

# Small Signal Stability Assessment of Power Systems With Increased Penetration of Photovoltaic Generation: A Case Study

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**Abstract**—The present paper investigates the effect of high penetration of photovoltaic (PV) systems on the small signal stability of a large power system. Reduced system inertia and altered power flow patterns as a result of the addition of the utility scale and residential rooftop PVs that replace a portion of conventional generation resources, may lead to decreased damping of the critical modes of the system. To identify the critical modes of the system and the effect of the high PV penetration on these modes, eigenvalue analysis is carried out on the aforementioned system under various PV penetration levels. To substantiate the results observed from the small signal analysis, transient analysis is carried out on the system under various PV penetration levels. The simulation results effectively identify the impact of high PV penetration on small signal stability of the studied system.

**Index Terms**—Distributed power generation, photovoltaic (PV) generation, power transmission, sensitivity, small signal stability, transient stability.

## I. INTRODUCTION

THE present renewable portfolio standards (RPS) which have been developed to address the need for clean, sustainable, and renewable energy, are expected to structurally alter the characteristics of the power systems [1]. The RPS mandates have required some of the states in the U.S. to increase their level of renewable generation to up to 20% by the year 2020. Hence, large-scale power generation from photovoltaic (PV) resources is no longer a vision but a foreseeable reality. As more PV systems are being installed in existing grids, more conventional generation units may be displaced due to their high operating costs and other factors such as aging. With these unprecedented rapid changes to the power grids, special measures may be required to accommodate the needs of the future power systems.

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The distinctive characteristics of the PV systems has resulted in new concerns regarding the reliability and security of the power systems with high levels of PV generation. Reduced system inertia and lack of reactive power support for residential rooftop units are the main contributors that could impact the response of power systems with increased PV penetration levels. Displacing part of the conventional generators with more PV resources translates to less rotating mass within the system, hence decreasing overall system inertia. Although this phenomenon is not much of a concern in terms of the steady state behavior of the power systems, it will be a major source of concern during system transients. The ability of the system to adjust the system operating conditions in response to various disturbances, and performing the required tasks in a timely manner is essential for a reliable system operation.

Presently practiced standards and recommended practices, e.g., IEEE 1547 [2] and UL 1741 [3], suggest that residential PV inverters should not actively regulate the voltage at the point of common coupling (PCC). Consequently, with the exception of the utility scale units, PV systems are mainly utilized as sources of active power and no reactive power is generated by these resources. Therefore, supplying adequate reactive power support to the customers is a problem that needs to be addressed as the PV penetration level increases.

While addition of the PV resources are certain to occur in the future, the exact effect of these systems, particularly on the transmission systems, are not adequately studied. The effects of high PV penetration on distribution systems have been studied in [4]–[7]. As these studies suggest, high PV penetration can affect the voltage profile depending on the loading conditions and the amount of PV penetration. Study results of these works suggest that presently available voltage regulation equipment is not capable of mitigating the adverse effects of PV system transients. These works have identified some problems associated with the PV systems that directly affect the distribution systems.

With increased PV penetration, the effect of these systems on the transmission systems can no longer be neglected. Researchers [8]–[10] have investigated the transient behavior of the transmission system in response to various disturbances related to PV generation. It has been shown in [8] that depending on the amount of PV solar generation and the point of interconnection (POI), PV generation can have both beneficial and detrimental impacts on transient voltages. However, the studied

cases are not actual representations of a large interconnected power system. Achilles *et al.* [10] have conducted various case studies on an IEEE 39 bus system to analyze transmission system performance with high PV penetration.

The aforementioned works have analyzed the system behavior in terms of the steady state and transient stability. Due to the reduced system inertia, small signal stability of the systems with high PV penetration could also be affected. In [11], small signal stability of a two area power system under increased PV generation is investigated to observe both the detrimental as well as the beneficial impacts of PV generation on the interarea modes of the system.

The authors in [13] study the voltage stability as well as angle stability of an IEEE 14 bus system while equipped with utility scale PV systems. The effect of PV units on inter-area modes of oscillation is also investigated in this work. In another work [12], the effect of PV systems on a power system represented as a single machine and an infinite bus is investigated. This work derives a mathematical model based on the  $V-I$  characteristics of the PV systems which is suited for the distribution system rather than a large-scale transmission system. It is also not apparent that considering the  $V-I$  characteristics significantly impacts electromechanical oscillations. The present paper investigates the small signal stability of a study system which is a portion of the Western Electricity Coordinating Council (WECC) under various PV penetration levels. The approach developed is proposed for any large realistic system without loss of generality even though the conclusions derived depend on the specific characteristics of the system analyzed. A comprehensive eigenvalue analysis of the system is carried out to identify the critical modes of the system. The results are compared to assess the stability of the system with increased PV penetration levels. The results derived from the small signal analysis of the system are further expanded by conducting transient simulations on the system. To perform the studies presented in this work, software packages such as DSATools and PSLF have been utilized.

