Leveraging Cooperative, Channel and Multiuser Diversities for Efficient Resource Allocation in Wireless Relay Networks

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***Abstract*—Relay stations can be deployed between mobile stations and base stations in a single-hop wireless network to extend its coverage and improve its capacity. In this paper, we exploit cooperative diversity, channel diversity and multiuser diversity gains in an OFDMA-based wireless relay network. We study a joint channel and relay assignment problem with the objective of maximizing a well-adopted utility function that can lead to a stable system. This problem turns out to be NP-hard. First, a mixed integer linear programming formulation is presented to provide optimal solutions. We then present three simple greedy algorithms to solve the problem in polynomial time, namely, Greedy-ChannelFirst, Greedy-RelayFirst and Greedy-Joint. We also perform a comprehensive theoretical analysis for the performance of the proposed algorithms. Our analytical results show the Greedy-ChannelFirst algorithm is a constant factor approximation algorithm which always provides a solution whose objective value is guaranteed to be no smaller than the optimal value multiplied by a constant less than 1; however, the other two algorithms do not provide a similar performance guarantee. Extensive simulation results have been presented to show that all three proposed algorithms perform very well on average cases.**

***Index Terms*—Wireless relay network, cooperative diversity, channel diversity, multiuser diversity, OFDMA, resource allocation.**

I. INTRODUCTION

Relay Stations (RSs) can be deployed between Mobile Stations (MSs) and Base Stations (BSs) in a single-hop wireless networks to extend its coverage and improve its capacity. The IEEE 802.16j task group [2] has been formed to extend the scope of IEEE 802.16e [1] to support Mobile Multihop Relay (MMR). In this paper, we consider a wireless relay network consisting of a Base Station (BS), a few Relay Stations (RSs) and a large number of Mobile Stations (MSs), as shown in Fig. 1. The BS serves as a gateway connecting the wireless network to external networks such as the Internet. RSs can be used to forward signals or packets received from a source node to the intended destination node. A WiMAX relay network [2] is a typical example of such a network.

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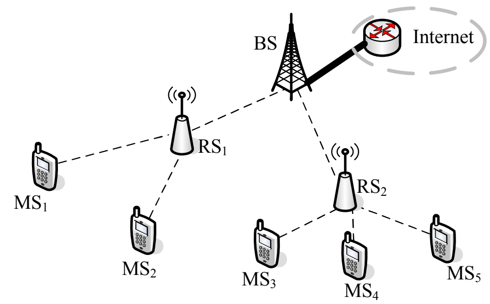

Fig. 1. A wireless relay network

Orthogonal Frequency Division Multiplexing Access (OFDMA) is an emerging OFDM-based multiple access technology. With OFDMA, the operating spectrum is divided into multiple sub-channels, each of which consists of multiple narrow frequency bands (a.k.a. sub-carriers). The multiple access is achieved by assigning sub-channels to different users in the network for simultaneous transmissions. Multipath fading and user mobility lead to independent fading across users for a channel. Therefore, the gains of a given channel for different users vary, which is referred to as *multiuser diversity* [16]. Moreover, different channels may experience different channel gains for an MS due to frequency selective fading, which is referred to as *channel diversity* [16]. Hence, sub-channels should be carefully assigned to proper users to achieve the maximum possible channel gains. In addition, an RS can leverage the wireless broadcast advantage in order to help intended receivers combat fading and improve Signal to Noise Ratio (SNR) through cooperative relaying, which is referred to as *cooperative diversity* [10]. Various cooperative communication schemes such as Amplify-and-Forward (AF) and Decode-and-Forward (DF), have been proposed in the literature. Regardless of cooperative schemes, the selection of relay nodes always plays a key role in system performance.

In this paper, we exploit cooperative, channel and multiuser diversities in an OFDMA-based wireless relay network. We study a joint relay and channel assignment problem with the objective of maximizing a well-adopted utility function that can lead to a stable system [3]. This problem turns out to be NP-hard. Even though both OFDMA-based resource allocation and cooperative communications have been well studied, few works jointly considered these three diversities. In addition, we are the first to present a comprehensive theoretical analysis for the worst-case performance of simple greedy algorithms. Specifically, our contributions are summarized as follows:

(1) We present a Mixed Integer Linear Programming (MILP) formulation to provide optimum solutions, which can serve as a benchmark for performance evaluation.
(2) We present three simple greedy algorithms to solve the problem in polynomial time, namely, Greedy-ChannelFirst, Greedy-RelayFirst and Greedy-Joint.
(3) We perform a comprehensive theoretical analysis for the performance of the proposed algorithms. The analytical results show that the Greedy-ChannelFirst algorithm is a constant factor approximation algorithm (i.e., if the problem is a maximization problem, then the objective value of a solution given by the algorithm is guaranteed to be no smaller than the optimal value multiplied by a constant less than 1). Furthermore, we demonstrate that the other two algorithms cannot provide a similar performance guarantee.
(4) Extensive simulation results are presented to justify the efficiency of the proposed algorithms.

The rest of this paper is organized as follows. We discuss related works in Section II. We describe the system model and present problem formulation in Sections III and IV, respectively. The proposed algorithms are presented in Section V. We present numerical results in Section VI and conclude the paper in Section VII.

