End-to-End Rate Allocation in Multi-Radio Wireless Mesh Networks: Cross-Layer Schemes

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Abstract— **In this paper, we study rate allocation for a set of end-to-end communication sessions in multi-radio wireless mesh networks. We propose cross-layer schemes which can jointly solve rate allocation, channel assignment, routing, scheduling and power control problems in multiple layers. Specifically, a Linear Programming (LP) based scheme is presented to compute end-to-end rate allocation with the goal of maximizing network throughput. As simple throughput maximization may lead to a severe bias on rate allocation, we take fairness into consideration based on a parameter named *Demand Satisfaction Factor (DSF)*, and two fairness models, a simplified max-min fairness model and the well-known proportional fairness model. We propose LP-based and Convex Programming (CP) based schemes to compute fair end-to-end rate allocation. Our schemes can provide upper bounds on achievable network throughput and max-min DSF values. Numerical results show that our proportional fair rate allocation scheme achieves a good tradeoff between throughput and fairness.**

Keywords: Wireless mesh network, cross-layer design, rate allocation, channel assignment, routing, scheduling, power control, fairness, QoS.

I. Introduction

A Wireless Mesh Network (WMN) is composed of wireless mesh routers and mesh clients ([1]). Wireless mesh routers form a multihop wireless network which serves as the backbone to provide network access for mesh clients. WMNs can be deployed in either the city or the rural areas to provide a large range of wireless coverage. In the future, various attractive applications, such as broadband Internet access, distributed information sharing and storage, and different multimedia applications, are expected to be provided at very low costs in WMNs. WMNs are quite different from the well studied mobile ad hoc networks and wireless sensor networks in which mobility support and power efficiency are always major concerns because wireless mesh routers are usually stationary and directly connected with AC power. The most critical issue of a WMN is network throughput since almost all of its potential applications require the network to deliver a high volume of traffic efficiently. Every time when we talk about throughput, fairness must be taken into consideration, otherwise we may end up with a serious bias on network resource allocation, which has been shown by previous research ([8]).

Compared with wired networks, a wireless network normally has lower network throughput due to the existence of wireless interference which prohibits simultaneous transmissions in a common neighborhood. An efficient method to improve throughput of wireless networks is to use multiple radios, i.e., to equip each wireless node with multiple Network Interface Cards (NICs) and tune them to different frequency channels ([15], [16]). Fortunately, there are multiple non-overlapping frequency channels available in either the 2.4GHz or the 5GHz band ([9], [10]). In a multi-radio wireless network, there is usually no interference among concurrent transmissions within a common neighborhood as long as they work on different channels. However, in order to make full use of available NICs and channels, we have to consider the channel assignment problem which has not been well addressed before and is a hard problem ([16]).

In this paper, we study a static network design problem, the end-to-end rate allocation problem, for a given set of communication sessions in multi-radio wireless networks with the objective of either maximizing network throughput or achieving certain fairness. We propose efficient cross-layer schemes to jointly compute rate allocations and corresponding channel assignments, flow allocations, transmission schedules and power assignments to achieve the computed rate allocations. To our best

This research is supported in part by ARO grant W911NF-04-1-0385 and NSF grant CCF-0431167. The information reported here does not reflect the position or the policy of the federal government.

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QShine'06 The Third International Conference on Quality of Service in Heterogeneous Wired/Wireless Networks
August 7-9 2006, Waterloo, ON, Canada © 2006 ACM 1-59593-472-3/06/08…$5.00

knowledge, this is the first work addressing the joint rate allocation, channel assignment, routing, scheduling and power control problems in multihop multi-radio WMNs and proposing efficient cross-layer schemes. We propose an LP-based scheme to solve the Maximum throughput Rate Allocation (MRA) problem. We use the *Demand Satisfaction Factor (DSF)* defined in [17] to characterize the fairness. Based on this, we define the Max-Min guaranteed Maximum throughput Rate Allocation (MMRA) and the Proportional fair Rate Allocation (PRA) problems. Correspondingly, we propose an LP-based scheme and a CP-based scheme to solve them respectively. Our schemes can provide upper bounds on achievable throughput and max-min DSF values. Moreover, for a given channel assignment and a set of given transmission modes, our schemes can provide corresponding optimal rate allocation, routing and scheduling solutions, which will be explained in detail later.

The rest of this paper is organized as follows. We discuss related work in Section II. We describe the system model in Section III and define the problems in Section IV. The three cross-layer schemes and the corresponding numerical results are presented in Section V and Section VI, respectively. We conclude the paper in Section VII.

II. Related Work

Multi-radio multihop wireless networks have attracted extensive research attentions recently due to its potential applications in the future. Draves *et al.* in [6] presented a new routing metric, Expected Transmission Time/Weighted Cumulative ETT (ETT/WCETT), and a corresponding Multi-Radio Link-Quality Source Routing (MR-LQSR) protocol to find high-throughput paths in multi-radio multihop wireless networks. In [15] and [16], the authors proposed one of the first 802.11-based multi-radio WMN architectures and developed a set of centralized and distributed channel assignment and routing heuristics. In [18], Tang *et al.* presented efficient schemes to compute maximum throughput and fair bandwidth allocation in multi-radio WMNs. In [2], Alicherry *et al.* proposed a constant-bound approximation algorithm to jointly compute channel assignment, routing and scheduling solutions for fair rate allocation in multi-radio WMNs. The authors of [11] studied a similar problem and derived upper bounds on the achievable throughput using a fast primal-dual algorithm. Based on that, they also proposed two channel assignment heuristics. Zhang *et al.* in [21] developed a novel column generation based approach to solve the joint routing and scheduling problem in multi-radio WMNs. In [12], the authors provided asymptotic bounds on the capacity of multi-channel wireless networks.

