Wireless Netw (2009) 15:53–64
DOI 10.1007/s11276-007-0024-y

Cross-layer optimization for end-to-end rate allocation in multi-radio wireless mesh networks

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Published online: 23 February 2007

Abstract In this paper, we study rate allocation for a set of end-to-end communication sessions in multi-radio wireless mesh networks. We propose cross-layer schemes to solve the joint rate allocation, routing, scheduling, power control and channel assignment problems with the goals of maximizing network throughput and achieving certain fairness. Fairness is addressed using both a simplified max-min fairness model and the well-known proportional fairness model. Our schemes can also offer performance upper bounds such as an upper bound on the maximum throughput. Numerical results show that our proportional fair rate allocation scheme achieves a good tradeoff between throughput and fairness.

Keywords Wireless mesh network · Cross-layer optimization · Rate allocation · Channel assignment · Fairness

1 Introduction

Wireless Mesh Networks (WMN) [1] are envisioned to provide various attractive applications in the future, including broadband Internet access, distributed information sharing and storage, and different multimedia applications at very low costs. Such networks can be deployed in both the urban and the rural areas to offer a large range of wireless coverage. A WMN consists of wireless mesh routers and mesh clients. Wireless mesh routers form a multihop wireless network which serves as the backbone to provide network access for mesh clients.

Although a WMN is a multihop wireless network, it is quite different from other multihop wireless networks, such as the well studied mobile ad hoc networks and wireless sensor networks. For example, a mobile ad hoc network is composed of a set of power constrained mobile nodes. However, wireless mesh routers in a WMN are usually stationary and directly connected with AC power. Therefore, major concerns of a mobile ad hoc network, such as mobility support and power efficiency, are not the most critical issues in a WMN.

All of the potential applications mentioned above demand the network to deliver a high volume of traffic efficiently. Hence, how to improve network throughput should be the most important design goal of WMNs. Moreover, every time when we talk about throughput, fairness must be taken into consideration, as otherwise we will end up with a serious bias on network resource allocation, which has been shown by previous research [11].

Compared with wired networks, a wireless network normally has lower network throughput due to the existence of interference which prohibits simultaneous transmissions in a common neighborhood. An effective method to improve throughput of wireless networks is to use multiple radios, i.e., to equip each wireless node with multiple Network Interface Cards (NICs) and tune them to different frequency channels [17, 18]. In a multi-radio wireless network, multiple transmissions within a common neighborhood can be concurrently conducted as long as they work on different

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channels. However, in order to make full use of the available NICs and channels, we have to carefully consider the channel assignment problem, which has not been well addressed before and is shown to be a challenging problem [18].

In this paper, we study a static network design problem, i.e., rate allocation, for a given set of end-to-end communication sessions in multi-radio wireless mesh networks. The objective is to jointly compute a rate allocation and the corresponding channel assignment, routing solution, transmission schedule and power assignment such that network throughput can be maximized or certain fairness can be achieved. We propose cross-layer schemes to solve the corresponding max throughput, max-min fair and proportional fair rate allocation problems. For a given channel assignment and a set of given transmission modes, our schemes offer the corresponding optimal rate allocation, routing and scheduling solutions, which will be explained in detail later. In addition, our schemes can provide upper bounds on performances such as the maximum throughput and the max-min rate value.

Many cross-layer schemes have been proposed for multihop wireless networks. We summarize our contributions and the differences between this work and previous related papers in the following.

- We are the first to cross four layers (the transport, network, MAC and physical layers) and study the joint rate control, routing, scheduling, power control and channel assignment problems in multi-radio WMNs. Among closely related works on multi-radio WMNs, power control was not addressed in [2, 8, 13, 17, 18, 20, 21, 23], and scheduling was not considered in [8, 17, 18, 20, 21].
- Different from previous works on cross-layer optimization in single-channel multihop wireless networks [4, 6, 7, 9, 11, 12, 16, 19, 22] in which channel assignment is not a concern, we consider the cross-layer optimization problems in the context of multi-radio WMNs and propose an efficient channel assignment algorithm.
- We study a WMN with a scheduling-based MAC layer which is believed to be a more suitable MAC solution for future WMNs [1] and adopted by the next generation wireless networking standard 802.16 [14]. In contrast, the 802.11-based MAC layer is assumed by most of related works [8, 17, 18, 20, 21].
- We addresses fairness under different models, including both a simplified max-min model and the well-known proportional fairness model. It differentiates this work from related works on multi-radio WMNs, in which either end-to-end fairness was not seriously considered [8, 13, 17, 18, 20, 23] or was only addressed based on the max-min fairness models [2, 21].

The rest of this paper is organized as follows. We discuss related works in Section 2. We describe the system model in Section 3 and define the problems in Section 4. The three cross-layer schemes and the corresponding numerical results are presented in Sections 5 and 6, respectively. We conclude the paper in Section 7.

2 Related work

Multi-radio WMNs have attracted extensive research attentions recently. Draves et al. [8] presented a new routing metric, Expected Transmission Time/Weighted Cumulative ETT (ETT/WCETT), and a corresponding Multi-Radio Link-Quality Source Routing (MR-LQSR) protocol to find high-throughput paths. In [17] and [18], the authors proposed one of the first 802.11-based multi-radio WMN architectures and developed a set of centralized and distributed channel assignment and routing heuristics. In [20], Tang et al. presented interference-aware topology control and routing schemes for QoS provisioning. Furthermore, they presented polynomial time optimal schemes to compute maximum throughput and fair bandwidth allocation in [21]. A constant-bound approximation algorithm was proposed in [2] to jointly compute channel assignment, routing and scheduling solutions for fair rate allocation. The authors of [13] studied a similar problem and derived upper bounds on the achievable throughput using a fast primal-dual algorithm. Based on that, they also proposed two channel assignment heuristics. Zhang et al. [23] developed a novel column generation based approach to solve the joint routing and scheduling problem.

