Cross-Layer Design for End-to-End Throughput and Fairness Enhancement in Multi-Channel Wireless Mesh Networks

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***Abstract*— In this paper, we study joint rate control, routing and scheduling in multi-channel Wireless Mesh Networks (WMNs), which are traditionally known as transport layer, network layer and MAC layer issues respectively. Our objective is to find a rate allocation along with a flow allocation and a transmission schedule for a set of end-to-end communication sessions such that the network throughput is maximized, which is formally defined as the Maximum throughput Rate Allocation (MRA) problem. As simple throughput maximization may result in a severe bias on rate allocation, we take account of fairness based on a simplified max-min fairness model and the proportional fairness models. We define the Max-min guaranteed Maximum throughput Rate Allocation (MMRA) problem and Proportional fair Rate Allocation (PRA) problem. We present efficient Linear Programming (LP) and Convex Programming (CP) based schemes to solve these problems. Numerical results show that proportional fair rate allocation schemes achieves a good tradeoff between throughput and fairness.**

***Index Terms*— Ad hoc wireless networks, cross-layer design, rate control, routing, scheduling, fairness.**

I. Introduction

A WIRELESS Mesh Network (WMN) [1] is composed of mesh routers and mesh clients. Wireless mesh routers form a multihop wireless network which serves as the backbone to provide network access for mesh clients. Mesh routers are usually stationary and connected with AC power. Most applications of WMNs, such as broadband Internet access, require the network to efficiently deliver a high volume of traffic. These features differentiate WMNs from the well-studied Mobile Ad hoc NETworks (MANETs): in stead of mobility and power efficiency, throughput and fairness are the most critical issues in WMNs.

Cross-layer schemes have been proposed to improve throughput and fairness for single-channel multihop wireless networks. In [5], [19], the joint rate control and scheduling problems have been studied for wireless ad hoc networks with either a scheduling-based MAC layer [5] or an Aloha-based MAC layer [19]. The cross-layer schemes presented in [5], [19] assume that the routes are given for the end-to-end communication sessions *a prior*. However, routing can significantly affect end-to-end performance and routing schemes without carefully addressing the impact of interference may perform very poorly [9]. In [16], Tang *et al.* proposed Integer Linear Programming formulations and a heuristic algorithm to solve the joint scheduling and power control problems in WMNs. Note that [16] focused on link layer (single hop) throughput and fairness, rather than the end-to-end (multihop) case studied in this paper.

Multi-channel multihop wireless networks can bring in dramatic throughput improvement compared to their single channel counterparts. Several recent works [6], [14], [15], [17], [18] studied 802.11-based multi-channel WMNs. In [14], [15], the authors proposed a set of heuristic algorithms for both channel assignment and routing. Draves *et al.* in [6] presented a new routing metric and a corresponding routing protocol to find high-throughput paths. In [17], Tang *et al.* presented an interference-aware channel assignment algorithm and a QoS routing algorithm. In [18], polynomial time algorithms were presented to compute max-min fair bandwidth allocation. It is well-known that random access based 802.11 MAC protocol does not perform well in terms of throughput and fairness [20]. If transmissions are carefully scheduled by making full use of space diversity, scheduling-based media access can guarantee collision-free transmissions and achieve a much higher throughput. Therefore, this approach is considered as a desirable solution for future WMNs [1].

To our best knowledge, this is the first work addressing joint rate control, routing and scheduling in multihop multi-channel WMNs with a scheduling-based MAC layer. We study a fundamental network design problem similar to the one studied in [9], i.e., given a specific traffic load, what is the maximum achievable throughput? We propose Linear Programming (LP) based and Convex Programming (CP) based schemes to seek a rate allocation along with a flow allocation and a transmission schedule such that throughput can be maximized and/or certain fairness can be achieved. Our contributions and the differences between our work and all previous related papers are summarized as follows.