Cross-layer approaches have been proposed to improve performance of single-channel multihop wireless networks by jointly solving problems in different layers. The authors of [5] formulated the joint design of congestion control and media access control as a utility maximization problem and presented two distributed algorithms to solve it. In [19], Wang and Kar proposed primal and dual based algorithms to compute proportional fair end-to-end rate allocation in a multihop Aloha-based wireless network. Li in [14] considered end-to-end rate allocation in wireless ad hoc networks and proposed algorithms to distribute resources among multihop flows with the objective of improving both throughput and fairness. In [8], Hou *et al.* developed a polynomial time algorithm, to calculate lexicographic max-min fair rate allocation in two-tiered wireless sensor networks. Wu *et al.* in [20] presented a cross-layer approach for multicast communications in wireless ad hoc networks.

III. System Model

In this paper, we study a multi-radio wireless mesh backbone network with n stationary wireless mesh routers in which there are totally C non-overlapping frequency channels and each node (mesh router) v is equipped with Q_v NICs ($1 < Q_v \leq W$). We consider static channel assignment schemes as in [2], [11], [15], [16], i.e., a channel assignment is pre-determined and will not be changed during communications.

We consider a scheduling-based MAC layer, i.e., the time domain is divided into time slots with equal constant durations, which are further grouped into frames of L time slots each. In the physical layer, omni-directional antennas are assumed to be used for communications. The transmission power of a NIC can be adjusted within a given range $[0, P_{max}]$. However, it will remain the same within one time slot. Each NIC transmits at the same fixed rate among all channels. Like all related work, we assume half-duplex operation at each NIC to prevent self-interference, and only consider unicast communications here. In addition, any two transmissions with a common receiver are not allowed to be made simultaneously, otherwise a collision will corrupt the packet reception. We use the physical model proposed in [7] to model the impact of interference. We say a transmission from a transmitter at node u can be successfully received by a receiver at node v on a certain channel at some time instant, if

$$\frac{G_{uv}P_{uv}}{N_0 + \sum_{(x,y)\in\tau\setminus\{(u,v)\}} G_{xv}P_{xy}} \geq \beta. \qquad (1)$$

In Inequality (1), τ stands for the set of concurrent transmissions; P_{uv} is the power level set at the transmitter of node u for transmission (u, v); G_{uv} is the channel gain for node pair (u, v) depending on path loss, channel fading and shadowing; β is a given threshold determined by some QoS requirements such as *Bit Error Rate (BER)*; N_0 is the thermal noise power at the receiver of node v which is usually a small constant. The left hand side of this inequality is normally called the *Signal to Interference and Noise Ratio (SINR)* at the receiver of node v. Note that the SINR constraint (Inequality (1)) is satisfied at each receiver implies that the half-duplexing, unicasting and collision-free constraints are satisfied at each receiver.

A directed graph $G(V, E)$ is used to model the considered network. Each vertex $v \in V$ corresponds to a wireless mesh node in the network with a known location. There is a directed link $e = (u, v) \in E$ connecting node u and node v if there exists a power level $P \in [0, P_{max}]$ such that $G_{uv}P/N_0 \geq \beta$, i.e., a transmission from node u to v can be successfully made if there is no interference from other transmissions at the same time.

Multiple available radios complicate the transmission scheduling. Two neighbor nodes may share more than one common channels, i.e., a link in G may work on different channels. So we have to use a link-channel tuple (e, i) to uniquely denote the transmission along link e on channel i. Note that even though we need to ensure half-duplex, unicast and collision-free communications in one NIC, two links sharing one or two common nodes can be active for transmission as long as they work on different channels. For a set of link-channel tuples having the same channel, we need to use the SINR constraint in (1) to determine if they can be active for transmissions concurrently. Once a network G and a corresponding channel assignment are given, we can easily identify the set EI of possible link-channel tuples in G. For example, suppose we have link $e = (u, v)$ and $\mathcal{A}(u) \bigcap \mathcal{A}(v) = \{i, j\}$, then we will obtain two possible link-channel tuples, (e, i) and (e, j).

IV. Problem Definition

Now we are ready to formally define the rate allocation problems. Suppose that we are given a network $G(V, E)$ and a set of K end-to-end communication sessions. Each session is specified by a triple (s, t, d), where s is its source node, t is its destination node and d is its traffic demand. Like mentioned before, the rate allocation problem is implicitly coupled with a channel assignment problem, a routing problem, a scheduling problem and a power control problem. Hence, in all of our optimization problems, we seek an end-to-end rate allocation vector $\mathbf{r}$ which specifies the rate r_k for each session k, along with a channel assignment $\mathcal{A}$ specifying channels assigned to each node, a flow allocation vector $\mathbf{f}$ specifying the amount of traffic f_{ei}^k of session k routed through link e on channel i, a frame length L, a transmission schedule which specifies the set of link-channel tuples active in each time slot and a power assignment vector specifying power level of each link-channel tuple in each time slot.