This paper is organized as follows. Section II describes the basic premises of small signal stability analysis and the motivations for this study. In Section III, a description of the studied system is presented followed by the modeling aspects of the PV systems. The analysis of small signal stability is presented in Section IV. Conclusions derived from these analyses are presented in Section V.

## II. SMALL SIGNAL STABILITY ANALYSIS

### A. Motivation for the Study

Small signal stability is defined as the ability of the system to maintain synchronism when it is subjected to small disturbances [14]. In this context, there are two types of instability that can occur: the steady state rotor angle increase due to lack of synchronizing torque and the increasing rotor oscillations due to insufficient damping torque are the two major types of small signal instability [15]. Generator–turbine inertia generally plays a key role in providing synchronizing capability to the synchronous generators whenever a disturbance results in a mismatch between the mechanical power input and the electrical

power output of a generator. In a system with high PV penetration, some of the synchronous generators are replaced with PV units. The authors in [16] have proposed a general approach for integration of variable energy resources. The method suggests that for every 3-MW addition of renewable generation to the system, there would be a 2-MW reduction in conventional generator commitment and 1-MW reduction in their dispatch. While the choice of the cited “1/3–2/3 rule” could be quite arbitrary, note that the overall system inertia is decreased, which can lead to potential small signal stability problems. Consequently, with displacing/rescheduling of conventional units as a result of the addition of PV generation, it is advantageous to determine if a particular generator’s inertia has significant impact on a particular inertial oscillation mode. This could be determined by performing sensitivity analysis with respect to generator inertia and performing decommitment/rescheduling using the sensitivity to inertia as a constraint.

### B. Proposed Approach

Small signal stability analysis is based on deriving a linear model of the nonlinear system model around a certain operating condition. The stability of the linearized system is determined by the eigenvalues ( $\lambda$ ) of the state matrix  $A$  while the participation of each system state in a specific eigenvalue is determined by the right eigenvector ( $\varphi$ ) and the left eigenvector ( $\psi$ ). The  $i$ th eigenvalue of the system matrix  $A$  and its corresponding eigenvectors are defined as

$$A\phi_i = \lambda_i\phi_i \quad (1)$$

$$\psi_i A = \lambda_i\psi_i. \quad (2)$$

For a complex eigenvalue that corresponds to an oscillatory mode of the system, the frequency of the oscillation in Hz ( $f$ ) and the damping ratio ( $\varsigma$ ) are expressed as

$$\lambda_i = \sigma_i \pm j\omega_i \quad (3)$$

$$f_i = \frac{\omega_i}{2\pi} \quad (4)$$

$$\varsigma_i = -\frac{\sigma_i}{\sqrt{\sigma_i^2 + \omega_i^2}}. \quad (5)$$

The damping ratio shown in (5), which is related to the real part of the eigenvalues, determines the rate at which the amplitude of the oscillations decreases [15]. Hence, a positive real part corresponds to oscillations with increased amplitudes while a negative real part corresponds to a damped oscillation. As the complex pole moves towards the right half plane (RHP), the damping of the system worsens.

The objective is to investigate whether the critical modes of the system are detrimentally or beneficially affected by increased PV penetration. The following steps are taken in the proposed approach:

- 1) Identify the most critical, i.e., poorly damped, modes of the system by performing eigenvalue analysis on the base case with no PV generation.
- 2) Perform eigenvalue analysis for the cases after introducing various levels of residential rooftop and utility scale PVs.
- 3) Compare the results of the eigenvalue analysis under different PV penetration levels to investigate the impact of

TABLE I  
SUMMARY OF THE STUDIED AREA

Total Load	MW	13277
	MVAr	2188
Total Generation	MW	21536
	MVAr	2238
Total Export	MW	7651
	MVAr	60
Total Losses	MW	608
Total Number of Generators		226
Total Number of Buses		2419
Total Number of Lines		1861

high PV penetration on small signal stability of the system under study.

- 4) Perform eigenvalue sensitivity analysis with respect to the displaced generators' inertia to validate the results achieved from the eigenvalue analysis.
- 5) Analyze the transient stability performance of the system and examine whether the identified critical modes can be excited by a large disturbance and substantiate the results obtained by eigenvalue analysis.