II. Related Work

Resource allocation in OFDMA-based wireless networks has been well studied recently. In [3], Andrews and Zhang considered several scheduling problems in single-hop OFDMA wireless networks. They analyzed the hardness of these problems and present several simple constant factor approximation algorithms to solve them. In a later work [4], they studied the problem of creating template-based schedules for such networks. They presented a general framework to study the delay performance of a multi-carrier template. They then described several deterministic and randomized scheduling algorithms for template creation and studied their delay performance via analysis and simulation. In [16], Sundaresan *et al.* showed that the scheduling problem to exploit diversity gains alone in 2-hop WiMAX relay networks is NP-hard, and provided polynomial time approximation algorithms to solve it. They also proposed a heuristic to exploit both spatial reuse and diversity gains. In [7], a similar scheduling problem was studied for OFDMA-based WiMAX relay networks. The authors provided an easy-to-compute upper bound. They also presented three heuristic algorithms and showed that they provide close-to-optimal solutions and outperform other existing algorithms by simulation results. Other recent works on this topic include [11], [17], [19].

Cooperative communications have attracted extensive research attention recently. In a well-known work [10], the authors outlined several strategies employed by cooperating radios, including fixed relaying schemes such as AF and DF, selection relaying schemes and incremental relaying schemes. They also developed performance characterizations in terms of outage events and associated outage probabilities. In [15], Shi *et al.* developed a polynomial time optimal algorithm to solve a relay node assignment problem in a multihop wireless network where multiple source-destination pairs compete for the same pool of relay nodes. In a very recent paper [13], the authors designed an iterative search algorithm to solve a relay station cooperation problem for throughput maximization.

In [5], Awad and Shen formulated a joint channel and relay assignment problem with QoS constraints in OFDMA-based 2-hop cooperative relay networks as an ILP problem and presented a simple heuristic algorithm to solve it. In a recent work [8], Han *et al.* formulated a power-optimization, subcarrier-allocation and relay-selection problem for OFDM networks, and presented an approximate closed-form solution for a two-user two-subcarrier case as well as a heuristic algorithm for a multiuser multiple-subcarrier case. Several other resource allocation problems have also been studied for OFDMA-based cooperative relay networks in [9], [12].

The difference between our work and these related works are summarized as follows: 1) We jointly consider channel, multiuser and cooperative diversities. However, most related works [3], [4], [7], [11], [16], [17], [19] did not address cooperative diversity and some papers [10], [13], [15] did not consider channel diversity. 2) We present a comprehensive theoretical analysis for the worst-case performance of the proposed algorithms and show one of them is a constant factor polynomial time approximation algorithm, whereas most related works [5], [8], [9], [12] presented heuristic algorithms that cannot provide any performance guarantees, or algorithms that may take a long time to converge to optimal. 3) We use a well-adopted utility function as the objective function, which is different from those considered in related works on OFDMA-based cooperative communications [5], [8], [9], [12].

III. System Model

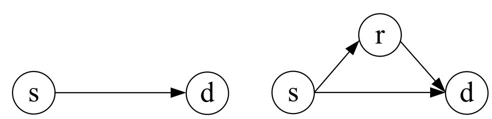

Fig. 2. Direct communication VS. cooperative communication

A pair of wireless nodes, the source node s and destination node d, can work in either the direct communication mode in which s directly communicates with d or the cooperative mode in which an RS r is used to relay signals or packets, which is shown in Fig. 2. We consider two cooperative communication schemes, AF and DF. With AF, the cooperative RS performs a linear operation on the signal received from the source node and forwards it to the destination node. With DF, the cooperative relay node decodes the received signal, re-encodes it and forwards it to the destination node. The transceiver at the RS works in the Time Division Multiplex (TDD) transmission mode and a well-adopted 2-timeslot approach is assumed to be used for scheduling [15]. Scheduling is conducted on a frame-by-frame basis and each frame includes 2 timeslots. In the first time slot, the source node makes a transmission to the destination node. This transmission is also overheard by the RS, due to the broadcast nature of wireless communications. In the second time slot, the RS forwards the signals or packets received in the first time slot to the destination node. In addition, the relay network system under consideration is an OFDMA-based system in which the operating spectrum is divided into multiple sub-channels and each of them consists of multiple sub-carriers. The capacity gains obtained by assigning RS r and sub-channel k to link (s, d) with AF and DF respectively are given as follows:

$$C_{AF}(s,d,r,k) = \frac{W}{2}\log_2(1 + SNR_{sd}^k + \frac{SNR_{sr}^k SNR_{rd}^k}{SNR_{sr}^k + SNR_{rd}^k + 1}); \quad (1)$$

$$SNR_{sr}^k = \frac{P_s}{\sigma^2}|h_{sr}^k|^2; \quad (2)$$

$$SNR_{sd}^k = \frac{P_s}{\sigma^2}|h_{sd}^k|^2; \quad (3)$$

$$SNR_{rd}^k = \frac{P_r}{\sigma^2}|h_{rd}^k|^2; \quad (4)$$

$$C_{DF}(s,d,r,k) = \frac{W}{2}\min\{\log_2(1 + SNR_{sr}^k), \log_2(1 + SNR_{sd}^k + SNR_{rd}^k)\}; \quad (5)$$

According to Shannon's theorem, the capacity gain of link (s, d) using sub-channel k and direct transmission without a relay is given by the following equation.

$$C_D(s,d,k) = W\log_2(1 + SNR_{sd}^k); \quad (6)$$

The major notations are summarized in Table I.

We consider a 2-hop relay networks with a BS v_0, m RSs $\{r_1, \ldots, r_j, \ldots, r_m\}$ and n MSs $\{v_1, \ldots, v_i, \ldots, v_n\}$. Centralized resource allocation is considered here. Each MS reports its Channel State Information (CSI) and traffic information (queue length) to the BS periodically. The central control unit at the BS runs a resource allocation algorithm to find a joint relay and sub-channel assignment for every MS, then sends it to all the MSs and RSs. A WiMAX system supports such a centralized resource allocation approach, whose details can be found in the IEEE 802.16e standard [1].