- Different from previous works on cross-layer optimization [5], [9], [16], [19] in single-channel wireless networks, our work crosses three (transport, network and MAC) layers and addresses the joint rate control, routing and scheduling problems which have never been done before, especially in the context of multi-channel WMNs.
- We consider a scheduling-based MAC layer instead of the traditional 802.11-based MAC layer which is assumed by most of previous works [6], [14], [15], [17], [18]. The MAC layer used in our paper is believed to be a more suitable MAC solution for future WMNs [1], and the next generation wireless networking standard 802.16 actually adopts a scheduling-based MAC layer [7].
- Different from related works on multi-channel WMNs, in

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which either end-to-end fairness was not seriously considered [6], [11], [14], [15], [17] or was only addressed based on the max-min fairness models [2], [18], we take account of fairness under different models including both a simplified max-min model and the well-known proportional fairness model.
- We present optimal schemes for throughput maximization and fairness problems, instead of heuristic or approximation algorithms as in [2], [6], [11], [14], [15].

II. System Model

We consider a stationary multi-channel WMN with W non-overlapping frequency channels. Each node v is equipped with Q_v Network Interface Cards (NICs), where $1 < Q_v \leq W$. For efficient resource usage, a *channel assignment* $\mathcal{A}$ assigns a certain channel to each NIC and a set $\mathcal{A}(v)$ of Q_v different channels to each node v, where $\mathcal{A}(v) \subseteq \{1, 2, \ldots, W\}$. We assume that every node v transmits at a fixed transmission power P_v, whose corresponding transmission range is denoted as R_v and whose interference range is denoted as R_v^I (R_v^I is typically 2 to 3 times of R_v [14]).

We assume that a channel assignment is given *a prior*, based on which we can determine a network *topology*, denoted as $G(V, E)$ in which each vertex $v \in V$ corresponds to a wireless node, and that there is a directed link $e = (u, v; l)$ *on channel* $\lambda(e) = l$ from node u to node v in G if $d(u, v) \leq R_u$ and $\lambda(e) \in \mathcal{A}(u) \bigcap \mathcal{A}(v)$, where $d(u, v)$ is the Euclidean distance between nodes u and v. Because two or more common channels may be shared by a pair of neighboring nodes, G could be a *multi-graph* which includes *multi-links* connecting the same pair of nodes.

In a multi-channel wireless network, two wireless links can be active at the same time as long as they work on different channels. We say two wireless links $(u, v; l)$ and $(x, y; h)$ interfere with each other if $l = h$ and either $d(u, y) \leq R_u^I$ or $d(x, v) \leq R_x^I$, since concurrent transmissions along both links will lead to collisions/conflicts. The above definition implies half-duplex operation (i.e., one NIC can only transmit or receive at one time) as well as unicast communication (i.e., one transmission has only one intended receiver). However, when one NIC at a node is transmitting or receiving data on one channel, another NIC at the same node can simultaneously transmit or receive data *on a different channel*.

We assume a scheduling-based MAC layer and define a set of *transmission modes*, each of which includes a subset of links in the given topology G that can be active simultaneously. We use a $T \times m$ matrix Γ to represent the set of transmission modes, where m is the number of links in G, and T is the number of transmission modes. Each row of the matrix corresponds to a transmission mode. If transmission mode t includes link e, we have $\Gamma_{te} = 1$. Otherwise, $\Gamma_{te} = 0$. We call Γ the *scheduling matrix*. The average transmission rate of link e can be obtained as $\displaystyle\sum_{t:\Gamma_{te}=1} p_t c_e$, where p_t is the fraction of time that transmission mode t is activated, and c_e is the link capacity which is usually a constant.

All the links in a transmission mode can be activated simultaneously in a time slot. Suppose that all possible transmission modes are given. The scheduling problem in the MAC layer is to determine the frame length and the number of active time slots of each transmission mode in one frame. If we know the value of p_t for every transmission mode t, we can calculate a frame length by finding the smallest positive integer L such that $p_t * L$ is an integer for every transmission mode. Correspondingly, transmission mode t should be activated in $p_t * L$ time slots. In this way, the scheduling problem is transformed into a problem of determining the time fraction p_t for each transmission mode t.

III. Problem Definition

Suppose that we are given a network topology G, the corresponding scheduling matrix Γ, and the source and destination nodes of K end-to-end communication sessions. We want to find a rate allocation vector $\mathbf{r}$ specifying the rate r_k for each session k, along with a flow allocation vector $\mathbf{f}$ specifying the amount of traffic f_k^e of session k routed through link e in each time unit, and a transmission schedule vector $\mathbf{p}$ specifying time fraction p_t for each transmission mode t. A flow allocation vector and a transmission schedule vector are said to be *feasible* if (a) the net amount of flow going out of the source node of a session is equal to the end-to-end session rate; (b) for each session, the flow conservation constraint is satisfied at every node except the source and the destination; (c) on each link, the aggregated flow is no more than the average link transmission rate; and (d) the summation of all elements in a transmission schedule vector is equal to 1. A rate allocation vector is said to be *feasible* if we can find a corresponding flow allocation vector and a corresponding transmission schedule vector that are feasible.