Cross-layer approaches have been proposed to improve the performance of single-channel multihop wireless networks. The authors of [6] formulated the joint design of congestion control and media access control as a utility maximization problem and presented two distributed algorithms to solve it. In [22], Wang and Kar proposed primal and dual based algorithms to compute proportional fair end-to-end rate allocation in a multihop Aloha-based wireless network. Li in [16] considered end-to-end rate allocation in wireless ad hoc networks and proposed algorithms to distribute resources among multihop flows with the objective of improving both throughput and fairness. In [12], the authors studied the impact of interference on routing using a conflict graph. They derived upper and lower bounds on the optimal network throughput. In [11], Hou et al. developed a polynomial time algorithm to jointly calculate lexicographic max-min fair rate allocation and routing solutions in two-tiered wireless sensor networks. Cross-layer congestion control and power control design in wireless ad hoc networks have been studied by Chiang in [7]. The authors of [9] studied the joint scheduling and power control problem in the wireless ad hoc and proposed a simple two-phase heuristic to minimize the total power consumption. Similar cross-layer design problems have also been studied by [4] and [19] in which the authors

presented both integer linear programming formulations and fast heuristics for the considered problems.

3 System model

In this paper, we study a multi-radio wireless mesh backbone network with n stationary wireless mesh routers in which there are C non-overlapping frequency channels and each node (mesh router) v is equipped with Q_v NICs ($1 < Q_v \leq W$). We consider static channel assignment schemes as in [2, 13, 17, 18], i.e., a channel assignment is pre-determined and will not be changed during communications. We consider a scheduling-based MAC layer, i.e., the time domain is divided into time slots with equal constant durations, which are further grouped into frames of L time slots each. In the physical layer, omni-directional antennas are used for communications. The transmission power of a NIC can be adjusted within a given range $[0, P_{\max}]$. However, it will remain the same within one time slot. Each NIC transmits at the same fixed rate among all channels. As in all related works, we assume half-duplex operation at each NIC to prevent self-interference, and only consider unicast communications. In addition, two transmissions with a common receiver are not allowed to be made simultaneously, otherwise a collision will happen and corrupt the packet reception. We use the physical model proposed in [10] to model the impact of interference. We say a transmission from a transmitter in node u can be successfully received by a receiver in node v on a certain channel at some time instant, if

$$\frac{G_{uv} P_{uv}}{N_0 + \sum_{(x,y)\in\tau\setminus\{(u,v)\}} G_{xv} P_{xy}} \geq \beta. \tag{1}$$

In Inequality (1), τ stands for the set of concurrent transmissions; P_{uv} is the power level set at the transmitter of node u for transmission (u, v); G_{uv} is the channel gain for node pair (u, v) depending on path loss, channel fading and shadowing; β is a given threshold determined by some QoS requirements such as *Bit Error Rate (BER)*; N_0 is the thermal noise power at the receiver of node v which is usually a small constant. The left hand side of this inequality is normally called the *Signal to Interference and Noise Ratio (SINR)* at the receiver of node v. Note that the SINR constraint (Inequality (1)) is satisfied at each receiver implies that the half-duplexing, unicasting and collision-free constraints are satisfied at each receiver.

A directed graph $G(V, E)$ is used to model the considered network. Each vertex $v \in V$ corresponds to a wireless mesh node in the network with a known location. There is a directed link $e = (u, v) \in E$ connecting node u and node v if there exists a power level $P \in [0, P_{\max}]$ such that $G_{uv} P/N_0 \geq \beta$, i.e., a transmission from node u to v can be successfully made if there is no interference from other transmissions at the same time.

Multiple available radios complicate the transmission scheduling. Two neighbor nodes may share more than one common channels, i.e., a link in G may work on different channels. So we have to use a link-channel tuple (e, i) to uniquely denote the transmission along link e on channel i. Note that even though we need to ensure half-duplex, unicast and collision-free communications in one NIC, two links sharing one or two common nodes can be active simultaneously as long as they work on different channels. For a set of link-channel tuples having the same channel, we need to use the SINR constraint in Inequality (1) to determine if they can be active for transmissions concurrently. Once a network G and a corresponding channel assignment are given, we can easily identify the set EI of possible link-channel tuples in G. For example, suppose we have link $e = (u, v)$ and $\mathcal{A}(u) \cap \mathcal{A}(v) = \{i, j\}$, then we will obtain two possible link-channel tuples, (e, i) and (e, j), where $\mathrm{A}(\cdot)$ represents the set of channels assigned to the given node.

4 Problem definition

Now we are ready to formally define the rate allocation problems. Suppose that we are given a network $G(V, E)$ and a set of K end-to-end communication sessions. Each session is specified by a triple (s, t, d), where s is its source node, t is its destination node and d is its traffic demand. Like mentioned before, the rate allocation problem is implicitly coupled with a routing problem, a scheduling problem, a power control problem and a channel assignment problem. Hence, in all of the problems to be studied, we seek an end-to-end rate allocation vector $\mathbf{r}$ which specifies the rate r_k for each end-to-end session k, along with a channel assignment $\mathcal{A}$ specifying channels assigned to each node, a flow allocation vector $\mathbf{f}$ specifying the amount of traffic f_{ei}^k of session k routed through link e on channel i, a frame length L, a transmission schedule which specifies the set of link-channel tuples which are active in each time slot, and a power assignment vector specifying the power level of each link-channel tuple in each time slot.