A channel assignment, a flow allocation vector, a transmission schedule and a power assignment vector are said to be *feasible* if (a) the channel assignment $\mathcal{A}$ assigns a certain channel to each NIC and a set $\mathcal{A}(v)$ of Q_v different channels to each node v, where $\mathcal{A}(v) \subseteq \{1, 2, \ldots, W\}$; (b) for each session, the net amount of flow going out of the source node is equal to the corresponding end-to-end session rate; (c) for each session, the flow conservation constraint is satisfied at every node except the source and destination nodes; (d) on each available link-channel tuple, the aggregated flow is no more than the mean link data rate; (e) in each time slot and on each assigned channel, there exists a power assignment vector, such that every power level is in the range $[0, P_{max}]$ and the SINR constraint in (1) is satisfied at the receiving node of each link. A rate allocation vector is said to be *feasible* if we can find such a channel assignment, a flow allocation vector, a transmission schedule, a frame length L and a power assignment vector that are feasible.

First, we give the definition for the Maximum throughput Rate Allocation (MRA) as follows.

Definition 1 (MRA): The **Maximum throughput Rate Allocation (MRA)** problem seeks a feasible rate allocation vector $\mathbf{r} = [r_1, r_2, \ldots, r_K]$, along with a feasible channel assignment, a feasible flow allocation vector, a feasible transmission schedule, a frame length L and a feasible power assignment vector such that the network *throughput* $\sum_{k=1}^{K} r_k$ is maximized.

Because there is a traffic demand associated with each communication session, we define a new variable called Demand Satisfaction Factor (DSF) to address the fairness. The DSF of a session is defined as the ratio between the rate allocated to that session and its traffic demand, which indicates how much a traffic demand is satisfied based on a rate allocation vector. So for each given rate allocation vector $\mathbf{r} = [r_1, r_2, \ldots, r_K]$, we have a corresponding DSF vector $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_K]$, where $\alpha_k = r_k/d_k$, $1 \leq k \leq K$. The corresponding fair rate allocation problems are defined in the following.

Definition 2 (MMRA): A feasible rate allocation vector $\mathbf{r} = [r_1, r_2, \ldots, r_K]$ ($\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_K]$) is said to be a feasible **max-min guaranteed rate allocation vector** if for any other feasible rate allocation vector $\mathbf{r}' = [r'_1, r'_2, \ldots, r'_K]$ ($\alpha' = [\alpha'_1, \alpha'_2, \ldots, \alpha'_K]$), $min\{\alpha_k | 1 \leq k \leq K\} \geq min\{\alpha'_k | 1 \leq k \leq K\}$, where α and α' are the DSF vector corresponding to $\mathbf{r}$ and $\mathbf{r}'$ respectively. The **Max-Min guaranteed maximum throughput rate allocation (MMRA)** problem seeks a feasible max-min guaranteed rate allocation vector, along with a feasible channel assignment, a feasible flow allocation vector, a feasible transmission schedule, a frame length L and a feasible power assignment vector such that the network *throughput* $\sum_{k=1}^{K} r_k$ is maximized.

Definition 3 (PRA): The **Proportional fair Rate Allocation (PRA)** problem seeks a feasible rate allocation vector $\mathbf{r} = [r_1, r_2, \ldots, r_K]$ ($\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_K]$), along with a feasible channel assignment, a feasible flow allocation vector, a feasible transmission schedule, a frame length L and a feasible power assignment vector such that the utility function $\sum_{k=1}^{K} \log(\alpha_k)$ is maximized, where α is the DSF vector corresponding to $\mathbf{r}$.

V. Proposed Cross-layer Schemes

The optimization problems defined in the last section involve four network layers. Moreover, without knowing the channel assignment, it is hard to determine interference between transmissions to which transmission scheduling and power control are highly related. Therefore, we propose 3-step heuristic methods to solve the optimization problems. In the first step, we propose LP and CP formulations for the MRA, MMRA and PRA problems including the constraints placed by the nodes, channels, NIC, and the flow and rate feasibility constraints. Note that the computed rate allocation may not be achievable because the wireless interference is not given full consideration in this phase. However, it can provide an upper bound on the achievable network throughput. Under the guidance of the flow allocation computed by solving the formulated LPs or CP, we propose a channel assignment heuristic. Once the channel assignment is determined, we can identify possible transmission modes and compute their corresponding power assignments in the second step. Here, each transmission mode corresponds to a set of link-channel tuples which can be scheduled for transmissions in one time slot. The concept of transmission mode is proposed to assist the computation of transmission schedule. In the third step, we use LP and CP formulations to provide optimal rate allocation, routing and scheduling solutions based on the channel assignment and transmission modes computed in the previous steps.

A. Channel Assignment

In this section, we will first present LP and CP formulations which can be used as the guidance of channel assignment. Then we will present the corresponding channel assignment heuristic.

