, we would have $L_k C = L_{k,\mu} C_\mu^T$. This can also be interpreted as having all $L_{k,m}$ equal to the zero column vector for sensors $m \neq \mu$. Thus, we assume that the measurement matrix C is constant and the scheduling of the sensors is captured by the columns of the estimator gains L_k, in the sense that if $L_{k,m}$ is a zero vector then at time k the mth sensor does not transmit a measurement to the fusion center.

Motivated by [3], in this work we search for optimal time-periodic sensor schedules, i.e., we seek sequences $\{L_k\}_{k=0,\ldots,K-1}$ and $\{P_k\}_{k=0,\ldots,K-1}$ that satisfy

$$L_{k+K} = L_k, \quad P_{k+K} = P_k, \quad (4)$$

where K is a given period. Note that the choice of K is not a part of the optimization problem considered in this paper. One possible algorithm for choosing K is proposed in [3].

Next, we formulate the optimal periodic sensor scheduling problem considered in this work, and then elaborate on the details of our formulation. We pose the optimal sensor scheduling problem as follows

$$\begin{array}{ll} \text{minimize} & \displaystyle\sum_{k=0}^{K-1} \operatorname{tr}(P_k) \\ \text{subject to} & \bullet \text{ Lyapunov recursion (3) for } k = 0, \ldots, K-1, \\ & \bullet \text{ periodicity condition (4)}, \\ & \bullet \displaystyle\sum_{k=0}^{K-1} \operatorname{card}\big(\|L_{k,m}\|_2\big) = \eta, \quad m = 1, 2, \ldots, M, \end{array} \quad (5)$$

where $\{L_k\}_{k=0,\cdots,K-1}$ are the optimization variables. In (5), $\|\cdot\|_2$ denotes the 2-norm, $\operatorname{card}(\cdot)$ stands for the cardinality function, i.e., the number of nonzero elements of a vector. In the equality constraint in (5), we refer to η as the measurement frequency bound which limits the number of times each sensor can acquire and transmit measurements over a time period of length K. Similar constraints have been considered in [5], [6]. We also remark here that even if an inequality constraint $\sum_{k=0}^{K-1} \operatorname{card}\big(\|L_{k,m}\|_2\big) \leq \eta$ is considered, the measurement frequency bound η is always achieved for improving the estimation performance [12].

[1] In the system theory literature, $\hat{x_k}$ and P_k are often denoted by $\hat{x}_{k|k-1}$ and $P_{k|k-1}$; here we use $\hat{x}_k$ and P_k for simplicity of notation.

Note that (5) is a combinatorial problem [13], and computationally intractable. Motivated by [9], in the next section, we employ the alternating direction method of multipliers (ADMM) algorithm to solve (5), and demonstrate that it leads to a pair of efficiently solvable subproblems.

III. Optimal Periodic Sensor Scheduling using ADMM

We begin by reformulating the optimization problem in (5) in a way that lends itself to the application of ADMM. References [7]–[9] demonstrate the effectiveness of ADMM for solving optimization problems that include cardinality functions. For P_k that satisfies the Lyapunov recursion in (3), it is easy to show that $\operatorname{tr}(P_k)$ can be expressed as a function f_k of $\{L_k\}_{k=0,\cdots,K-1}$ after invoking the periodicity of L_k.

We introduce the indicator function [8] of the constraint set and a set of auxiliary variables $\{G_k\}$ to arrive at the equivalent formulation,

$$\begin{array}{ll} \text{minimize} & \displaystyle\sum_{k=0}^{K-1} f_k(\{L_k\}) + \mathcal{I}(\{G_k\}) \\ \text{subject to} & L_k = G_k, \quad k = 0, 1, \ldots, K-1, \end{array} \quad (6)$$

where

$$\mathcal{I}(\{L_k\}) = \begin{cases} 0 & \text{if } \sum_{k=0}^{K-1} \operatorname{card}\big(\|L_{k,m}\|_2\big) = \eta \\ & \text{for } m = 1, 2, \ldots, M \\ +\infty & \text{otherwise} \end{cases}.$$

The augmented Lagrangian [7] corresponding to optimization problem (6) is given by

$$\begin{aligned} \mathscr{L}(\{L_k\}, \{G_k\}, \{\Lambda_k\}) &= \sum_{k=0}^{K-1} f_k(\{L_k\}) + \mathcal{I}(\{G_k\}) \\ &+ \sum_{k=0}^{K-1} \operatorname{tr}[\Lambda_k (L_k - G_k)] + \frac{\rho}{2} \sum_{k=0}^{K-1} \|L_k - G_k\|_F^2, \end{aligned} \quad (7)$$

where the matrices $\{\Lambda_k\}$ are the Lagrange multipliers, the scalar $\rho > 0$ is a penalty weight, and $\|\cdot\|_F$ denotes the Frobenius norm of a matrix, $\|X\|_F^2 = \operatorname{tr}(X^T X)$.