TABLE I
MAJOR NOTATIONS

C_{ij}^k	The capacity gain obtained by assigning RS r_j and sub-channel k to MS v_i;
$C_{AF}(s,d,r,k)$	The capacity gain obtained by assigning RS r and sub-channel k to link (s,d) and using AF;
$C_{DF}(s,d,r,k)$	The capacity gain obtained by assigning RS r and sub-channel k to link (s,d) and using DF;
$C_D(s,d,k)$	The capacity gain obtained by assigning sub-channel k to link (s,d) and using direct communications;
$\|h_{sd}^k\|^2$	The channel gain between nodes s and d on sub-channel k (which captures pass loss, shadowing and fading.);
K	The number of sub-channels in the network;
n/m	The number of MSs/RSs in the network;
P_s/P_r	The transmit power levels at source node and relay node respectively;
$\mathbf{Q}$	The queue length vector, $\mathbf{Q} = [q_1, \ldots, q_i, \ldots, q_m]$, where q_i is the queue length of MS v_i at the beginning of a frame;
SNR_{sd}^k	The signal-to-noise ratio for transmissions from node s to node d on sub-channel k;
σ^2	Background noise;
W	The bandwidth of a sub-channel;
x_{ij}^k	A decision variable for the MILP, $x_{ij}^k = 1$ if RS r_j and sub-channel k are assigned to MS v_i, and $x_{ij}^k = 0$, otherwise;
y_{ij}	A decision variable for the MILP, $y_{ij} = 1$ if RS r_j is assigned to MS v_i, and $y_{ij} = 0$, otherwise;
z_i	A decision variable for the MILP, which gives the effective utility of MS v_i.

IV. PROBLEM FORMULATION

We define the resource allocation problem as follows.

Definition 1: Given n MSs, m RSs K sub-channels, the queue length vector $\mathbf{Q}$, the resource allocation problem seeks a joint relay and channel assignment that assigns a subset S_i of sub-channels and an RS to each MS v_i such that the utility function $\sum_{i=1}^m q_i \min\{q_i, \sum_{k\in B_i} c_i^k\}$ is maximized, where c_i^k is the capacity gain obtained by MS v_i using sub-channel k and the assigned RS.

We choose the above objective function because it is known that it can lead to a stable system, i.e., keep the length of each queue finite [3]. In addition, as pointed out by [3], a commonly used weighted sum throughput function ignores user's demands (queue lengths), which has the potential shortcoming of assigning more transmission capacity to a user than it can actually use. The objective function considered here offers a natural fix by only counting part of transmission capacity that can actually be used, therefore, it is more accurate and reasonable. This problem turns out to be NP-hard since a special case of this problem where there is only one RS is the same as one of the problems studied in [3] which has been shown to be NP-hard. This problem can be formulated as an MILP problem, which is given as follows.

MILP:

$$\max \sum_{i=1}^n q_i z_i \quad (7)$$

subject to:

$$z_i \le q_i, \qquad i \in \{1, \ldots, n\}; \quad (8)$$

$$z_i \le \sum_{j=0}^m \sum_{k=1}^K C_{ij}^k x_{ij}^k, \qquad i \in \{1, \ldots, n\}; \quad (9)$$

$$\sum_{j=0}^m \sum_{i=1}^n x_{ij}^k \le 1, \qquad k \in \{1, \ldots, K\}; \quad (10)$$

$$\sum_{j=1}^m y_{ij} \le 1, \qquad i \in \{1, \ldots, n\}; \quad (11)$$

$$x_{ij}^k \le y_{ij}, \quad i \in \{1, \ldots, n\}, j \in \{1, \ldots, m\}, k \in \{1, \ldots, K\}; \quad (12)$$

$$x_{ij}^k = \{0, 1\}, \quad i \in \{1, \ldots, n\}, j \in \{1, \ldots, m\} \; k \in \{1, \ldots, K\}; \quad (13)$$

$$y_{ij} = \{0, 1\}, \quad i \in \{1, \ldots, n\}, j \in \{1, \ldots, m\}. \quad (14)$$

In this formulation, $C_{i0}^k = C_D(v_i, v_0, k)$ ($j = 0$ means no RS is used.) $C_{ij}^k = C_{AF}(v_i, v_0, r_j, k)$ if we consider this problem for a relay network using AF, and $C_{ij}^k = C_{DF}(v_i, v_0, r_j, k)$ if using DF. Constraints (8) and (9) make sure that the decision variables z_i give the effective utility of MS v_i. Constraints (10) ensures that a sub-channel can be assigned to at most one pair of MS and RS. In addition, an MS can only be associated with at most one RS, which is guaranteed by constraints (11).

The MILP problem can be solved using the branch-and-bound algorithm [18] to obtain optimal solutions, which can serve as the benchmark for performance evaluation. However, it is known that solving an MILP problem may take an exponentially long time. Therefore, this approach cannot be used in a real-time manner on larger cases. Hence, we also propose three polynomial time algorithms for this problem.

V. Proposed Algorithms

In this section, we present three simple greedy algorithms, namely, *Greedy-ChannelFirst*, *Greedy-RelayFirst*, and *Greedy-Joint*, to solve the resource allocation problem in polynomial time. We also theoretically analyze the worst-case performance of the proposed algorithms.

A. The Greedy-ChannelFirst Algorithm

The Greedy-ChannelFirst algorithm is based on a greedy approach that first decides how to allocate sub-channels among MSs and then assigns each MS to the RS that best serves it using its allocated sub-channels. Specifically, each MS v_i is allocated a set S_i of sub-channels, and then each MS simply chooses the best RS to use based on S_i.