A channel assignment, a flow allocation vector, a transmission schedule and a power assignment vector are said to be *feasible* if (a) the channel assignment $\mathcal{A}$ assigns a certain channel to each NIC and a set $\mathcal{A}(v)$ of Q_v different channels to each node v, where $\mathcal{A}(v) \subseteq \{1, 2, \ldots, W\}$; (b) for each session, the net amount of flow going out of the source node is equal to the corresponding end-to-end session rate; (c) for each session, the flow conservation constraint is satisfied at every node except the source and destination nodes; (d) on each available link-channel tuple, the aggregated flow is no more than the mean link data rate which

can be supported by the transmission schedule; and (e) in each time slot and on each assigned channel, there exists a power assignment vector, such that every power level is in the range $[0, P_{\max}]$ and the SINR constraint in Inequality 1 is satisfied at the receiving node of each link. A rate allocation vector is said to be *feasible* if we can find such a channel assignment, a flow allocation vector, a transmission schedule, a frame length L and a power assignment vector that are feasible.

First, we define the Maximum throughput End-to-end Rate Allocation (MERA) problem as follows.

Definition 1 (MERA). The **Maximum throughput End-to-end Rate Allocation (MERA)** problem seeks a feasible end-to-end rate allocation vector $\mathbf{r} = [r_1, r_2, \ldots, r_K]$, along with a feasible channel assignment, a feasible flow allocation vector, a feasible transmission schedule, a frame length L and a feasible power assignment vector such that the network *throughput* $\sum_{k=1}^{K} r_k$ is maximized.

Because there is a traffic demand associated with each end-to-end communication session, which may or may not be completely satisfied, we define a new variable called *Demand Satisfaction Factor (DSF)* to address the fairness. The DSF of a session is defined as the ratio between the rate actually allocated to that session and its traffic demand, which indicates how much a traffic demand is satisfied based on a rate allocation vector. So for each rate allocation vector $\mathbf{r} = [r_1, r_2, \ldots, r_K]$, we have a corresponding DSF vector $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_K]$, where $\alpha_k = r_k/d_k$, $1 \le k \le K$. The corresponding fair rate allocation problems are defined in the following.

Definition 2 (MMERA). A feasible end-to-end rate allocation vector $\mathbf{r} = [r_1, r_2, \ldots, r_K]$ $(\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_K])$ is said to be a feasible **max-min guaranteed end-to-end rate allocation vector** if for any other feasible end-to-end rate allocation vector $\mathbf{r}' = [r'_1, r'_2, \ldots, r'_K]$ $(\alpha' = [\alpha'_1, \alpha'_2, \ldots, \alpha'_K])$, $\min\{\alpha_k | 1 \le k \le K\} \ge \min\{\alpha'_k | 1 \le k \le K\}$, where α and α' are the DSF vector corresponding to $\mathbf{r}$ and $\mathbf{r}'$ respectively. The **Max-Min guaranteed maximum throughput End-to-end Rate Allocation (MMERA)** problem seeks a feasible max-min guaranteed end-to-end rate allocation vector, along with a feasible channel assignment, a feasible flow allocation vector, a feasible transmission schedule, a frame length L and a feasible power assignment vector such that the network *throughput* $\sum_{k=1}^{K} r_k$ is maximized.

Definition 3 (PERA). The **Proportional fair End-to-end Rate Allocation (PERA)** problem seeks a end-to-end feasible rate allocation vector $\mathbf{r} = [r_1, r_2, \ldots, r_K]$ $(\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_K])$, along with a feasible channel assignment, a feasible flow allocation vector, a feasible transmission schedule, a frame length L and a feasible power assignment vector such that the utility function $\sum_{k=1}^{K} \log(\alpha_k)$ is maximized, where α is the DSF vector corresponding to $\mathbf{r}$.

5 Proposed cross-layer schemes

The optimization problems defined in the last section involve four network layers. Without knowing the channel assignment, it is hard to determine interference among transmissions to which transmission scheduling and power control are highly related. Therefore, we propose 3-step heuristic methods to solve the problems: *In the first step*, we propose Linear Programming (LP) and Convex Programming (CP) formulations for the MERA, MMERA and PERA problems including the constraints placed by the nodes, channels, NIC, and the flow and rate feasibility constraints. Note that the rate allocation given by these LP and CP formulations may not be achievable because the wireless interference is not given full consideration in this phase. However, they can give upper bounds on performances such as the maximum throughput and the max-min DSF. More importantly, the flow allocation computed by solving the LPs or CP, can provide a guidance for the channel assignment. So based on them, we propose a channel assignment algorithm. *In the second step*, we identify a set of transmission modes and compute their corresponding power assignments based on the channel assignment determined in the first step. Here, each transmission mode corresponds to a set of link-channel tuples which can be scheduled for transmissions in one time slot. The concept of transmission mode is proposed to assist the computation of transmission schedule. *In the third step*, we present LP and CP formulations to provide rate allocation, routing and scheduling solutions based on the channel assignment and the transmission modes computed in the previous two steps.

5.1 Channel assignment

In this section, we will first present LP and CP formulations to provide solutions which can be used as the guidance for channel assignment. Then we will present the corresponding channel assignment algorithm.

We define rate allocation variables r_k and DSF variables α_k to specify rate allocated to communication session k and the corresponding DSF value respectively. We also have flow allocation variables f_{ei}^k specifying the amount of flow for session k going through link e on channel i. We only allow non-negative values for those variables.

We present an LP formulation LP1 to obtain approximate solutions of the MERA problem. In this formulation, E_v^{out}, E_v^{in} and E_v denote the set of outgoing, incoming and incident edges of node $v \in V$. c_e, d_k and Q_v are given parameters,

and represent the capacity of link e, the traffic demand of session k and the number of NICs in node v respectively.