Definition 1 (MRA): The **Maximum throughput Rate Allocation (MRA)** problem seeks a feasible rate allocation vector $\mathbf{r} = [r_1, r_2, \ldots, r_K]$, along with a feasible flow allocation vector and a feasible transmission schedule vector such that the throughput $\sum_{k=1}^K r_k$ is maximized.

Definition 2 (MMRA): A feasible rate allocation vector $\mathbf{r}$ is a feasible **max-min guaranteed rate allocation vector** if for any other feasible rate allocation vector $\mathbf{r}' = [r'_1, r'_2, \ldots, r'_K]$, $min\{r_k | 1 \leq k \leq K\} \geq min\{r'_k | 1 \leq k \leq K\}$. The **Max-Min guaranteed maximum throughput Rate Allocation (MMRA)** problem seeks a feasible max-min guaranteed rate allocation vector, along with a feasible flow allocation vector and a feasible transmission schedule vector such that the throughput $\sum_{k=1}^K r_k$ is maximized.

Definition 3 (PRA): The **Proportional fair Rate Allocation (PRA)** problem seeks a feasible rate allocation vector $\mathbf{r}$, along with a feasible flow allocation vector and a feasible transmission schedule vector such that the utility function $\sum_{k=1}^K \log(r_k)$ is maximized.

IV. Proposed Cross-layer Schemes

We proposed cross-layer schemes to solve the problems defined in the previous section. All of them follow the same basic idea: in the first step, we identify all possible transmission modes or a subset of transmission modes; in the second step, we formulate the problems as LPs and CPs based on the transmission modes found in the first step and apply existing algorithms to solve them.

Firstly, we present the methods to compute transmission modes. The well-known *contention graph* ([12]) is used to assist computation. In a contention graph $G_C(V_C, E_C)$ of network topology $G(V, E)$, every vertex corresponds to a wireless link in G. There is an undirected edge connecting two vertices in G_C if their corresponding links in G interfere with each other. A transmission mode actually corresponds to an *independent set* in the contention graph. Obviously, we only need to consider those transmission modes corresponding to Maximal Independent Sets (MISs) of G_C. The algorithm in [10] can be used to find all MISs in a contention graph. However, it is well-known that the number of MISs grows exponentially with the increase of the graph size. Therefore, we may only consider a subset of MISs if the contention graph is relatively large. We propose a polynomial time heuristic (Algorithm 1) to compute a subset of transmission modes in a given contention graph. Intuitively, a good subset should cover all vertices in G_C and the number of times every vertices is included in certain transmission modes should be evenly distributed.

Algorithm 1 Generating Transmission Modes

```
Step_1  Z := ∅; i := 1;
        W[v] := 0, ∀v ∈ V_C
Step_2  while (i <= ω)
           T_M := ∅;
           forall v ∈ V_C
               Add v to T_M; W[v] := W[v] + 1;
               do Add vertex u ≠ v to T_M, s.t. W[u] is
                  minimum among all vertices which are
                  not identical with or adjacent to any
                  other existing vertices in T_M;
                  W[u] := W[u] + 1;
               until T_M becomes an MIS;
               if (T_M ∉ Z)
                  Z := Z ∪ {T_M};
               endif
           endforall
           i := i + 1;
        endwhile
Step_3  output Z;
```

In Algorithm 1, T_M represents an MIS (transmission mode). Z is output as the computed subset of transmission modes and Z covers all vertices in G_C due to **Step**_2. ω is a given tunable parameter. The larger the ω is, the more MISs will be added into Z. W is a weight array which records how many times each vertex has been included in certain MISs of Z during the execution of the algorithm. We try to make the aforementioned distribution as even as possible by always adding the vertex with the minimum weight value every time. The running time of Algorithm 1 is dominated by **Step**_2 and it can be done in $O(\omega^2 m + \omega m^3)$, where m is the number of links in G. After obtaining transmission modes, we can easily construct the scheduling matrix Γ. For ease of presentation, we always append a special all-zero row at the end of Γ which corresponds to a special transmission mode including no links.