Let $C_i^k = \max_j C_{ij}^k$. To assign sub-channels, we will use an optimistic estimate of the objective value achievable at each MS v_i with the set S_i of allocated sub-channels,

$$\text{opt}(i, S_i) = q_i \min(q_i, \sum_{k \in S_i} C_i^k). \tag{15}$$

This estimate optimistically assumes that there is a single relay that can provide the optimal capacity to MS v_i for all of the sub-channels in S_i. To decide whether to allocate a new sub-channel k to an MS v_i, we use the following function:

$$\text{gain}(k, i, S_i) = \text{opt}(i, S_i \cup \{k\}) - \text{opt}(i, S_i). \tag{16}$$

The idea of this algorithm is to initially let all sub-channels be unallocated and then assign them in a greedy fashion based on which unallocated sub-channel k and MS v_i yield the highest value of gain(k, i, S_i). The algorithm is formally presented as Algorithm 1.

The running time of the Greedy-ChannelFirst algorithm is dominated by Step 2 which takes $O(nmK)$ time, where n, m and K are the numbers of MSs, RSs and sub-channels respectively.

Algorithm 1 Greedy-ChannelFirst

Step 1 Set all sub-channels unallocated;
  Set all $S_i := \emptyset$;
  Set all $x_{ij}^k := 0, y_{ij} := 0$;
Step 2 **while** $\exists$ an unallocated sub-channel k
    Select a pair of unallocated sub-channel and MS $(k_{\max}, i_{\max})$ with the highest gain(k, i, S_i) and allocate sub-channel $k_{\max}$ to MS $v_{i_{\max}}$: $S_{i_{\max}} := S_{i_{\max}} \cup \{k\}$;
  endwhile
Step 3 **for** $i = 1$ **to** n
    Find an RS r_j that maximizes $\sum_{k \in S_i} C_{ij}^k$;
    $y_{ij} := 1$;
    forall $k \in S_i$ $x_{ij}^k := 1$; **endforall**
  endfor

B. The Greedy-RelayFirst Algorithm

The Greedy-RelayFirst algorithm is a greedy algorithm based on first deciding a relay assignment for each MS v_i and then greedily assigning sub-channels to MSs. Let

$$C_{ij} = \frac{1}{K} \sum_k C_{ij}^k \tag{17}$$

be the average capacity among all sub-channels for MS v_i and RS r_j. We assign RSs using the following rule: MS v_i is assigned to the RS that provides it the highest average capacity, $r(i) = \text{argmax}_j C_{ij}$. We modify the gain function slightly from (16):

$$\begin{aligned} \text{gain}(k, i, S_i) &= q_i \min(q_i, \sum_{k \in S_i \cup \{k\}} C_{i,r(i)}^k) \\ &\quad - q_i \min(q_i, \sum_{k \in S_i} C_{i,r(i)}^k). \end{aligned} \tag{18}$$

The algorithm proceeds by greedily choosing the unallocated sub-channel and MS that has the highest gain using (18) and allocating that sub-channel to that MS. The full details of the algorithm are provided below. The running time of this algorithm is $O(n(m + K))$, which is better than the Greedy-ChannelFirst algorithm.

C. The Greedy-Joint Algorithm

The Greedy-Joint algorithm is another greedy variant algorithm where the relay and channel assignment is computed jointly. If MS v_i has not yet been assigned any sub-channels ($S_i = \emptyset$), then it may use any RS. Once v_i is assigned an RS $r(i)$ it must keep using this relay. We modify the gain function again to reflect this:

$$\text{gain}(k, i, \emptyset) = q_i \min(q_i, \max_j C_{ij}^k); \tag{19}$$

$$\begin{aligned} \text{gain}(k, i, S_i) &= q_i \min(q_i, \sum_{k \in S_i \cup \{k\}} C_{i,r(i)}^k) \\ &\quad - q_i \min(q_i, \sum_{k \in S_i} C_{i,r(i)}^k). \end{aligned} \tag{20}$$

The algorithm proceeds by finding the unallocated sub-channel and MS that has the highest gain according to (19) and

Algorithm 2 Greedy-RelayFirst

Step 1 Set all sub-channels unallocated;
Set all $S_i := \emptyset$;
Set all $x_{ij}^k := 0, y_{ij} := 0$;
Step 2 **for** $i = 1$ **to** n
$r(i) := \text{argmax}_j C_{ij}$;
$y_{i,r(i)} := 1$;
endfor
Step 3 **while** $\exists$ an unallocated sub-channel k
Select a pair of unallocated sub-channel and MS $(k_{\max}, i_{\max})$ with the highest gain(k, i, S_i) and allocate sub-channel $k_{\max}$ to MS $v_{i_{\max}}$:
$S_{i_{\max}} := S_{i_{\max}} \cup \{k\}$;
endwhile
Step 4 **for** $i = 1$ **to** n
forall $k \in S_i$ $x_{i,r(i)}^k := 1$; **endforall**
endfor

(20) and greedily assigning that sub-channel to that MS. We formally present the algorithm in the following. Similar to the Greedy-ChannelFirst algorithm, this algorithm also has a time complexity of $O(nmK)$.