LP1:

$$\max \quad \sum_{k=1}^{K} r_k \tag{2}$$

subject to:

$$\sum_{e\in E_{s_k}^{\text{out}}} \sum_{i=1}^{C} f_{ei}^k - \sum_{e\in E_{s_k}^{\text{in}}} \sum_{i=1}^{C} f_{ei}^k = r_k, \quad 1 \le k \le K; \tag{3}$$

$$\sum_{e\in E_{v}^{\text{out}}} \sum_{i=1}^{C} f_{ei}^k - \sum_{e\in E_{v}^{\text{in}}} \sum_{i=1}^{C} f_{ei}^k = 0, \quad 1 \le k \le K, \forall v \in V \setminus \{s_k, t_k\}; \tag{4}$$

$$\sum_{e\in E_v} \frac{\sum_{k=1}^{K} f_{ei}^k}{c_e} \le 1, \quad \forall v \in V, 1 \le i \le C; \tag{5}$$

$$\sum_{i=1}^{C} \sum_{e\in E_v} \frac{\sum_{k=1}^{K} f_{ei}^k}{c_e} \le Q_v, \quad \forall v \in V; \tag{6}$$

$$f_{ei}^k \ge 0, \quad \forall e \in E, 1 \le i \le C, 1 \le k \le K; \tag{7}$$

$$0 \le r_k \le d_k, \quad 1 \le k \le K. \tag{8}$$

Constraints (3)–(4) in LP1 are corresponding to the flow feasibility constraints (b) and (c) described in the last section respectively. In order to explain the other two constraints, we define a new variable x_{ei}^t whose value is 1 if link e is active on channel i in time slot t, and whose value is 0, otherwise. Then we shall have the following two inequalities.

$$\sum_{e\in E_v} x_{ei}^t \le 1, \quad 1 \le t \le L, 1 \le i \le C, \forall v \in V; \tag{9}$$

$$\sum_{i=1}^{C} \sum_{e\in E_v} x_{ei}^t \le Q_v, \quad 1 \le t \le L, \forall v \in V. \tag{10}$$

Inequality (9), i.e., in each node, at most one link can be active for transmissions on a certain channel at one time, is due to the fact that channels assigned to NICs in one node must be different, and the half-duplexing, unicasting and collision-free constraints in each NIC we mentioned before. We have Inequality (10) because there are Q_v NICs in node v. The mean flow over link e on channel i is given by $f_{ei} = \sum_{k=1}^{K} f_{ei}^k = c_e \cdot (\sum_{t=1}^{L} x_{ei}^t)/L$. Hence, we have Constraints (5) and (6) in LP1. The objective of LP1 is to maximize network throughput. Here, the scheduling and power control constraints are not included in the formulation because the channel assignment is not known so far. We use this formulation to provide an upper bound on the maximum achievable throughput and provide an approximate flow allocation to guide channel assignment.

LP2:

$$\max \quad \alpha \tag{11}$$

subject to:

Constraints (4)–(7);

$$\left(\sum_{e\in E_{s_k}^{\text{out}}} \sum_{i=1}^{C} f_{ei}^k - \sum_{e\in E_{s_k}^{\text{in}}} \sum_{i=1}^{C} f_{ei}^k \right) \Big/ d_k = \alpha_k, \quad 1 \le k \le K; \tag{12}$$

$$\alpha \le \alpha_k \le 1, \quad 1 \le k \le K. \tag{13}$$

LP3(α):

$$\max \quad \sum_{k=1}^{K} r_k$$

subject to:

Constraints (3)–(7);

$$\alpha d_k \le r_k \le d_k, \quad 1 \le k \le K. \tag{14}$$

In order to obtain an approximate solution of the MMERA problem, we need to solve two LPs sequentially. First, we solve LP2 and obtain the max-min DSF value α which is ensured by Constraint (13) and the objective function of LP2. Similar to the previous formulation, the computed α can serve as an upper bound on the max-min DSF value. Then we feed this max-min DSF value α as a parameter to LP3(α) to obtain an approximate MMERA solution.

The PERA problem has almost the same set of linear constraints as the MERA and MMERA problems and its objective is to maximize a concave utility function. Therefore, we can formulate and solve a CP (CP1) to obtain the approximate solution.

CP1:

$$\max \quad \sum_{k=1}^{K} \log(\alpha_k) \tag{15}$$

subject to:

Constraints (4)–(7), (12);

$$0 \le \alpha_k \le 1, \quad 1 \le k \le K. \tag{16}$$

The channel assignment algorithm is formally presented as Algorithm 1. In this algorithm, f_e denotes the aggregated flow on link e which is given by the flow allocation solution computed by solving the above LPs or CP. During

the execution of the algorithm, N_C records the number of common channels in nodes u and v, and N_u and N_v record the number of available NICs in node u and v respectively. N represents the number of required channels which is determined by the corresponding flow allocation value in the selected link e and the numbers of available NICs in its end nodes. The *interference weight* of a particular channel is defined as $W_I(i) = \sum_{l \in E \setminus \{e\}} f_l^i \cdot G_{s(l)t(e)}$, where f_l^i records the flow through link l on channel i; $s(l)$, $t(e)$ and $G_{s(l)t(e)}$ denote the transmitting node of link l, the receiving node of link e and the corresponding channel gain respectively. Every time when assigning a channel i to a link e, we imagine c_e^i amount of flow is allocated on link-channel tuple (e, i). The link-channel tuple flow values (f_l^i) are all initialized to 0 and will be updated during the execution of the channel assignment algorithm. The purpose of choosing channels with the smallest interference weight is to make the channels assigned to spatially close nodes as different as possible. Note that in the replacement procedure of Step 2, we always use the selected channel i to replace a channel with the largest interference weight. In the worst-case, the algorithm will eventually stop after passing through all n nodes and return to node u.

5.2 Transmission modes and power control

Based on the channel assignment computed by Algorithm 1, we can easily identify all link-channel tuples in the network G. We denote such link-channel tuple set as EI. In order to compute the transmission schedule, we define a set of *transmission modes*, each of which includes a subset of link-channel tuples that can be active for concurrent transmissions. Here, we introduce a $T \times m$ *scheduling matrix* Γ to

Algorithm 1. Channel assignment

Step 1 $\mathcal{A}(v) := \emptyset, \forall\, v \in V$.