### III. ATTRIBUTES OF THE TEST BED SYSTEM

#### A. Description of the Test Bed System

Small signal stability analysis as well as transient analysis is carried out on a test bed that represents the entire Western Electricity Coordinating Council (WECC) with transmission voltage levels ranging from 34.5 to 500 kV. The PV systems are added to an area of the WECC system that has a high potential for PV installations and has a relatively large amount of power export to the other areas. The synchronous generators within the WECC system are equipped with power system stabilizers (PSS), excitation systems, as well as governors for frequency regulation. Each area is equipped with a single slack bus for power flow analysis. Table I summarizes the attributes of the studied area which has been chosen for incorporating a high concentration of utility scale and rooftop PVs. The active load in the case studied is about 13 GW while the summer peak load of the studied area is close to 20 GW. Therefore, these operating conditions are valid representations of a case with peak PV outputs and light loads. Under these conditions, specifically during the daylight off peak hours, the load at a particular location might be less than the PV output generated. Hence, the excess PV generation can contribute to the reversal of the power flow from the load side to the transmission system and disruption in system operating conditions may occur.

Two types of PV systems are included in the system considered. Table II lists important characteristics of these units [10]. "Rooftop" PVs are aggregated at the transmission/subtransmission voltage level of 69 kV based on actual penetration provided using a zip code delineation across the studied area. Rooftop units in a given zip code are lumped at the closest 69 kV bus and are modeled on the "system side" (upstream) of the inverters as

TABLE II  
PV SYSTEM DESCRIPTION AND CAPABILITIES

PV Type	MW Generation	MVAr Generation	Q-Enabled	Voltage Control	Frequency Control
Utility scale	50	17	Yes	Yes	No
Rooftop	Varies	0	No	No	No

TABLE III  
SUMMARY OF PV PENETRATION LEVELS

PV generation (MW)	2158	4316	6475	8633	10792
PV penetration level (%) (based on (6))	10	20	30	40	50
PV penetration level (%) (load based)	11.2	22.5	34	45	56
PV penetration level (%) (by energy)	2.5	5	7.5	10	12.5

negative loads. Additional details of these units can be found in [10]. Utility scale PVs are fixed at 600 MW of active power generation while rooftop PVs are varied under various PV generation levels. The PV penetration definition used is based on the available generation in the base case and is calculated as

$$\text{PV Penetration (\%)} = \frac{\text{Total PV generation (MW)}}{\text{Total generation (MW)}}. \quad (6)$$

Alternate definitions of PV penetration levels such as the definition based on the system peak load [9] or by the amount of energy served (e.g., in one year) can also be found in the literature. The equivalent active power generation for each generation scenario is illustrated in Table III. Calculation of the PV penetration percentages is carried out based on (6). For comparison purposes, PV penetration levels are also defined based on the other methods in Table III. Note that 50% PV penetration level defined based on (6) approximately corresponds to 12.5% PV penetration by energy, which is below the RPS requirements for some states in the U.S. The definition of the PV penetration by energy is an approximation that calculates the PV penetration by energy based on (7). The capacity factor of the PV systems is assumed to be 25% throughout this work

$$\text{PV}_{\text{penetration}}(\text{energy}) = \text{PV}_{\text{penetration}}(\text{name plate}) \times \text{Capacity Factor}. \quad (7)$$

As the level of PV penetration increases, more conventional generators are displaced by rooftop PV resources. However, the critical generators in terms of the reactive power support are retained in service. In order to keep the generation and load balanced, the output of these critical generators is scaled down to accommodate the addition of the PV resources to the system, for PV penetration levels of higher than 30%.

#### B. Modeling of the PV Systems

Presently practiced standards such as IEEE 1547 [2] do not allow PV inverters to regulate the voltages at the point of

common coupling. This limitation has resulted in PV installations being operated at a fixed power factor. Although the PV inverters can be operated at any fixed power factor, the majority of those systems hold a unity power factor. Therefore, these systems are modeled at unity power factor throughout this work. Being operated at unity power factor, residential rooftop PVs are modeled as constant current loads corresponding to negative power injections. This choice of modeling is justified by the fact that rooftop units are typically located at the distribution level. The rooftop units are also limited in their rating compared to utility scale units and in general there does not exist any coordinated control among the inverters of units located in the same neighborhood. As a result, the dynamics of each individual unit is unlikely to affect system dynamic behavior but the large penetration of rooftop units in a given neighborhood is likely to alter the power flows at the subtransmission level. The model considered accurately represents the manner in which the rooftop units alter the power flows.