The ADMM algorithm can be described as follows [7], [9]. For $i = 0, 1, \ldots$, we iteratively execute the following three steps

$$\{L_k^{i+1}\} := \underset{\{L_k\}}{\arg\min} \; \mathscr{L}(\{L_k\}, \{G_k^i\}, \{\Lambda_k^i\}), \quad (8)$$

$$\{G_k^{i+1}\} := \underset{\{G_k\}}{\arg\min} \; \mathscr{L}(\{L_k^{i+1}\}, \{G_k\}, \{\Lambda_k^i\}), \quad (9)$$

$$\Lambda_k^{i+1} := \Lambda_k^i + \rho(L_k^{i+1} - G_k^{i+1}), \quad k = 0, \ldots, K-1 \quad (10)$$

until both of the conditions

$$\sum_{k=0}^{K-1} \|L_k^{i+1} - G_k^{i+1}\|_F \leq \epsilon, \quad \sum_{k=0}^{K-1} \|G_k^{i+1} - G_k^i\|_F \leq \epsilon$$

are satisfied.

In the following subsections, we elaborate on solving each of the minimization problems (8) and (9), of which the former can be addressed using variational methods and descent algorithms and the latter can be solved analytically.

A. **L**-*minimization using the Anderson-Moore method*

Completing the squares with respect to $\{L_k\}$ in the augmented Lagrangian (7), the L-minimization step in (8) can be expressed as [7], [9]

$$\text{minimize} \quad \sum_{k=0}^{K-1} f_k(\{L_k\}) + \sum_{k=0}^{K-1} \frac{\rho}{2}||L_k - U_k^i||_F^2$$

where $U_k^i := G_k^i - (1/\rho)\Lambda_k^i$ for $k = 0, 1, \ldots, K-1$. Recalling the definition of f_k, the above problem can be equivalently written as

$$\begin{array}{ll} \text{minimize} & \phi(\{L_k\}) := \sum_{k=0}^{K-1} \text{tr}(P_k) + \sum_{k=0}^{K-1} \frac{\rho}{2}||L_k - U_k^i||_F^2 \\ \text{subject to} & \text{Lyapunov recursion (3) for } k = 0, \ldots, K-1, \\ & \text{periodicity condition (4).} \end{array}$$

Proposition 1: *The necessary conditions for optimality of a sequence* $\{L_k\}$ *can be expressed as the set of coupled matrix recursions*

$$\begin{aligned} P_{k+1} &= (A - L_k C) P_k (A - L_k C)^T + BQB^T + L_k R L_k^T \\ V_k &= (A - L_k C)^T V_{k+1} (A - L_k C) + I \\ 0 &= 2V_{k+1} L_k R - 2V_{k+1}(A - L_k C) P_k C^T + \rho(L_k - U_k^i) \end{aligned}$$

for $k = 0, \ldots, K-1$*, where* $U_k^i := G_k^i - (1/\rho)\Lambda_k^i$ *and* $L_K = L_0$*,* $P_K = P_0$*. The expression on the right of the last equation is the gradient of* ϕ *with respect to* L_k*.*

Proof: The proof is an extension to the time-periodic case of similar propositions in [9], [10]. The details are omitted for the sake of brevity and will be reported elsewhere. ■

We emphasize here that it is difficult to solve the above set of matrix equations due to their coupling. Thus, we apply what can be thought of as a *lifting* procedure [14] to take the periodicity out of these equations and express them in a form appropriate for the application of the Anderson-Moore method, which is an iterative technique for solving systems of coupled matrix equations efficiently. We refer the readers to [9] for more details.