We define rate allocation variables r_k and DSF variables α_k to specify rate allocated to communication session k and the corresponding DSF value respectively. We also have flow allocation variables f_{ei}^{k} specifying the amount of flow for session k going through link e on channel i. We only allow non-negative values for those variables.

We present an LP formulation ($LP1$) to obtain approximate solutions of the MRA problem. In this formulation, E_v^{out}, E_v^{in} and E_v denote the set of outgoing, incoming and incident edges of node $v \in V$. c_e is the capacity of link e.

$LP1$:

$$\max \quad \sum_{k=1}^{K} r_k \tag{2}$$

subject to:

$$\sum_{e \in E_{s_k}^{out}} \sum_{i=1}^{C} f_{ei}^k - \sum_{e \in E_{s_k}^{in}} \sum_{i=1}^{C} f_{ei}^k = r_k,\ 1 \le k \le K; \tag{3}$$

$$\sum_{e \in E_v^{out}} \sum_{i=1}^{C} f_{ei}^k - \sum_{e \in E_v^{in}} \sum_{i=1}^{C} f_{ei}^k = 0,$$
$$1 \le k \le K,\ \forall v \in V \setminus \{s_k, t_k\}; \tag{4}$$

$$\sum_{e \in E_v} \frac{\sum_{k=1}^{K} f_{ei}^k}{c_e} \le 1,\ \forall v \in V,\ 1 \le i \le C; \tag{5}$$

$$\sum_{i=1}^{C} \sum_{e \in E_v} \frac{\sum_{k=1}^{K} f_{ei}^k}{c_e} \le Q_v,\ \forall v \in V; \tag{6}$$

$$f_{ei}^k \ge 0,\ \forall e \in E,\ 1 \le i \le C,\ 1 \le k \le K; \tag{7}$$

$$0 \le r_k \le d_k,\ 1 \le k \le K. \tag{8}$$

Constraints (3)-(4) in $LP1$ are corresponding to the flow feasibility constraints (b) and (c) described in the last section respectively. In order to explain the other two constraints, we define a new variable x_{ei}^t whose value is 1 if link e is active on channel i in time slot t, and whose value is 0, otherwise. Then we shall have the following two inequalities.

$$\sum_{e \in E_v} x_{ei}^t \le 1,\ 1 \le t \le L, 1 \le i \le C, \forall v \in V; \tag{9}$$

$$\sum_{i=1}^{C} \sum_{e \in E_v} x_{ei}^t \le Q_v,\ 1 \le t \le L, \forall v \in V. \tag{10}$$

Inequality (9), i.e., in each node, at most one link can be active for transmissions on a certain channel at one time, is due to the fact that channels assigned to NICs in one node must be different, and the half-duplexing, unicasting and collision-free constraints in each NIC we mentioned before. We have Inequality (10) because there are Q_v NIC in node v. The mean flow over link e on channel i is given by $f_{ei} = \sum_{k=1}^{K} f_{ei}^k = c_e \cdot (\sum_{t=1}^{L} x_{ei}^t)/L$. Hence, we have Constraints (5) and (6) in $LP1$. The objective of $LP1$ is to maximize network throughput. Here, the scheduling and power control constraints are not included in the formulation because the channel assignment is not known so far. We use this formulation to provide an upper bound on the maximum achievable network throughput and provide an approximate flow allocation to guide channel assignment.

$LP2$:

$$\max \quad \alpha \tag{11}$$

subject to:

$$Constraints\ (4) - (7);$$
$$\Big(\sum_{e \in E_{s_k}^{out}} \sum_{i=1}^{C} f_{ei}^k - \sum_{e \in E_{s_k}^{in}} \sum_{i=1}^{C} f_{ei}^k\Big)/d_k = \alpha_k,$$
$$1 \le k \le K; \tag{12}$$

$$\alpha \le \alpha_k \le 1,\ 1 \le k \le K. \tag{13}$$

$LP3(\alpha)$:

$$\max \quad \sum_{k=1}^{K} r_k$$

subject to:

$$Constraints\ (3) - (7);$$
$$\alpha d_k \le r_k \le d_k,\ 1 \le k \le K. \tag{14}$$

In order to approximate optimal solutions of the MMRA problem, we need to solve two LPs sequentially. First, we solve $LP2$ and obtain the max-min DSF value α which is ensured by Constraint (13) and the objective function of $LP2$. Similar to the previous formulation, the computed α can serve as an upper bound on achievable max-min DSF values. Then we feed this max-min DSF value α to $LP3(\alpha)$ to obtain an approximate MMRA solution. The PRA problem has almost the same set of linear constraints as the MRA and MMRA problems and its objective is to maximize a concave utility function. Therefore, we can formulate and solve a CP ($CP1$) to approximate optimal solutions.

$CP1$:

$$\max \quad \sum_{k=1}^{K} \log(\alpha_k) \tag{15}$$

subject to:

$$Constraints\ (4) - (7), (12);$$
$$0 \le \alpha_k \le 1,\ 1 \le k \le K. \tag{16}$$

It is well known both LP and CP can be efficiently solved ([3], [4]). For simulation purpose, we use a famous LP/ILP solver, *CPLEX 9.0* ([13]) to solve all LPs. We apply the barrier method (Algorithm 11.1) introduced in Chapter 11 of [4] to solve all CPs.