Step 2 Select the links in G one by one in the descending order of their flow allocation values.

For each selected link $e = (u, v)$, update $\mathcal{A}(u)$ and $\mathcal{A}(v)$ as follows: $N_C := |\mathcal{A}(u) \cap \mathcal{A}(v)|$;
$N := \min\{|\lceil f_e/c_e \rceil - N_C|, |Q_u - N_C|, |Q_v - N_C|\}$; $N_u := Q_u - |\mathcal{A}(u)|$; $N_v := Q_v - |\mathcal{A}(v)|$;
if ($N > 0$ and $N_u > 0$ and $N_v > 0$)
  $N_{\min} := \min\{N_u, N_v, N\}$; Add $N_{\min}$ channels with the smallest interference weights to $A(v)$ and $A(u)$; $N := N - N_{\min}$; $N_u := N_u - N_{\min}$; $N_v := N_v - N_{\min}$;
end if
if ($N > 0$ and $N_u > 0$ and $N_v = 0$)
  $N_{\min} := \min\{N_u, N, |\mathcal{A}(v) \setminus \mathcal{A}(u)|\}$; Add $N_{\min}$ channels in N_v with the smallest interference weights to $A(u)$; $N := N - N_{\min}$; $N_u := N_u - N_{\min}$;
else if ($N > 0$ and $N_v > 0$ and $N_u = 0$)
  $N_{\min} := \min\{N_v, N, |\mathcal{A}(u) \setminus \mathcal{A}(v)|\}$; Add $N_{\min}$ channels in N_u with the smallest interference weights to $A(v)$; $N := N - N_{\min}$; $N_v := N_v - N_{\min}$;
end if
if ($N > 0$ and $N_u = 0$ and $N_v = 0$)
  while ($N > 0$)
    Let i be the channel with the smallest interference weight among channels in $\mathcal{A}(u) \bigcup \mathcal{A}(v)$. WLOG, assume that $i \in \mathcal{A}(u)$. Let $i' \neq i$ be a channel in $\mathcal{A}(v)$ with the largest interference weights. Replace i' in $\mathcal{A}(v)$ by i. For every link (v, w) already considered such that the change of $\mathcal{A}(v)$ makes $\mathcal{A}(v) \bigcap \mathcal{A}(w) = \emptyset$ (this implies $i' \in \mathcal{A}(w)$), replace i' in $\mathcal{A}(w)$ by i. This replacement may be performed multiple times.
    $N := N - 1$;
  endwhile
end if

Step 3 Assign nodes having unassigned NICs with the channels having the smallest interference weights among channels assigned to their neighboring nodes.

represent the set of transmission modes, where T is the number of transmission modes and m is the cardinality of the complete link-channel tuple set. Each row of the matrix corresponds to a transmission mode. If transmission mode t includes link-channel (e, i), we have $\Gamma^t_{ei} = 1$. Otherwise, $\Gamma^t_{ei} = 0$. We always append a special all-zero row at the end of Γ which corresponds to a transmission mode including no link-channel tuple.

The mean data rate of link-tuple (e, i) can be obtained as $\sum_{t:\Gamma^t_{ei}=1} p_t c_e$, where p_t is the fraction of time that transmission mode t is activated and c_e is the capacity of link e. According to a scheduling-based MAC protocol, a transmission mode is activated in each time slot. Suppose that we know all possible transmission modes. The transmission scheduling problem in the MAC layer is to determine the frame length L and the number of active time slots in one frame for each transmission mode. Actually, we can calculate a frame length by finding the smallest positive integer L such that $p_t \cdot L$ is an integer for every transmission mode. Correspondingly, transmission mode t should be activated in $p_t \cdot L$ time slots. Therefore, the scheduling problem is further transformed into a problem of computing the time fraction for each transmission mode. However, it may be impossible to find such an integer L since p_t could be an irrational number. In this case, p_t can be rounded up to obtain an frame length L, which will be a very close approximation.

The number of all possible transmission modes grows exponentially with the number of link-channel tuples. To our best knowledge, there does not exist a polynomial time algorithm which can identify all possible transmission modes based on the physical interference model in the literature. Therefore, we present a heuristic algorithm, Algorithm 2, to find a "good" subset of all possible transmission modes. Intuitively speaking, a good subset should cover all link-channel tuples and the number of times every tuple is included in certain transmission modes should be evenly distributed.

In Algorithm 2, T_M represents a transmission mode. Z is output as the computed subset of all possible transmission modes. Step 2 makes sure that every available link-channel tuple is covered by Z at least once. Furthermore, there is a weight variable, W^i_e, associated with each link-channel tuple, and recording how many times it is included in Z during the execution of the algorithm. The link-channel tuple with the smallest weight value will be selected into T_M, which helps to create a relatively even distribution of the number of times a particular link-channel tuple is selected.

LP4(h, E_j) is used to verify if link h and the existing set of links in T_M working on channel j (E_j) can be simultaneously active on channel j. Eventually, the solution given by LP4(h, E_j) can be used as the power assignment for the corresponding transmission mode. Even though we only need a feasible power assignment or need to test if there exists a feasible solution, it is always good to minimize the total power consumption which is achieved by the objective function (17). Constraint (18) is the SINR constraint (1) which is described in the system model. Here, $s(l)$ and $t(l)$ stand for the transmitting and receiving nodes of link l respectively. Constraint (19) ensures that each computed power level is in the range $[0, P_{\max}]$.