In order to present LP and CP formulations to solve the problems defined in Section III, we need to construct a simple digraph $G'(V', E')$ which is equivalent to the given topology $G(V, E)$. For each node $v \in V$, V' contains Q_v nodes $v^{\lambda_1(v)}, v^{\lambda_2(v)}, \ldots, v^{\lambda_{Q_v}(v)}$, where $\lambda_1(v) < \lambda_2(v) < \cdots < \lambda_{Q_v}(v)$ are the Q_v channels assigned to Q_v NICs in node v. Hence, a node in G' corresponds to a NIC. For each directed link $(u, v; h) \in E$, E' contains a directed link (u^h, v^h). Such links are called *inter-node* links. For each $v \in V$ and $1 \le i < Q_v$, E' includes a pair of opposite directed links between $v^{\lambda_i(v)}$ and $v^{\lambda_{i+1}(v)}$. We call such links *intra-node* links. E^I and E^O denote the set of intra-node and inter-node links respectively. Clearly, $E' = E^I \bigcup E^O$. If multiple nodes in V' correspond to a node in V which is not incident with *multi-links*, we can shrink them to a single node in G' and eliminate all related intra-node links. We select $s_k^{\lambda_1(s_k)}$ ($d_k^{\lambda_1(d_k)}$), as the corresponding source node s'_k (destination node d'_k) for session k.

Both the MRA and MMRA problems can be formulated as LPs. We have the aforementioned rate allocation variables r_k, flow allocation variables f_e^k and the transmission schedule variables p_t.

$LP1$: MRA

$$\max \quad \sum_{k=1}^{K} r_k \tag{1}$$

subject to:

$$\sum_{e \in E_{s'_k}^{out}} f_e^k - \sum_{e \in E_{s'_k}^{in}} f_e^k = r_k, \qquad 1 \le k \le K; \tag{2}$$

$$\sum_{e \in E_v^{out}} f_e^k - \sum_{e \in E_v^{in}} f_e^k = 0, \quad 1 \le k \le K, \forall v \in V' \setminus \{s'_k, d'_k\}; \tag{3}$$

$$\sum_{k=1}^{K} f_e^k \le \sum_{t:\Gamma_{te}=1} p_t c_e, \qquad \forall e \in E^O; \tag{4}$$

$$\sum_{t=1}^{T} p_t = 1; \tag{5}$$

$$f_e^k \ge 0, \qquad 1 \le k \le K, \forall e \in E'; \tag{6}$$

$$p_t \ge 0, \qquad 1 \le t \le T; \tag{7}$$

$$r_k \ge 0, \qquad 1 \le k \le K. \tag{8}$$

$LP2$: Max-Min

$$\max \quad \theta \tag{9}$$

subject to: $\quad Constraints\ (2) - (7);$

$$r_k \ge \theta, \quad 1 \le k \le K. \tag{10}$$

$LP3(\theta)$: MMRA

$$\max \quad \sum_{k=1}^{K} r_k$$

subject to: $\quad Constraints\ (2) - (7), (10).$

In the above formulations, E_v^{out} and E_v^{in} denote the set of outgoing and incoming edges of node $v \in G'$ respectively. c_e is the capacity of link e. Constraints (2)–(5) in $LP1$ are corresponding to the feasibility constraints (a)–(d) described in

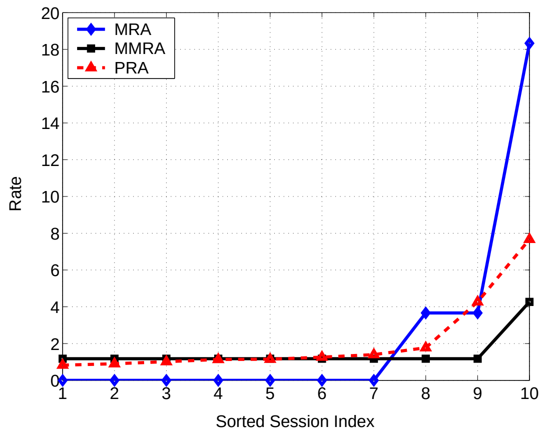


Fig. 1. Scenario 1: rate allocation with $C = 3, Q = 2, c = 11$.