The channel assignment algorithm is formally presented as Algorithm 1. In this algorithm, f_e denotes the aggregated flow on link e which is given by the computed flow allocation solution. N_C represents the number of common channels in nodes u and v. N_u and N_v record the number of available NICs in node u and v, respectively. N stands for the number of required channels determined by the corresponding flow allocation value in the selected link e and the numbers of available NICs in its end nodes. The *interference weight* of a specific channel is defined as $IW(i) = \sum_{l \in E \setminus \{e\}} f_l^i \cdot G_{s(l)t(e)}$, where f_l^i records the flow through link l on channel i; $s(l)$ and $t(e)$ denote the transmitting node of link l and the receiving node of link e respectively. Every time when assigning a channel i to a link e, we imagine c_e^i amount of flow is allocated on link-channel tuple (e, i). The link-channel tuple flow values (f_l^i) are all initialized to 0 and will be updated during the execution of the channel assignment algorithm. The purpose of choosing channels with the smallest interference weights is to make the channels assigned to spatially close nodes as different as possible. Note that in the replacement procedure of **Step 2**, we always use the selected channel i to replace a channel with the largest interference weight. In the worst-case, the algorithm will eventually stop after passing through all n nodes and return to node u.

B. Transmission Modes and Power Control

Based on the channel assignment computed by Algorithm 1, we can easily identify all link-channel tuples in the network G. We denote such link-channel tuple set as EI. In order to compute the transmission schedule, we define a set of *transmission modes*, each of which includes a subset of link-channel tuples that can be active for concurrent transmissions. Here, we introduce a $T \times m$ *scheduling matrix* Γ to represent the set of transmission modes, where T is the number of transmission modes and m is the cardinality of the link-channel tuple set. Each row of the matrix corresponds to a transmission mode. If transmission mode t includes link-channel (e, i), we have $\Gamma_{ei}^t = 1$. Otherwise,

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Algorithm 1 Channel Assignment

Step 1 $\mathcal{A}(v) := \emptyset$, $\forall\ v \in V$.

Step 2 Select the link in G one by one in the descending order of its flow values. For each selected link $e = (u, v)$, update $\mathcal{A}(u)$ and $\mathcal{A}(v)$ as follows:

$N_C := |\mathcal{A}(u) \bigcap \mathcal{A}(v)|$;
$N := min\{|\lceil f_e/c_e \rceil - N_C|, |Q_u - N_C|, |Q_v - N_C|\}$;
$N_u := Q_u - |\mathcal{A}(u)|$; $N_v := Q_v - |\mathcal{A}(v)|$;
if $(N > 0$ and $N_u > 0$ and $N_v > 0)$
 $N_{min} := min\{N_u, N_v, N\}$;
 Add N_{min} channels with the smallest interference weights to $A(v)$ and $A(u)$;
 $N := N - N_{min}$;
 $N_u := N_u - N_{min}$; $N_v := N_v - N_{min}$;
endif
if $(N > 0$ and $N_u > 0$ and $N_v = 0)$
 $N_{min} := min\{N_u, N, |\mathcal{A}(v) \setminus \mathcal{A}(u)|\}$;
 Add N_{min} channels in N_v with the smallest interference weights to $A(u)$;
 $N := N - N_{min}$; $N_u := N_u - N_{min}$;
else if $(N > 0$ and $N_v > 0$ and $N_u = 0)$
 $N_{min} := min\{N_v, N, |\mathcal{A}(u) \setminus \mathcal{A}(v)|\}$;
 Add N_{min} channels in N_u with the smallest interference weights to $A(v)$;
 $N := N - N_{min}$; $N_v := N_v - N_{min}$;
endif
if $(N > 0$ and $N_u = 0$ and $N_v = 0)$
 while $(N > 0)$
  Let i be the channel with the smallest interference weight among channels in $\mathcal{A}(u) \bigcup \mathcal{A}(v)$. WLOG, assume that $i \in \mathcal{A}(u)$. Let $i' \neq i$ be a channel in $\mathcal{A}(v)$ with the largest interference weights. Replace i' in $\mathcal{A}(v)$ by i. For every link (v, w) already considered such that the change of $\mathcal{A}(v)$ makes $\mathcal{A}(v) \bigcap \mathcal{A}(w) = \emptyset$ (this implies $i' \in \mathcal{A}(w)$), replace i' in $\mathcal{A}(w)$ by i. This replacement may be performed multiple times.
  $N := N - 1$;
 endwhile
endif

Step 3 Assign nodes having unassigned NICs with the channels having the smallest interference weights among assigned channels from their neighboring nodes.

$\Gamma_{ei}^t = 0$. We always append a special all-zero row at the end of Γ which corresponds to a transmission mode including no link-channel tuple.