Algorithm 2. *Finding transmission mode set*

Step 1 $Z := \emptyset; i := 1; W^i_e := 0, \forall (e, i) \in EI$;
Step 2 **while** $(i <= \omega)$
  $T_M := \emptyset$;
  forall $((e, i) \in EI)$
    Add (e, i) to T_M; $W^i_e := W^i_e + 1$;
    do Add $(h, j) \neq (e, i)$ to T_M,
    s.t. $LP4(h, E_j)$ has a feasible solution
      and W^j_h is minimum among all
      link-channel tuples not in T_M;
      $W^j_h := W^j_h + 1$;
    until no more link-channel tuple can be added to
      T_M;
    if $(T_M \notin Z)$
      $Z := Z \cup \{T_M\}$;
    end if
  endforall
  $i := i + 1$;
  endwhile
Step 3 **output** Z;

LP4(h, E_j):

$$\min \sum_{l \in E_j \cup \{h\}} P_l \tag{17}$$

subject to:

$$G_{s(l)t(l)} P_l - \beta \sum_{q \in E_j \cup \{h\} \setminus \{l\}} G_{s(q)t(l)} P_q - \beta N_0 \geq 0, \quad \forall l \in E_j \cup \{h\}; \tag{18}$$

$$0 \leq P_l \leq P_{\max}, \quad \forall l \in E_j \cup \{h\}. \tag{19}$$

In Step 2 of Algorithm 2, ω is a tunable parameter. We observe that the larger the ω is, the more transmission modes will be added into Z, which will make the final solutions closer to the optimal ones at the cost of increasing the time complexities of our schemes.

5.3 Rate allocation

In this section, we present LP and CP formulations for the MERA, MMERA and PERA problems based on the set of transmission modes computed in the second step and the channel assignment computed in the first step.

LP5: MERA

$$\max \sum_{k=1}^{K} r_k$$

subject to:

$$\left(\sum_{(e,i)\in EI_{s_k}^{\text{out}}} f_{ei}^k - \sum_{(e,i)\in EI_{s_k}^{\text{in}}} f_{ei}^k \right) = r_k, \quad 1 \le k \le K; \tag{20}$$

$$\sum_{(e,i)\in EI_v^{\text{out}}} f_{ei}^k - \sum_{(e,i)\in EI_v^{\text{in}}} f_{ei}^k = 0, \quad 1 \le k \le K,\ \forall v \in V \setminus \{s_k, t_k\}; \tag{21}$$

$$\sum_{k=1}^{K} f_{ei}^k \le \sum_{t:\Gamma_{ei}^t=1} p_t c_e, \quad \forall (e,i) \in EI; \tag{22}$$

$$\sum_{t=1}^{T} p_t = 1; \tag{23}$$

$$f_{ei}^k \ge 0, \quad \forall (e,i) \in EI,\ 1 \le k \le K; \tag{24}$$

$$0 \le r_k \le d_k, \quad 1 \le k \le K.$$

In the above LP formulation, we have the aforementioned rate allocation variables r_k and flow allocation variables f_{ei}^k and the transmission schedule variables p_t. EI_v^{out}, EI_v^{in} and EI_v denote the set of link-channel tuples whose corresponding links are the outgoing, incoming and incident links of node $v \in V$ respectively. Similar to LP1, Constraints (20) and (21) correspond to the flow feasibility constraints. Constraint (22) guarantees that the mean data rate of a specific link on a certain channel given by the transmission schedule is large enough to support the amount of traffic going through that link on that channel. Obviously, since p_t is the fraction of time using transmission mode t, the summation of the values of p_t should be equal to 1, which is ensured by Constraint (23).

Similar as LP2, LP3, we use LP6 to find max-min rate allocation value α first and then solve LP7(α) which takes α as a parameter to compute the max-min guaranteed maximum throughput rate allocation. In addition, we present a CP formulation, CP2, to compute the proportional fair rate allocation.

LP6:

$$\max \ \alpha$$

subject to:

Constraints (21)−(24);

$$\left(\sum_{(e,i)\in EI_{s_k}^{\text{out}}} f_{ei}^k - \sum_{(e,i)\in EI_{s_k}^{\text{in}}} f_{ei}^k \right) \Big/ d_k = \alpha_k, \quad 1 \le k \le K; \tag{25}$$

$$\alpha \le \alpha_k \le 1, \quad 1 \le k \le K.$$

LP7(α): *MMERA*

$$\max \sum_{k=1}^{K} r_k$$

subject to:

Constraints (20)−(24);

$$\alpha d_k \le r_k \le d_k, \quad 1 \le k \le K.$$

CP2: *PERA*

$$\max \sum_{k=1}^{K} \log(\alpha_k)$$

subject to:

Constraints (21)−(25);

$$0 \le \alpha_k \le 1, \quad 1 \le k \le K.$$

In summary, our MERA scheme is to 1) apply Algorithm 1 to compute a channel assignment based on the flow allocation given by solving LP1; 2) use Algorithm 2 to find a set of transmission modes and their corresponding power assignments according to the channel assignment computed in 1); 3) solve LP5 to find a rate allocation, a flow allocation and a scheduling solution for the MERA problem. Similarly, our MMERA (PERA) scheme is to apply Algorithm 1 based on LP2 and LP3 (CP1), use Algorithm 2, and then solve LP6 and LP7 (CP2).

6 Numerical results

In this section, we illustrate the performances of the three cross-layer schemes by numerical results. In our simulations, we consider wireless networks with static nodes randomly located in a 1200×1200 m^2 region. The maximum transmission power is set to $P_{\max} = 300$ mW. The thermal noise power is set to $N_0 = -90$ dBm. The SINR threshold is set to $\beta = 10$ dB. The channel gain, G_{uv} is set to $1/d_{uv}^4$, where d_{uv} is the Euclidean distance between node u and node v. For each simulation scenario, we generate end-to-end communication sessions with random source and destination nodes. The traffic demand for each communication session (d_k) is given by a random number uniformly distributed in $[0.2c, 0.6c]$, where c is the link capacity. We solve all LPs using *CPLEX 9.0* [15]. We implement the barrier method introduced in Chapter 11 of [5] to solve all CPs and set the related parameters as follows: $\epsilon = 10^{-3}$, $\mu = 120$ and $t^{(0)} = 2$.