Unlike the rooftop PVs, utility scale PV systems are equipped with converters that can regulate the voltage. Hence, utility scale PV units are not operated at unity power factor and possess the capability to generate reactive power. As a result, a detailed model including the representation of the inverter is required for utility scale PV system representation for both power flow as well as dynamic studies. Details of the power flow modeling can be found in [10] and [17]. Although utility scale PV systems are only a small portion of the total PV generation, due to the fact that small signal analysis of the studied system necessitates dynamic modeling of the PV systems, a brief description of these dynamic models is presented. The PV model used for the small signal analysis is the same as the models used for the transient analysis mainly because the DSATools utilize the same dynamic data file for both the transient stability analysis and small signal stability analysis.

To better represent the dynamic behavior of the utility scale PV units, a representation of the converter model is required. As suggested by NERC and WECC working group on PV modeling, [17], [18], a converter model suitable for electromechanical transients that is available both in DSATools and PSLF can approximately represent the full converter models. In the model of the converter considered, the dynamics are dominated by the rapid response due to the electrical controls through the converter. However, the parameters of the aforementioned converter model should be modified to fully represent the utility scale PV resources [18]. It should be noted that due to the time frame of the stability analyses, the dc side dynamics, which generally exhibit slower response, are neglected in this model. Fig. 1 presents a schematic of the WECC generic model proposed for representation of the converter characteristics. As seen from Fig. 1, two main components contribute to the dynamic behavior of the full converter: *converter model* and the *converter control*. The converter model represents the dynamic behavior of the utility scale PV systems. The control model has the required modules for representing the active and reactive power control, in addition to representing current control capabilities of the converters.

The reactive power control block shown in Fig. 1 determines the reactive power reference value for the electrical control

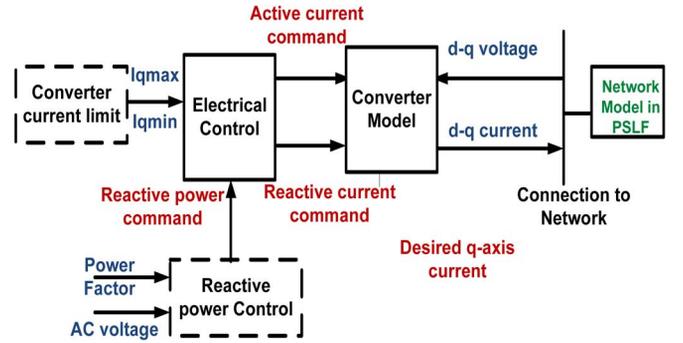


Fig. 1. Schematic of the PSLF model used to represent utility scale PVs.

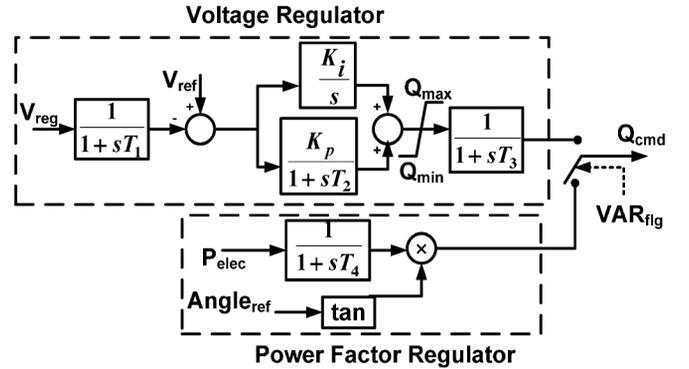


Fig. 2. Reactive power control model in PSLF used to represent a reactive power controller at utility scale PVs.

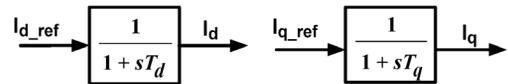


Fig. 3. Converter model used to represent the dynamics of the PV inverter.

block. The basic structure of the reactive power controller is presented in Fig. 2. This controller consists of two major parts: the voltage regulator and the power factor controller. Depending on the required control task, each of these control units can be activated in PSLF by specifying a proper flag. The details of the model presented in Fig. 2 can be found in [19]. In this work, voltage regulation has been selected for the controller options.

Various works have investigated the models for representing the inverter dynamics. The authors in [20] have proposed a first-order transfer function to model the inverters. This model has been used in this study and is presented in Fig. 3.

#### IV. IMPACTS ON POWER SYSTEM SMALL SIGNAL STABILITY

The objective of the small signal stability analysis is to examine the stability of the system under various PV penetration levels. In order to locate the critical modes of the system, an eigenvalue analysis is conducted for the modes within the frequency range of 0.01–2 Hz and damping of less than 10%. This analysis is performed utilizing the Small Signal Analysis Toolbox (SSAT) which is part of the DSATools software package [21]. The analysis is conducted for various PV penetration levels defined in Table III. The critical modes of the base case that are also present in all PV penetration cases studied are presented in Table IV.