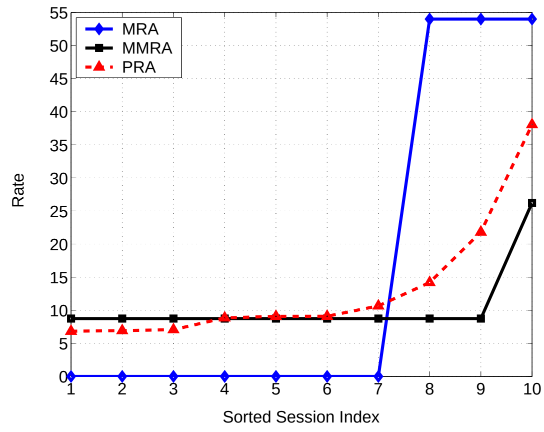


Fig. 2. Scenario 2: rate allocation with $C = 12, Q = 2, c = 54$.

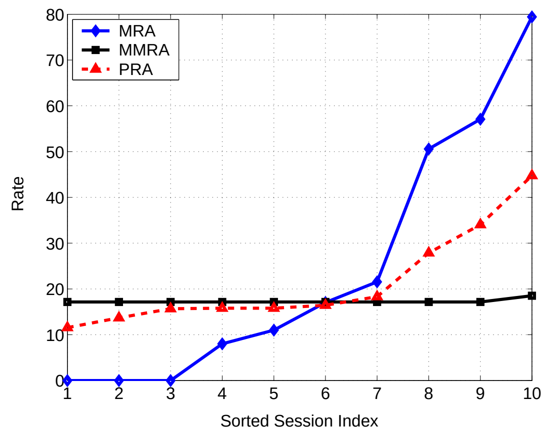


Fig. 3. Scenario 3: rate allocation with $C = 12, Q = 3, c = 54$.

Section III respectively. The objective of $LP1$ is to maximize network throughput.

For the MMRA problem, we need to solve two LPs sequentially. First, we solve $LP2$ and obtain a max-min rate value θ. Due to Constraint (10) and the objective function of $LP2$, we know that for any feasible rate allocation vector $\mathbf{r}'$, $\min\{r'_k|1 \leq k \leq K\} \leq \theta$. Next, we feed θ as a parameter to $LP3$. Constraint (10) in $LP3$ makes sure that for the computed rate allocation vector $\mathbf{r}$, $\min\{r_k|1 \leq k \leq K\} \geq \theta \geq \min\{r'_k|1 \leq k \leq K\}$. The objective of $LP3$ is also to maximize the throughput. Therefore, solving $LP2$ and $LP3(\theta)$ together can give an MMRA solution.

The PRA problem can be formulated as a convex program since it has the same linear constraints as the MRA problem and the objective is to maximize a concave utility function.

$CP1$: PRA

$$\max \quad \sum_{k=1}^{K} \log(r_k) \tag{11}$$

subject to: $\quad Constraints\ (2) - (8).$

There are efficient algorithms for solving LPs and CPs [3], [4]. In our simulations, we used a famous LP/ILP solver, CPLEX 9.0 [8] to solve all LPs. We implemented the barrier method (Algorithm 11.1) introduced in Chapter 11 of [4] to solve the CPs. The proposed schemes provide optimal solutions for the defined problems if all possible transmission modes are identified in the first step.