The mean data rate of link-tuple (e, i) can be obtained as $\sum_{t:\Gamma_{ei}^t=1} p_t c_e$, where p_t is the fraction of time that transmission mode t is activated and c_e is the capacity of link e. According to a scheduling-based MAC protocol, a transmission mode is activated in each time slot. Suppose that we know all possible transmission modes. The transmission scheduling problem in the MAC layer is to determine the frame length L and the number of active time slots in one frame for each transmission mode. Actually, we can calculate a frame length by finding the smallest positive integer L such that $p_t \cdot L$ is an integer for every transmission mode. Correspondingly, transmission mode t should be activated in $p_t \cdot L$ time slots. Therefore, the scheduling problem is further transformed into a problem of computing the time fraction for each transmission mode. However, it may be impossible to find such an integer L since p_t could be an irrational number. In this case, p_t can be rounded to obtain an approximated frame length L, which will be a very close approximation.

The number of all possible transmission modes grows exponentially with the number of link-channel tuples. Therefore, we present Algorithm 2, a heuristic algorithm, to find a subset of all possible transmission modes.

In Algorithm 2, T_M represents a transmission mode. Z is output as the computed subset of all possible transmission modes. **Step 2** makes sure that every available link-channel tuple is covered by Z at least once. Furthermore, there is a weight variable, W_e^i, associated with each link-channel tuple, and recording how many times it is included in Z during the execution of the algorithm. The link-channel tuple with the smallest weight value will be selected into T_M, which helps to create a relatively even distribution of the number of times a specific link-channel tuple is selected.

$LP4(h, E_j)$ is used to verify if link h and the existing set of links in T_M working on channel j (E_j) can be simultaneously active on channel j. Eventually, the solutions given by $LP4(h, E_j)$s can be used as the power assignment for the corresponding transmission mode. Even though we only need a feasible power assignment or need to test if there exists a feasible solution, it is always good to minimize the total power consumption which is achieved by the objective function (17). Constraint (18) is the SINR constraint (1) which is described in the system model. Here, $s(l)$ and $t(l)$ stand for the transmitting and receiving nodes of link l respectively. Constraint (18) ensures that each computed power level is in the range $[0, P_{max}]$.

Algorithm 2 Finding Transmission Mode Set

Step 1 $Z := \emptyset$; $i := 1$; $W_e^i := 0, \forall (e, i) \in EI$;
Step 2 **while** $(i <= \omega)$
 $T_M := \emptyset$;
 forall $((e, i) \in EI)$
  Add (e, i) to T_M;
  $W_e^i := W_e^i + 1$;
  do Add $(h, j) \neq (e, i)$ to T_M,
  s.t. $LP4(h, E_j)$ has a feasible solution
   and W_h^j is minimum among all
   link-channel tuples not in T_M;
   $W_h^j := W_h^j + 1$;
  until no more link-channel tuple can
   be added to T_M;
 endforall
 if $(T_M \notin Z)$
  $Z := Z \cup \{T_M\}$;
 endif
 $i := i + 1$;
endwhile
Step 3 **output** Z;

$LP4(h, E_j)$:

$$\min \sum_{l \in E_j \bigcup \{h\}} P_l \tag{17}$$

subject to:

$$G_{s(l)t(l)} P_l - \beta \sum_{q \in E_j \bigcup \{h\} \setminus \{l\}} G_{s(q)t(l)} P_q - \beta N_0 \geq 0,$$
$$\forall l \in E_j \bigcup \{h\}; \tag{18}$$
$$0 \leq P_l \leq P_{max}, \ \forall l \in E_j \bigcup \{h\}. \tag{19}$$

In **Step 2** of Algorithm 2, ω is a tunable parameter. We observe that the larger the ω is, the more transmission modes will be added into Z, which will make the final solutions closer to the optimal ones at the cost of increasing the time complexities of our schemes.

C. Rate Allocation

In this section, we present LP and CP formulations for the MRA, MMRA and PRA problems based on the transmission modes computed in the second step and channel assignments computed in the first step.

$LP5$: MRA

$$\max \quad \sum_{k=1}^{K} r_k$$

subject to:

$$\Big(\sum_{(e,i)\in EI_{s_k}^{out}} f_{ei}^k - \sum_{(e,i)\in EI_{s_k}^{in}} f_{ei}^k\Big) = r_k,\ 1 \le k \le K; \quad (20)$$

$$\sum_{(e,i)\in EI_{v}^{out}} f_{ei}^k - \sum_{(e,i)\in EI_{v}^{in}} f_{ei}^k = 0,$$
$$1 \le k \le K,\ \forall v \in V \setminus \{s_k, t_k\}; \quad (21)$$

$$\sum_{k=1}^{K} f_{ei}^k \le \sum_{t:\Gamma_{ei}^t=1} p_t c_e,\ \forall (e,i) \in EI; \quad (22)$$

$$\sum_{t=1}^{T} p_t = 1; \quad (23)$$

$$f_{ei}^k \ge 0,\ \forall (e,i) \in EI,\ 1 \le k \le K; \quad (24)$$
$$0 \le r_k \le d_k,\ 1 \le k \le K.$$

In the above LP formulation, we have the aforementioned rate allocation variables r_k and flow allocation variables f_{ei}^k and the transmission schedule variables p_t. EI_v^{out}, EI_v^{in} and EI_v denote the set of link-channel tuples whose corresponding links are the outgoing, incoming and incident links of node $v \in V$. Similar to $LP1$, Constraints (20) and (21) correspond to the flow feasibility constraints. Constraints (23) guarantees that the mean data rate of a specific link on a certain channel given by the transmission schedule is large enough to support the amount of traffic going through that link on that channel. Obviously, the summation of the values of p_t should be equal to 1 (Constraint(23)) since p_t is the fraction of time using transmission mode t.