Using a permutation matrix in a block cyclic form

$$\mathcal{T} = \begin{bmatrix} 0 & & & I \\ I & \ddots & & \\ & \ddots & \ddots & \\ & & I & 0 \end{bmatrix},$$

we define

$$\mathcal{L} := \mathcal{T}\,\text{diag}\{L_k\} = \begin{bmatrix} 0 & & & L_{K-1} \\ L_0 & & & \\ & \ddots & \ddots & \\ & & L_{K-2} & 0 \end{bmatrix},$$

$$\begin{aligned} &\mathcal{P} := \text{diag}\{P_k\}, \quad \mathcal{V} := \text{diag}\{V_k\}, \quad \mathcal{U}^i := \mathcal{T}\text{diag}\{U_k^i\}, \\ &\mathcal{Q} := \text{diag}\{Q\}, \quad \mathcal{R} := \text{diag}\{R\}, \quad \mathcal{I} := \text{diag}\{I\}, \\ &\mathcal{A} := \mathcal{T}\text{diag}\{A\}, \quad \mathcal{B} := \text{diag}\{B\}, \quad \mathcal{C} := \text{diag}\{C\}. \end{aligned}$$

The recursive equations in the statement of Proposition 1 can now be rewritten in the time-independent form

$$\mathcal{P} = (\mathcal{A} - \mathcal{LC})\mathcal{P}(\mathcal{A} - \mathcal{LC})^T + \mathcal{BQB}^T + \mathcal{LRL}^T \tag{11}$$

$$\mathcal{V} = (\mathcal{A} - \mathcal{LC})^T \mathcal{V}(\mathcal{A} - \mathcal{LC}) + \mathcal{I} \tag{12}$$

$$0 = 2\mathcal{VLR} - 2\mathcal{V}(\mathcal{A} - \mathcal{LC})\mathcal{PC}^T + \rho(\mathcal{L} - \mathcal{U}^i) \tag{13}$$

Furthermore, it can be shown that the right side of (13) gives the gradient direction (defined by $\nabla\Phi := \mathcal{T}\text{diag}\{\nabla_{L_k}\phi\}$) for each of the L_k, $k = 0, 1, \ldots, K-1$.

The Anderson-Moore method alternates between keeping the values of $\mathcal{L}$ fixed and solving (11) and (12) for $\mathcal{P}$ and $\mathcal{V}$, and then keeping $\mathcal{P}$ and $\mathcal{V}$ fixed and solving (13) for a new value $\mathcal{L}_{new}$ of $\mathcal{L}$. It can be shown that the difference $\tilde{\mathcal{L}} := \mathcal{L}_{new} - \mathcal{L}$ between the values of $\mathcal{L}$ over two consecutive iterations constitutes a descent direction for $\phi(\{L_k\})$, see [9] for a related result. Thus, standard line search [13] can be employed to determine the step-size s in $\mathcal{L} + s\tilde{\mathcal{L}}$ to guarantee the convergence to a stationary point of ϕ (i.e., the optimal sequence $\{L_k\}$).

B. **G**-*minimization*

Completing the squares with respect to $\{G_k\}$ in the augmented Lagrangian (7) and recalling the definition of $\mathcal{I}(\{G_k\})$, the G-minimization step in (9) can be expressed as [7], [9]

$$\begin{array}{ll} \text{minimize} & \sum_{k=0}^{K-1} \frac{\rho}{2}||G_k - S_k^i||_F^2 \\ \text{subject to} & \sum_{k=0}^{K-1} \text{card}\big(\|G_{k,m}\|_2\big) = \eta, \;\; m = 1, 2, \ldots, M, \end{array}$$

where $S_k^i := L_k^{i+1} + (1/\rho)\Lambda_k^i$ for $k = 0, 1, \ldots, K-1$.

Replacing $||G_k - S_k^i||_F^2$ with $\sum_{m=1}^{M} \|G_{k,m} - S_{k,m}^i\|_2^2$, then the G-minimization problem decomposes into subproblems

$$\begin{array}{ll} \text{minimize} & \sum_{k=0}^{K-1} \frac{\rho}{2}\|G_{k,m} - S_{k,m}^i\|_2^2 \\ \text{subject to} & \sum_{k=0}^{K-1} \text{card}\big(\|G_{k,m}\|_2\big) = \eta, \end{array} \tag{14}$$

which can be solved separately for $m = 1, 2, \ldots, M$.

Therefore, the optimal sequence $\{G_k\}$ can be obtained by its *column-wise* solution from (14).