Algorithm 3 Greedy-Joint

Step 1 Set all sub-channels unallocated;
Set all $S_i := \emptyset$;
Set all $x_{ij}^k := 0, y_{ij} := 0$;
Set all $r(i) := 0$ (default value);
Step 2 **while** $\exists$ an unallocated sub-channel k
Select a pair of unallocated sub-channel and MS $(k_{\max}, i_{\max})$ with the highest gain(k, i, S_i):
if $(S_i = \emptyset)$, $r(i) := \text{argmax}_j C_{ij}^k$; **endif**
Allocate sub-channel $k_{\max}$ to MS $v_{i_{\max}}$:
$S_{i_{\max}} := S_{i_{\max}} \cup \{k\}$;
endwhile
Step 3 **for** $i = 1$ **to** n
$y_{i,r(i)} := 1$;
forall $k \in S_i$ $x_{i,r(i)}^k := 1$; **endforall**
endfor

D. Optional Secondary Optimization

After running the Greedy-RelayFirst algorithm or the Greedy-Joint algorithm, it may be the case that an MS can be better served by another relay (without changing its assigned sub-channels) rather than the one that it was originally assigned. Algorithm 4 checks for this situation and tries to make some improvement. In the algorithm, S_i is the set of sub-channels assigned to MS v_i given by the Greedy-RelayFirst algorithm or the Greedy-Joint algorithm. Note that it is not necessary to run Algorithm 4 after the Greedy-ChannelFirst algorithm, since the RSs are picked after the channels are assigned thus no MS will prefer another RS.

E. Performance Analysis

First, we show that the Greedy-ChannelFirst algorithm has a constant approximation ratio. To analyze its performance, we

Algorithm 4 Post-Optimization

Step 1 Set all $x_{ij}^k := 0, y_{ij} := 0$;
Step 2 **for** $i = 1$ **to** n
$r(i) := \text{argmax}_j \, q_i \min(q_i, \sum_{k \in S_i} C_{ij}^k)$
$y_{i,r(i)} := 1$
forall $k \in S_i$ $x_{i,r(i)}^k := 1$ **endforall**
endfor

define a notion of a *minimal best relay cover* B_i for an MS v_i: this is a set of RSs such that $\forall k, \exists j \in B_i$; $C_{ij}^k = C_i^k$ and $|B_i|$ is minimized. Let $B = \max_i |B_i|$. Typically, we would expect B to be quite small (e.g. $B = 1$), which has been verified by the simulation.

Theorem 1: The Greedy-ChannelFirst algorithm is a $\frac{1}{2B}$-approximation algorithm.

Proof: Let $(x_{ij}^{k*}, y_{ij}^*, z_i^*)$ be an optimal solution to the original MILP that achieves total utility $\sum_{i=1}^n q_i z_i^*$. Step 2 of the Greedy-ChannelFirst algorithm uses a greedy approach to solve a relaxed version of the original MILP where constraints (9) are replaced with:

$$z_i \leq \sum_{j=0}^{m} \sum_{k=1}^{K} C_i^k x_{ij}^k, \;\; i \in \{1, \ldots, n\}. \tag{21}$$

This new constraint relaxes the original constraint since each $C_{ij}^k \leq C_i^k$. Thus, $(x_{ij}^{k*}, y_{ij}^*, z_i^*)$ is a feasible solution to the relaxed MILP. Let $(x_{ij}^{k*r}, y_{ij}^{*r}, z_i^{*r})$ be an optimal solution to the relaxed MILP. It follows that

$$\sum_{i=1}^{n} q_i z_i^* \leq \sum_{i=1}^{n} q_i z_i^{*r}. \tag{22}$$

We claim that the greedy strategy employed by Step 2 of the Greedy-ChannelFirst algorithm provides a solution to the relaxed MILP $({x'}_{ij}^k, y'_{ij}, z'_i)$ whose objective value is at least half of optimal. The assignment of relays does not affect the value of the objective function to this relaxed MILP (e.g., we can assume that $y'_{i1} = y_{i1}^{*r} = 1$ for all i). The other variables are set by the rules: ${x'}_{ij}^k = 1 \Leftrightarrow k \in S_i \wedge j = 1$, $z'_i = \min(q_i, \sum_{k \in S_i} C_i^k)$. In the optimal solution to the relaxed MILP, at least one of the inequalities (8) and (21) must be tight. We may assume that (10) is tight as well (any unused sub-channels can be assigned arbitrarily). Let $S_i^{*r} = \{k | x_{i1}^{k*r} = 1\}$ and define i_k^* by $k \in S_{i_k^*}^{*r}$. Also, let $S_{i|k}^{*r} = S_i^{*r} \cap \{1, 2, \ldots, k\}$, and let $S_{i|0}^{*r} = \emptyset$. Let $g_k^{*r} = \text{gain}(k, i_k^*, S_{i|k-1}^{*r})$. Then,

$$\begin{aligned}
\sum_{i=1}^{n} q_i z_i^{*r} &= \sum_{i=1}^{n} q_i \min(q_i, \sum_{k=1}^{K} C_i^k x_{i1}^{k*r}) \\
&= \sum_{i=1}^{n} q_i \min(q_i, \sum_{k \in S_i^{*r}} C_i^k) \\
&= \sum_{i=1}^{n} \sum_{k \in S_i^{*r}} g_k^{*r} = \sum_{k=1}^{K} g_k^{*r}.
\end{aligned} \tag{23}$$

Similarly, we consider the S_i (channel allocation done by Step 2) and define i'_k by $k \in S_{i'_k}$. Also, let $S_{i|k} = S_i \cap$

$\{1, 2, \ldots, k\}$, and let $S_{i|0} = \emptyset$. Let $g'_k = \text{gain}(k, i'_k, S_{i|k-1})$. As above, we have

$$\sum_{i=1}^{n} q_i z'_i = \sum_{k=1}^{K} g'_k. \tag{24}$$

Let $G = \{k | g'_k \geq g^{*r}_k\}$ and $L = \{k | g'_k < g^{*r}_k\}$. We have

$$\begin{aligned}\sum_{i=1}^{n} q_i z^{*r}_i &= \sum_{k \in G} g^{*r}_k + \sum_{k \in L} g^{*r}_k \\ &\leq \sum_{k \in G} g'_k + \sum_{k \in L} (g^{*r}_k - g'_k + g'_k) \\ &= \sum_{k=1}^{K} g'_k + \sum_{k \in L} (g^{*r}_k - g'_k). \end{aligned} \tag{25}$$