Similar to $LP2$, $LP3$, we use $LP6$ to find max-min rate allocation value α first and then solve $LP7(\alpha)$ to compute max-min guaranteed maximum throughput rate allocation. In addition, we present a CP formulation, $CP2$, to compute the proportional fair rate allocation.

$LP6$:

$$\max \quad \alpha$$

subject to:

$$Constraints\ (21)-(24);$$
$$\Big(\sum_{(e,i)\in EI_{s_k}^{out}} f_{ei}^k - \sum_{(e,i)\in EI_{s_k}^{in}} f_{ei}^k\Big)/d_k = \alpha_k,$$
$$1 \le k \le K; \quad (25)$$
$$\alpha \le \alpha_k \le 1,\ 1 \le k \le K.$$

$LP7(\alpha)$: MMRA

$$\max \quad \sum_{k=1}^{K} r_k$$

subject to:

$$Constraints\ (20)-(24);$$
$$\alpha d_k \le r_k \le d_k,\ 1 \le k \le K.$$

$CP2$: PRA

$$\max \quad \sum_{k=1}^{K} \log(\alpha_k)$$

subject to:

$$Constraints\ (21)-(24),(25);$$
$$0 \le \alpha_k \le 1,\ 1 \le k \le K.$$

In summary, our MRA scheme is to $1)$ apply Algorithm 1 to compute channel assignment based on the flow allocation given by solving $LP1$; $2)$ use Algorithm 2 to find a set of transmission modes and their corresponding power assignments according to the computed channel assignment; $3)$ solve $LP5$ to find a rate allocation, a flow allocation and a scheduling solution for the MRA problem. Similarly, our MMRA (PRA) scheme is to apply Algorithm 1 based on $LP2$ and $LP3$ ($CP1$), use Algorithm 2, and then solve $LP6$ and $LP7$ ($CP2$).

VI. Numerical Results

In our simulations, we consider wireless networks with static nodes randomly located in a $1200 \times 1200\ m^2$ region. The maximum transmission power $P_{max} = 300mW$, the thermal noise power $N_0 = -90dBm$, and the SINR threshold $\beta = 10dB$. The channel gain, G_{uv} is set to $1/d_{uv}^4$, where d_{uv} is the Euclidean distance between node u and node

v. For each simulation scenario, we generate 15 communication sessions with random source and destination nodes. The traffic demand for each communication session (d_k) is also given by a random number uniformly distributed in $[0.2c, 0.6c]$, where c is the link capacity. We solve all LPs using *CPLEX 9.0* ([13]) and apply the barrier method (Algorithm 11.1) introduced in Chapter 11 of [4] to solve all CPs by setting the related parameters as follows: $\epsilon = 10^{-3}$, $\mu = 120$ and $t^{(0)} = 2$.

We evaluate the performance of the three rate allocation schemes in terms of DSF of each session (α_k) and total throughput ($\sum_{k=1}^{K} r_k$). We also compare the computed throughput and max-min DSF values against the corresponding upper bounds computed by solving $LP1$, $LP2/LP3$ and $CP1$.

We perform simulations on networks with n nodes and C channels. Each node is equipped with Q NICs, each of which can transmit at the rate of c Mbps. n, C, Q and c will be set to different values under different scenarios. The simulation results are presented in the following figures.