Proposition 2: The solution of (14) is given by

$$G_{k,m} = \begin{cases} S_{k,m}^i & ||S_{k,m}^i||_2 \geq ||[\mathcal{S}_m^i]_\eta||_2 \text{ and } \eta \neq 0, \\ 0 & \text{otherwise,} \end{cases}$$

for $k = 0, 1, \cdots, K-1$, where $S_k^i := L_k^{i+1} + (1/\rho)\Lambda_k^i$, $\mathcal{S}_m^i := [S_{0,m}^i, \cdots, S_{K-1,m}^i]$, $[\mathcal{S}_m^i]_\eta$ denotes the ηth largest column of $\mathcal{S}_m^i$ in the 2-norm sense, and $G_{k,m}$, $S_{k,m}^i$ denote the mth columns of G_k, S_k^i, respectively.

Proof: The proof is similar to [8, Appendix B]. We omit the details for brevity. ■

IV. Simulation Results

We consider an example in which $M = 4$ sensors are deployed on a 4×4 discrete lattice to estimate a diffusion process [11]. In system (1)-(2), we select $T = 0.5$, $B = I$, $Q = 0.25I$ and $R = I$, where T is the sampling interval and I is the identity matrix of appropriate size. For the measurement frequency bound in (5), we assume that every sensor is selected at most η times, $\eta \in \{1, \ldots, 6\}$, during a period of length $K = 6$. The ADMM parameters are selected as $\epsilon = 10^{-3}$ and $\rho = 10$. In our computations, ADMM converges for $\rho \geq 10$ and its required number of iterations is approximately 15.

In Fig. 1, we compare the difference between the performance of our approach and that of the random strategy (which refers to randomly selected sensor schedules that satisfy the measurement frequency constraint) with the performance of the globally optimal sensor schedule (obtained via an exhaustive search). Simulation results show that our proposed sensor schedule assures near optimal performance for all values of the measurement frequency bound η. For very small or very large values of η, any scheduling strategy yields estimation performances that are close to each other since there are only a few choices of sensor schedules over time. An extreme case is $\eta = 0$ or 6 where every sensor is imposed to be off or on for every time step in a period.

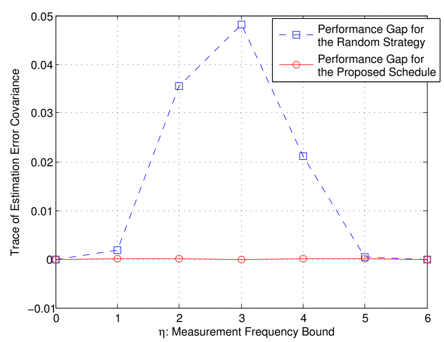

Fig. 1: Performance gaps of the random strategy and our proposed scheme with respect to the optimal schedule

In Fig. 2, we use the ADMM algorithm to obtain the sensor schedule over a time period of length $K = 6$ for $\eta \in \{1, 2, 3, 4\}$; the subplots represent increasing values of η from the left top to the right bottom. In each subplot, the horizontal axis represents discrete time, the vertical axis represents sensor indices, and circles represent activated sensors. As we can see, the schedule of sensors tends to be activated in a staggered way. In addition, although the periodicity of the sensor schedule was a fixed *priori* at the value $K = 6$, as η increases computational results demonstrate repetitive patterns in the optimal sensor schedule. This indicates that the value of the sensing period K can be made smaller than 6.

V. Conclusions

In this paper, we studied the problem of sensor scheduling for dynamic field estimation. We established a correspondence between active sensors and nonzero columns of the estimator gain, and proposed a novel framework for the joint design of optimal sensor activation schedules and optimal estimator gains via ADMM. We demonstrated that the application of ADMM yields a pair of efficiently solvable subproblems. In the future, we aim to seek the optimal tradeoff between estimation accuracy and sensor activations, and to investigate the effectiveness of our approach in large-scale sensor networks.

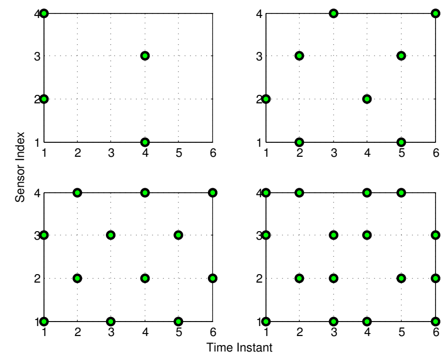

Fig. 2: Proposed optimal sensor schedules

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