V. Numerical Results

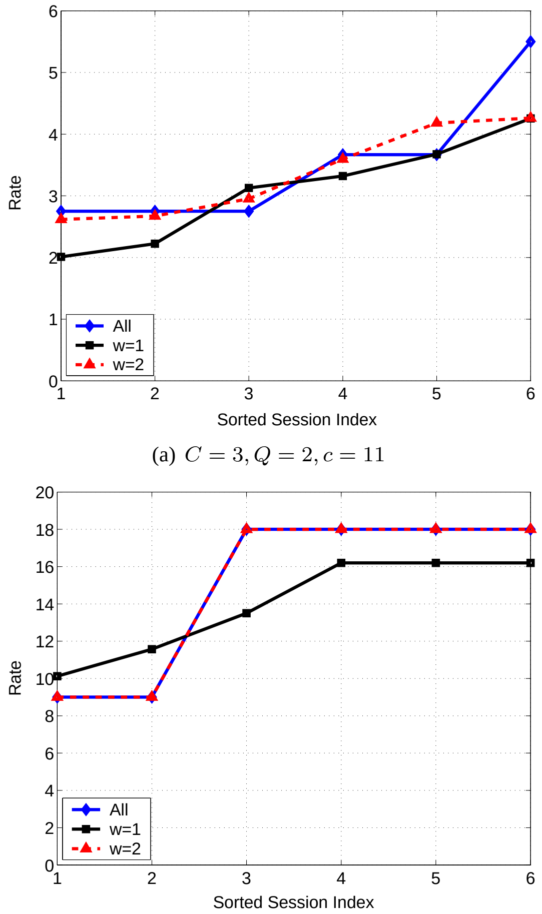
In the simulation, we randomly generate WMNs with n nodes located in a $800 \times 800 m^2$ region. The transmission range and corresponding interference range of each wireless mesh node are set to $250m$ and $500m$, respectively ([14]). We apply the channel assignment algorithm in [17] to assign channels for each NIC and ensure 2-connectivity of the resulting topologies.

The performance of the three rate allocation schemes are shown in terms of rate allocated to each session (r_k) and the throughput ($\sum_{k=1}^{K} r_k$). The sessions are sorted in the non-descending order of their rate values. For the PRA scheme, we also show its convergence speed in terms of the number of iterations.

In the first part of the simulations (Scenarios $1-3$), we test our schemes on networks with 15 nodes. In each scenario, 10 communication sessions are generated with random source and destination nodes. We evaluate the performance of the proposed schemes under different settings, i.e., different number of available channels (C), different link capacity (c), and different number of NICs (Q) in each node. They are set to typical values specified in 802.11 standards. We generate transmission modes using Algorithm 1 by setting $\omega = 1$. The results are presented in Figs. 1–3 and Table I. As expected, we observe that the MRA scheme achieves the highest throughput but results in a severe unfairness on rate allocation in all simulation scenarios. Rates allocated to some sessions (more than half of sessions in the first two scenarios) are equal to zero but the others obtain very high rates. The MMRA scheme performs best in terms of fairness since rates allocated to all sessions are almost the same. However, it offers very low network throughput which is only 68% of the maximum throughput on average. The average throughput given by the PRA scheme is about 85% of the corresponding maximum throughput. With regards to fairness, the PRA scheme is much better than the MRA scheme even though it does not achieve an absolutely even distribution. Furthermore, We can see from the table that the PRA scheme converges to optimal solutions

TABLE I
THROUGHPUT AND ITERATIONS UNDER DIFFERENT SCENARIOS

Scenario	MRA	MMRA	PRA	Iterations (PRA)
1	25.67	14.86	21.39	60
2	162.00	104.91	132.51	57
3	244.67	172.93	213.95	54



(a) $C = 3, Q = 2, c = 11$

(b) $C = 12, Q = 2, c = 54$

Fig. 4. Scenario 4: rate allocation based on different transmission mode sets.

very quickly in all scenarios (no more than 60 iterations). In addition, we notice that more NICs or more channels lead to much higher throughput as expected, however, with regards to fairness, rate allocation given by a particular scheme follows a similar pattern in different settings.

In the second part (Scenario 4), we evaluate the efficiency of our heuristic algorithm for transmission mode generation (Algorithm 1). The algorithm in [10] (labeled as "All" in the figures) is employed to generate all possible transmission modes and our Algorithm 1 is used to generate different subsets of transmission modes by setting the parameter ω to different values. In this case, we only run simulations on smaller networks with 6 nodes and 6 sessions, and apply our PRA scheme to compute rate allocation. The results are presented in Fig. 4. In addition, the throughput obtained for the three cases (All, $\omega = 1$, $\omega = 2$) are 21.08, 18.61 and 20.28 respectively in Scenario $4(a)$ and 90.00, 83.80 and 90.00 respectively in Scenario $4(b)$.

We observe that the rate allocations computed by using only transmission modes generated by our heuristic are very close to those computed by using all possible transmission modes in terms of both fairness and throughput.

VI. CONCLUSIONS

In this paper, we have studied joint rate control, routing and scheduling in multi-channel WMNs. We proposed LP and CP based schemes to compute maximum throughput, max-min fair and proportional fair solutions. Numerical results showed that the proportional fair rate allocation scheme achieves a good tradeoff between throughput and fairness.

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