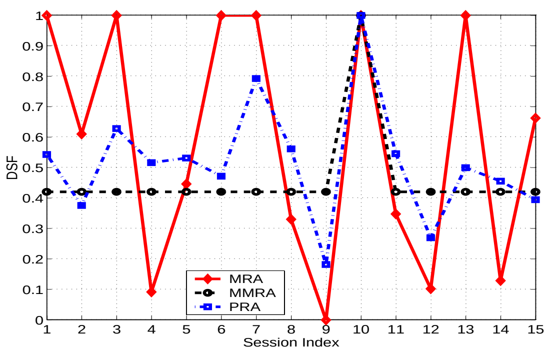

Fig. 1. DSF: Scenario 1 $(n = 10, C = 3, Q = 2, c = 11)$

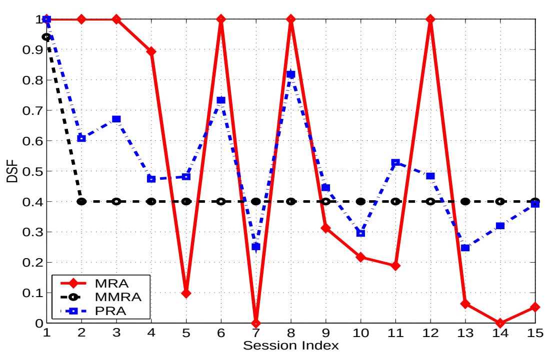

Fig. 2. DSF: Scenario 2 $(n = 15, C = 3, Q = 2, c = 11)$

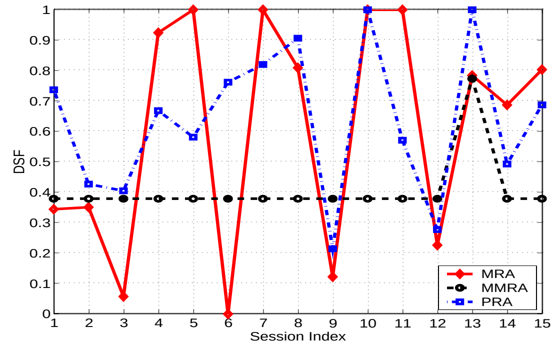

Fig. 3. DSF: Scenario 3 $(n = 10, C = 5, Q = 2, c = 54)$

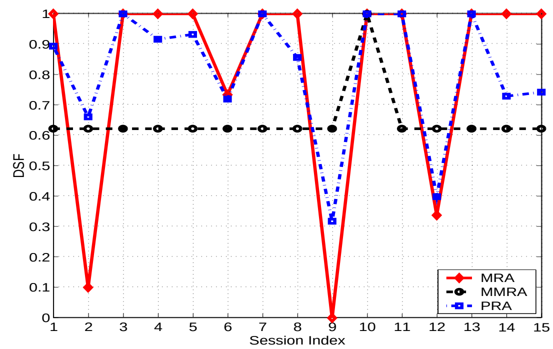

Fig. 4. DSF: Scenario 4 $(n = 10, C = 5, Q = 3, c = 54)$

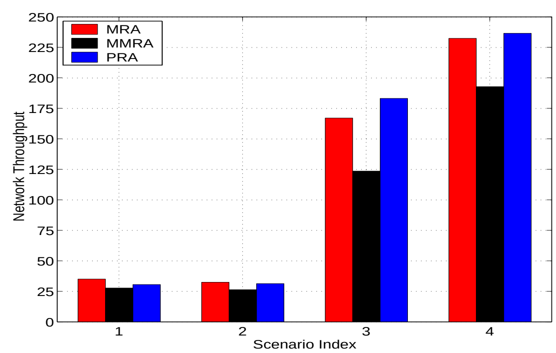

Fig. 5. Network throughput

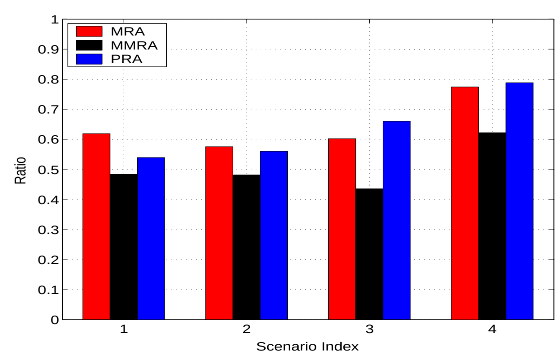

Fig. 6. Upper bound ratio

In Fig. 1–Fig. 4, we use DSF vector values to illustrate the fairness achieved by the three schemes. As expected, we observe that the MRA scheme results in a severe unfairness on rate allocation in all simulation scenarios. For example, in Scenario 2 (Fig. 2), demands of some sessions, such as session 1, 2, 3, 6, 8, 12, are completely satisfied ($\alpha_k = 1$). However, the DSFs of some other sessions, such as

5, 7, 13, 14 and 15, are close to 0. The MMRA scheme achieves fairest rate allocation since the DSFs of all sessions are almost the same. In addition, we can see that the PRA scheme obtains much fairer rate allocation than the MRA scheme. With regards to throughput, the PRA scheme is surprisingly good. Compared with our MRA scheme, the PRA scheme provides comparable throughput in Scenario 1 and 2, and achieves even higher throughput in Scenario 3 and 4. This is possible because our MRA scheme is a heuristic scheme which does not guarantee to find the maximum throughput solutions even its objective is to maximize throughput. However, the MMRA scheme obtains very low network throughput in every scenario. Its average throughput is only 77% of that given by the PRA scheme. For the MRA and PRA scheme, the upper bound ratio is defined as the ratio between the computed throughput and its upper bound. For the MMRA scheme, it is defined as the ratio between the computed max-min DSF value and its upper bound. From Fig. 6, we observe that both our MRA and PRA schemes perform very well in terms of the upper bound ratio. For the cases in which the networks have relative rich resources (Scenario 3 and 4), the ratios given by those two schemes are larger than 0.6 and become even close to 0.8 in Scenario 4.

VII. CONCLUSIONS

In this paper, we have studied the joint rate allocation, channel assignment, routing, scheduling and power control problems in multi-radio WMNs. We presented three efficient cross-layer schemes to solve the Maximum throughput Rate Allocation (MRA), Max-Min guaranteed maximum throughput Rate Allocation (MMRA) and Proportional fair Rate Allocation (PRA) problems respectively. Our schemes can provide not only upper bounds on achievable throughput or max-min DSF values but also optimal rate allocation, routing and scheduling solutions for the given transmission modes and the channel assignment. The numerical results show that the PRA scheme achieves a good tradeoff between throughput and fairness. Moreover, the throughput given by our MRA and PRA schemes are close to the corresponding upper bounds.

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