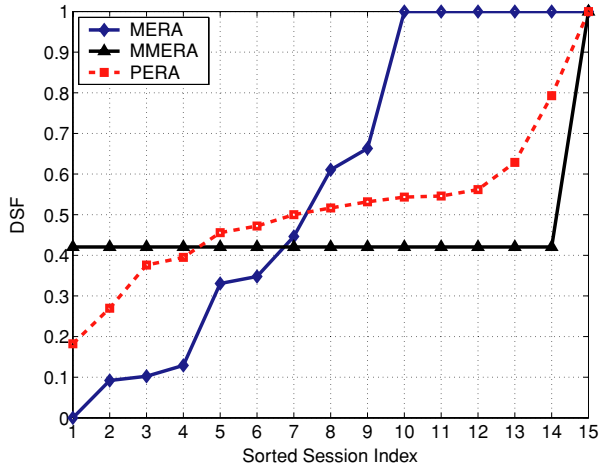

Fig. 1 DSF: Scenario 1 ($n = 10$, $K = 15$, $C = 3$, $Q = 2$, $c = 11$)

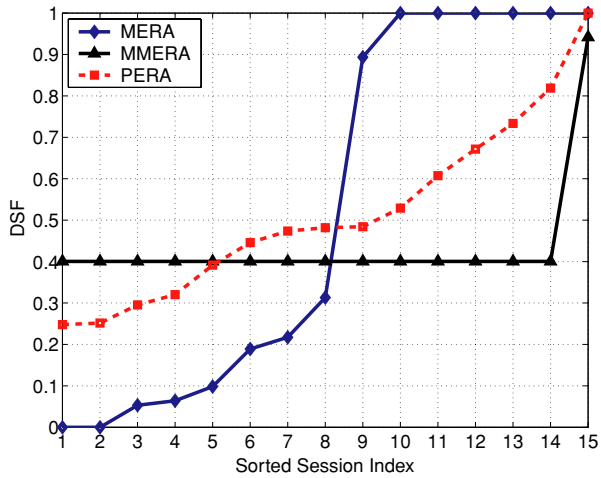

Fig. 2 DSF: Scenario 2 ($n = 15$, $K = 15$, $C = 3$, $Q = 2$, $c = 11$)

We evaluate the performance of the three rate allocation schemes in terms of the DSF value of each session (α_k) and network throughput ($\sum_{k=1}^{K} r_k$). We also compare the computed throughput and max-min DSF values against the corresponding upper bounds. For the MERA and PERA scheme, the upper bound ratio is defined as the ratio between the actual throughput and its corresponding upper bound obtained by solving LP1 or CP1. For the MMERA scheme, the upper bound ratio is defined as the ratio between the actual max-min DSF value and its corresponding upper bound given by solving LP2 and LP3.

We evaluate the performance of the proposed schemes under different settings, i.e., different network sizes (n), different number of end-to-end sessions (K), different numbers of available channels (C), different numbers of NICs (Q) in each node, and different link capacities (c). The simulation results are presented in the following figures. In Figs. 1–5,

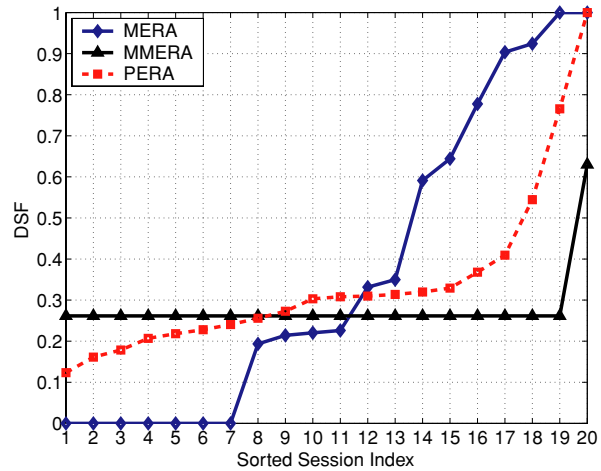

Fig. 3 DSF: Scenario 3 ($n = 15$, $K = 20$, $C = 3$, $Q = 2$, $c = 11$)

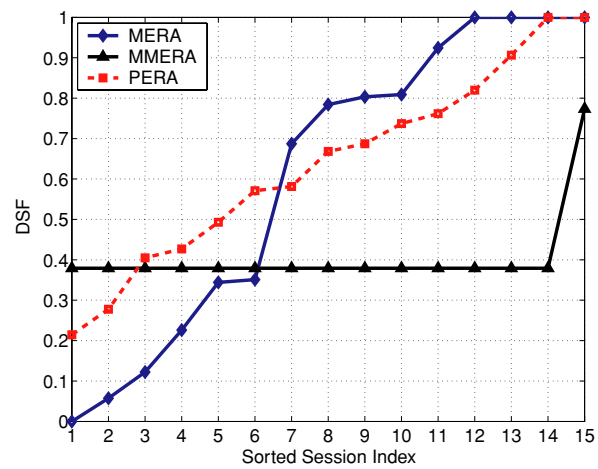

Fig. 4 DSF: Scenario 4 ($n = 10$, $K = 15$, $C = 5$, $Q = 2$, $c = 54$)

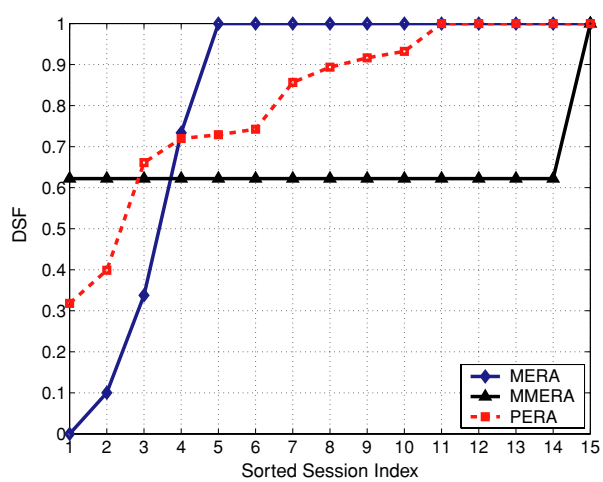

Fig. 5 DSF: Scenario 5 ($n = 10$, $K = 15$, $C = 5$, $Q = 3$, $c = 54$)