We consider the final sum in (25). Let $L_i = S^{*r}_i \cap L$ and suppose $k \in L_i$. We have

$$\begin{aligned}\text{gain}(k, i, S^{*r}_{i|k-1}) &= g^{*r}_k > g'_k \\ &= \text{gain}(k, i'_k, S_{i_k|k-1}) \\ &\geq \text{gain}(k, i, S_{i|k-1}), \end{aligned} \tag{26}$$

where the final inequality in (26) follows from the fact that when sub-channel k was allocated in Step 2, it could have been allocated to MS v_i. This means that sub-channel k provides strictly less gain to MS v_i than it does in the optimal solution. This only happens when MS v_i is close to full utility already and

$$q_i \min(q_i, \sum_{k \in S_{i|k-1}} C^k_i) + \text{gain}(k, i, S_{i|k-1}) = q_i^2. \tag{27}$$

Next, we would like to bound the final term in (25). If $L_i \neq \emptyset$, let κ_i be some element in the set. Then

$$\begin{aligned}\sum_{k \in L} (g^{*r}_k - g'_k) &= \sum_{\{i | L_i \neq \emptyset\}} \sum_{k \in L_i} (g^{*r}_k - g'_k) \\ &\leq \sum_{\{i | L_i \neq \emptyset\}} \sum_{k \in L_i} (g^{*r}_k - \text{gain}(k, i, S_{i|k-1})) \\ &\leq \sum_{\{i | L_i \neq \emptyset\}} (\sum_{k \in L_i} g^{*r}_k - \text{gain}(\kappa_i, i, S_{i|\kappa_i - 1})) \\ &\leq \sum_{\{i | L_i \neq \emptyset\}} (q_i^2 - \text{gain}(\kappa_i, i, S_{i|\kappa_i - 1})) \\ &= \sum_{\{i | L_i \neq \emptyset\}} q_i \min(q_i, \sum_{k \in S_{i|\kappa_i - 1}} C^k_i) \\ &\leq \sum_{i=1}^{n} q_i \min(q_i, \sum_{k \in S_i} C^k_i) = \sum_{i=1}^{n} q_i z'_i. \end{aligned} \tag{28}$$

Note that the first inequality above follows from (26) and the final equality follows from (27). We can now use the bound established by (28) to bound the final term in (25) and obtain

$$\sum_{i=1}^{n} q_i z^{*r}_i \leq 2 \sum_{i=1}^{n} q_i z'_i. \tag{29}$$

Next, we analyze Step 3. Since $\sum_{k \in S_i} C^k_i \leq \sum_{j \in B_i} \sum_{k \in S_i} C^k_{ij}$, this implies there exists a $j_i \in B_i$ such that $\sum_{k \in S_i} C^k_i \leq |B_i| \sum_{k \in S_i} C^k_{ij_i}$. Since Step 3 chooses the best relay r_j available given the allocated sub-channels S_i, $\sum_{k \in S_i} C^k_{ij} \geq \frac{1}{B} \sum_{k \in S_i} C^k_i$. Thus

$$\begin{aligned} q_i z'_i &= q_i \min(q_i, \sum_{k \in S_i} C^k_i) \\ &\leq q_i \min(q_i, B \sum_{k \in S_i} C^k_{ij}) \leq B q_i z_i. \end{aligned} \tag{30}$$

Summing (30) over i yields

$$\sum_{i=1}^{n} q_i z'_i \leq B \sum_{i=1}^{n} q_i z_i. \tag{31}$$

Combining (22), (29) and (31) yields

$$\sum_{i=1}^{n} q_i z^*_i \leq 2B \sum_{i=1}^{n} q_i z_i, \tag{32}$$

which completes the proof. ■

Unfortunately, neither the Greedy-Joint algorithm nor the Greedy-RelayFirst algorithm has a similar approximation ratio. In fact, they can have arbitrarily poor approximation ratios on certain *extreme* input scenarios (that are admittedly somewhat contrived and will not often occur). However, it will be shown via simulations later that both algorithms performs slightly better than the Greedy-ChannelFirst algorithm on average cases. Next, we describe those "bad" input scenarios. We make the assumption that the capacities can be 0 to simplify discussion and assume that there are n MSs and two relays r_1 and r_2.

Bad input scenario for the Greedy-Joint algorithm: In this scenario, we will assume there are cn available channels, where c is an arbitrary positive integer. Each consecutive block of c channels is useful for only two MSs; block $B_i = [1 + (i-1)c, 2 + (i-1)c, \ldots, ic]$ is useful only to v_i and $v_{(i \mod n)+1}$. Let $\epsilon > 0$. Let $q_i = c$. We define

$$C^k_{i,1} = \begin{cases} 1 + \epsilon & \text{if } k = 1 + (i-1)c; \\ \epsilon & \text{if } k \in B_{(i \mod n)+1}; \\ 0 & \text{otherwise.} \end{cases}$$

and

$$C^k_{i,2} = \begin{cases} 1 & \text{if } k \in B_i; \\ 0 & \text{otherwise.} \end{cases}$$

It is easy to see that the Greedy-Joint algorithm will pick relay r_1 for each MS to use. MS v_i will be allocated the single channel $1 + (i-1)c$ from B_i and $c - 1$ channels from the previous block for a net utility of $q_i(1 + c\epsilon)$. Furthermore, there are no favorable relay swaps (since using relay r_2 only yields a utility of q_i with the existing channel assignment). On the other hand, the optimal solution is to assign each v_i to use relay r_2 and allocate it all of the channels in B_i. This provides each MS a net utility of $q_i c$. Thus, the approximation ratio provided by the Greedy-Joint algorithm is $(1 + c\epsilon)/c = 1/c + \epsilon$ which can be made arbitrarily small by choosing ϵ and c accordingly.

Bad input scenario for the Greedy-RelayFirst algorithm: We modify the above scenario slightly by adding an additional channel $(cn+1)$. Let $C^{(cn+1)}_{i,1} = c - 1$ and $C^{(cn+1)}_{i,2} = 0$. For each v_i, the average channel capacity with relay r_1 is $c(1+\epsilon)/(cn+1)$ and with relay r_2 is $c/(cn+1)$ so each

v_i is assigned r_1 and as before there are no favorable relay swaps. One MS is allocated channel $(cn+1)$, so has a utility of $c(1+\epsilon)$, the rest get a utility of $(1+c\epsilon)$, as before. So the total utility achieved by the Greedy-RelayFirst algorithm is $c(1+\epsilon)+(n-1)(1+c\epsilon)$. The optimal solution is to assign just one MS to use relay r_1 and for the remainder to use relay r_2. This yields a total utility of $c(n+\epsilon)$. The approximation ratio is $\frac{c(1+\epsilon)+(n-1)(1+c\epsilon)}{c(n+\epsilon)}$. As $n \to \infty$ and $\epsilon \to 0$ this expression approaches $1/c$ and so again can be made arbitrarily small.

VI. Numerical Results

We also evaluated the performance of the proposed algorithms via simulation. In the simulation, n MSs were randomly deployed within a circle with a radius of 600m, n RSs were randomly placed within a circle with a radius of 300m, and the BS was always placed at the center of the circle. The operating frequency was set to 5.725-5.825GHz, in which K subchannels were evenly distributed. The post-optimization algorithm (Algorithm 4) was run for both the Greedy-Relay and Greedy-Joint algorithms in the simulation. Both the large-scale fading and the small-scale fading were considered for generating channel gains for Equations (2)(3)(4). For the large-scale fading, the widely used approach in [14] was employed, where the underlying assumption is that the coherence bandwidth is less than the inter-channel spacing, enabling the channel gains to be treated as independently and identically distributed random variables. Such channels generally exhibit log-normal fading characteristics. The small scale fading is characterized by the Rayleigh distribution typical of mobile radio channels dominated by multipath effects. The narrow subchannel bandwidth allows for the flat, or frequency-independent small-scale fading. In the simulation, we calculated the channel gain by using the free space path loss (with the path loss exponent set to 4) to establish the mean power level with an additional random component derived from a Gaussian distributed random variable, where the variance was set to 6dB (consistent with widely reported channel measurements [14]). The common simulation settings are summarized in the following table. In this table, AVG gives an average over all possible link capacity values (capacity values given by all possible MS-RS-channel (i,j,k) combinations).

TABLE II
Common Simulation Settings

Deployment region	A circle with a radius of 600m;
Gain of an MS antenna	2dB;
Gain of an RS antenna	10dB;
Operating frequency	5.725-5.825GHz;
Queue length of each MS	Uniformly distributed in [AVG, $\beta *$ AVG];
Transmit power of each MS (P_s)	1W;
Transmit power of each RS (P_r)	5W;
Noise power	-174dBm/Hz;
Sub-channel bandwidth W	0.59MHz;
Variance (σ^2)	6dB

The utility value defined by the objective function (7) of the MILP formulation was used as the performance metric. We evaluated the performance of the proposed algorithms in simulation scenarios with different numbers of MSs (n), different numbers of RSs (m), different numbers of subchannels (K), different queue lengths (β). In each scenario, we changed the value of one parameter and fixed the values of the others. We summarize our 6 scenarios in the following. Note that in the first two scenarios (with relatively small input sizes), we solved the MILP formulation to obtain optimal solutions using CPLEX 10.1 [6]. The simulation results are presented in Figs. 3–8.

Scenario 1: Change n from 3 to 18 with a step size of 3. Fix $m=3$, $K=40$ and $\beta=8$.
Scenario 2: Change β from 5 to 10 with a step size of 1. Fix $n=10$, $m=3$ and $K=40$.
Scenario 3: Change n from 10 to 60 with a step size of 10. Fix $m=5$, $K=80$ and $\beta=8$.
Scenario 4: Change m from 3 to 8 with a step size of 1. Fix $n=30$, $K=80$ and $\beta=8$.
Scenario 5: Change K from 50 to 100 with a step size of 10. Fix $n=30$, $m=5$ and $\beta=8$.
Scenario 6: Change β from 5 to 10 with a step size of 1. Fix $n=30$, $m=5$ and $K=80$.