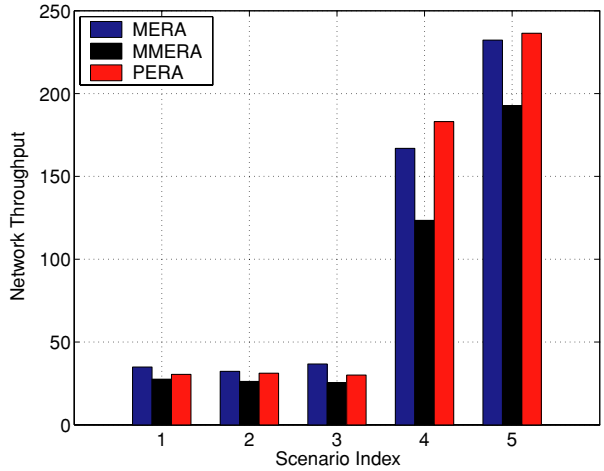

Fig. 6 Network throughput

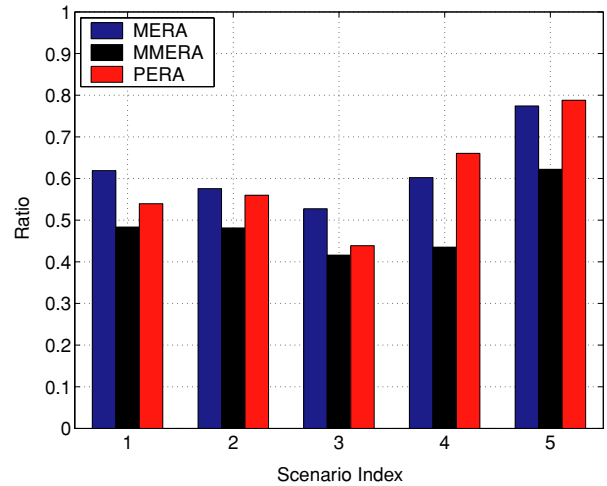

Fig. 7 Upper bound ratio

the communication sessions are sorted in the non-descending order of their DSF values.

We make the following observations from the numerical results:

- From Figs. 1–5, we observe that the MERA scheme results in a severe unfairness on rate allocation in all scenarios as expected. In each scenario, some sessions obtain very large DSF values which are equal to or close to 1, however, other sessions obtain very small DSF values which are equal to or close to 0. For example, from Fig. 2, we can see that the DSF values of about half of the sessions are below 0.3. However, the other half of the sessions achieve very large DSF values (DSF values of 6 sessions are equal to 1). A very sharp increase can be clearly seen from the corresponding curve in the figure. The MMERA scheme performs best in terms of fairness. As expected, it achieves the max-min DSF values among these three schemes and the DSF of every session is almost the same. In addition, we observe that the PERA scheme obtains much fairer rate allocation than the MERA scheme. In all scenarios, the minimum DSF values obtained by the PERA scheme are always larger than those obtained by the MERA scheme. Furthermore, the curves corresponding to the PERA scheme are much smoother than those corresponding to the MERA scheme.
- With regards to network throughput, the PERA scheme is surprisingly good, which can be seen from Fig. 6. Compared with the MERA scheme, the PERA scheme provides comparable throughput in Scenarios 1, 2 and 3, and achieves even higher throughput in Scenarios 4 and 5. This is possible because our MERA scheme is a heuristic scheme which does not guarantee to find the maximum throughput solutions even though its objective is to maximize the throughput. On average, these two schemes lead to almost the same network throughput. However, the MMERA scheme performs poorly in every scenario. Its average throughput is only 77% of that of the PERA scheme.
- From Figs. 1, 4 and 5, we can see that the network throughput and the rate allocated to each session are substantially improved with the increase of network resources such as NIC, number of channels and link capacity. However, with the network resources fixed, increasing the network size or the number of sessions will not lead to noticeable change on network throughput, which can be observed from Figs. 1–3.
- From Fig. 7, we can see that both the MERA and PERA schemes perform very well in terms of the upper bound ratio. For the cases in which the networks have relative rich resources (Scenarios 4 and 5), the ratios given by them are larger than 0.6 and become even close to 0.8 in Scenario 5.

7 Conclusions

In this paper, we have studied the joint rate allocation, routing, scheduling, power control and channel assignment problems in multi-radio WMNs. We presented three efficient cross-layer schemes to solve the Maximum throughput Rate Allocation (MERA), Max-Min guaranteed maximum throughput Rate Allocation (MMERA) and Proportional fair Rate Allocation (PERA) problems respectively. Our schemes can provide not only upper bounds on the maximum throughput and max-min DSF values, but also optimal joint rate allocation, routing and scheduling solutions if the transmission modes and the channel assignment are given. Numerical results show that the PERA scheme achieves a good tradeoff between throughput and fairness. Moreover, the throughput obtained by our MERA and PERA schemes are quite close to the corresponding upper bounds.

Acknowledgments This research is supported in part by ARO grant W911NF-04-1-0385 and NSF grant CCF-0431167. The information reported here does not reflect the position or the policy of the federal government.

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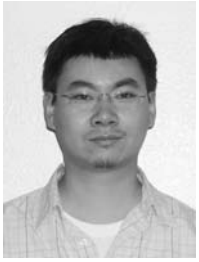

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