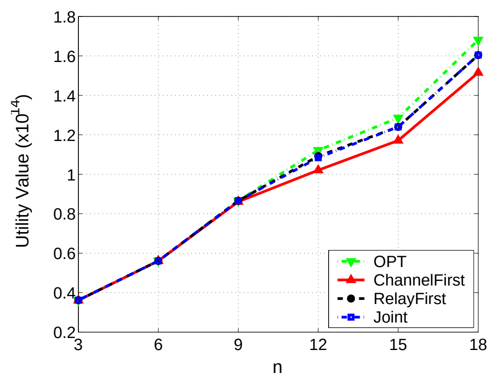

Fig. 3. *Scenario 1: Varying the number of MSs (small cases)*

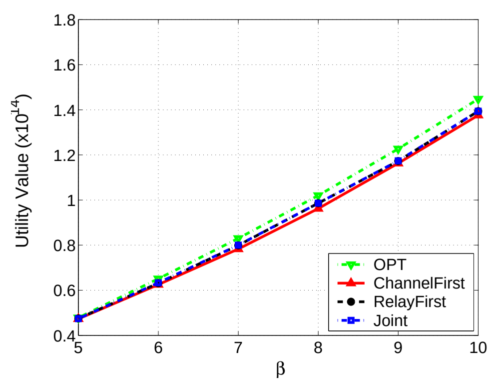

Fig. 4. *Scenario 2: Varying the average queue length (small cases)*

We make the following observations from the simulation results:

1) From Figs. 3 and 4, we can see that the utility values given by the three algorithms are all very close to the optimal

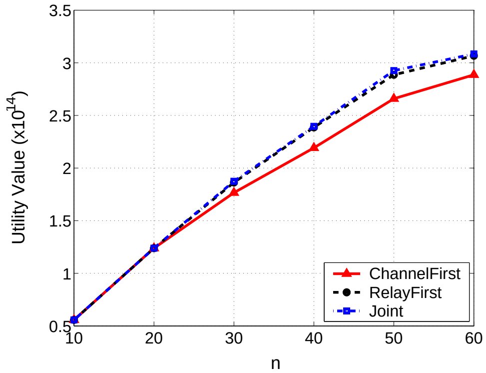

Fig. 5. *Scenario* 3*: Varying the number of MSs (larger cases)*

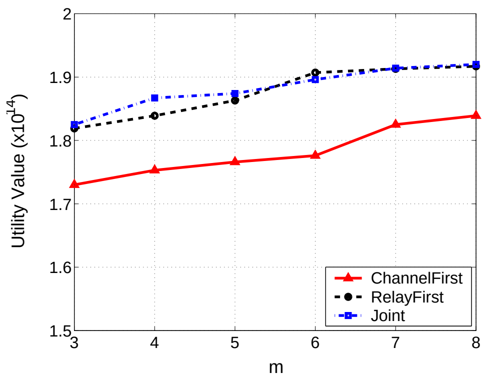

Fig. 6. *Scenario* 4*: Varying the number of RSs (larger cases)*

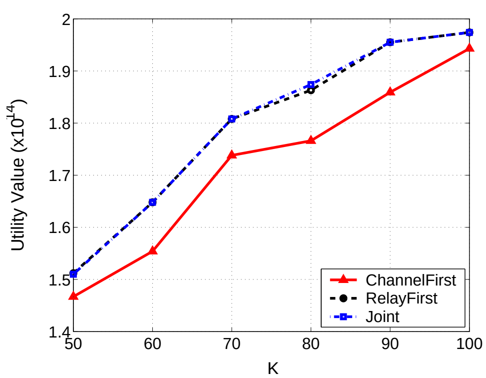

Fig. 7. *Scenario* 5*: Varying the number of channels (larger cases)*

ones. The utility values given by the Greedy-Joint algorithm and the Greedy-RelayFirst algorithm are almost the same as the optimal ones. The Greedy-ChannelFirst algorithm is slightly worse than the other two algorithms. However, the average difference from the optimal is only 4.6%.

2) In all the simulation scenarios, the Greedy-Joint algorithm performed similarly to the Greedy-RelayFirst algorithm, and slightly outperformed the Greedy-ChannelFirst algorithm. Even though the Greedy-ChannelFirst algorithm performs slightly worse than the other two algorithms on average (randomly generated) cases, it provides a certain worst-case performance guarantee, which is not possible for the other two algorithms.

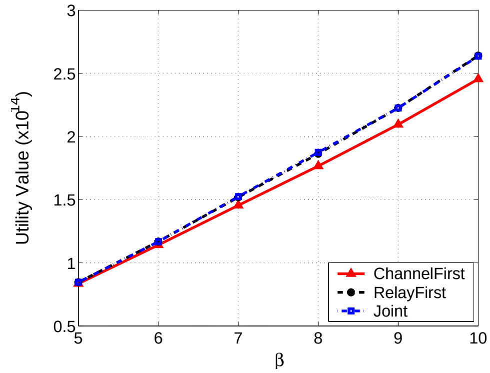

Fig. 8. *Scenario* 6*: Varying the average queue length (larger cases)*

3) No matter which algorithm was used, the utility value monotonically increases with the number of MSs (n), the number of RSs (m) or the number of sub-channels (K) since more resources certainly should lead to higher utility values. As can be seen in Fig. 7, as the number of channels increases, there is a diminishing increase in the overall utility, which is due to the fact with additional channels available, some MSs begin to have all of their demands (queues) fully satisfied and they cannot contribute to increasing the overall utility any further. Moreover, the utility value increases almost linearly with the queue length (β) since heavier traffic loads will also result in higher utility values.

VII. Conclusions

In this paper, we studied a joint channel and relay assignment problem in OFDMA-based cooperative relay networks with the objective of maximizing a well-adopted utility function. An MILP formulation was presented to provide optimal solutions. We also presented three simple greedy algorithms to solve this problem in polynomial time, namely, Greedy-ChannelFirst, Greedy-RelayFirst and Greedy-Joint. A comprehensive theoretical analysis was then performed to show that the Greedy-ChannelFirst algorithm has a constant approximation ratio while the other two algorithms cannot provide a similar performance guarantee. Extensive simulation results demonstrated that all the algorithms provide close-to-optimal solutions in relatively small input scenarios, and they all perform very well on average cases.

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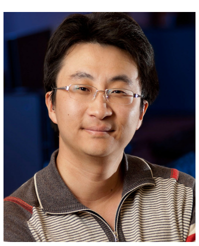

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