TABLE IV  
CRITICAL MODES PRESENT IN THE BASE CASE

Real Part (1/s)	Imaginary Part (rad/s)	Frequency (Hz)	Damping Ratio (%)
-0.4229	12.5101	1.991	3.38
-1.0926	11.2221	1.786	9.69
-0.7454	9.5445	1.5191	7.79
-0.92	10.83	1.72	8.43

TABLE V  
CRITICAL MODE DETRIMENTALLY AFFECTED BY HIGH PV GENERATION

PV Penetration (%)	Real Part (1/s)	Imaginary Part (rad/s)	Frequency (Hz)	Damping Ratio (%)
0%	-1.0926	11.2221	1.786	9.69
20%	-0.9135	11.1828	1.7798	8.14
30%	-0.9423	11.1346	1.7721	8.43
40%	-0.5173	11.0029	1.7512	4.7
50%	-0.4876	10.9947	1.7499	4.43

Among the modes presented in Table IV, the second mode, i.e.  $-1.09 \pm j11.21$ , is detrimentally affected by increasing the level of PV penetration. For comparison purposes, the characteristics of the aforementioned mode for various PV penetration levels are shown in Table V. As seen from the results presented in Table V, the oscillatory mode has been adversely impacted by increasing the level of PV penetration. This is evident by comparing the real parts of the eigenvalues that have moved closer to the RHP as the PV penetration increases. This is also reflected in the reduction in the damping ratio. The significant reduction of the damping ratio from 30% to 40% PV penetration is mainly caused by displacing a number of large conventional generators that were in service prior to 40% PV penetration level. By displacing those generators, the overall inertia of the studied system dramatically reduces and hence the damping ratio of the adversely impacted mode, which is closer to those units, decreases.

In order to substantiate the results observed by the eigenvalue analysis, a sensitivity analysis is carried out corresponding to the mode which is detrimentally impacted with increased PV penetration. As more conventional generators are displaced with distributed PV resources, the overall system inertia is reduced. Sensitivity assessment of the critical modes of the system, with respect to inertia, is a means of explaining the detrimental impacts of the PV generation on small signal stability of the power systems. The eigenvalue sensitivity with respect to inertia of the  $j$ th generator ( $H_j$ ) is expressed as [22]

$$\frac{\partial \lambda_i}{\partial H_j} = \frac{\psi_i \frac{\partial A}{\partial H_j} \phi_i^T}{\psi_i^T \phi_i} \quad (8)$$

The sensitivity of the detrimentally impacted critical mode of the system with respect to the inertia of the system generators that are being displaced due to the higher PV penetration levels

TABLE VI  
EIGENVALUE SENSITIVITY CORRESPONDING TO THE INERTIA OF THE DISPLACED GENERATORS IN THE 20% PV PENETRATION CASE

Gen. Bus Number	Base Value of Inertia (s)	Real Part Sensitivity (1/s <sup>2</sup> )
469	3.54	-0.0129
471	3.54	-0.015
555	2.59	-0.0486
556	2.59	-0.048
809	1.3106	-0.0013
184	2.93	-0.0007

TABLE VII  
EIGENVALUE SENSITIVITY CORRESPONDING TO THE INERTIA OF THE DISPLACED GENERATORS IN THE 40% PV PENETRATION CASE

Gen. Bus Number	Base Value of Inertia (s)	Real Part Sensitivity (1/s <sup>2</sup> )
806	3.13	-0.0062
807	3.13	-0.0062
164	2.3	-0.0023
165	2.3	-0.0023
2164	4.3	-0.0023

is computed. These computations are performed with the aid of the SSAT software that calculates sensitivity of a particular mode to variations of a system parameter. Inertia of the conventional generators is the perturbed system parameter in this case. Tables VI and VII present a summary of the sensitivity of the critical mode to inertia variations of the conventional generators being displaced by rooftop PVs in the 20% and 40% penetration levels, respectively. Considering the fact that the damping of the system modes is determined by the real part of the eigenvalues, the real part sensitivity of the critical mode is presented in the aforementioned tables.

The negative real part sensitivities of the eigenvalues shown in Tables VI and VII illustrate the adverse impact of high PV penetration on system damping. By displacing with rooftop PVs or reducing the inertia of the conventional generators listed in those tables, the real part of the studied mode moves closer to the RHP. Therefore, the sensitivity analysis results corroborate the results derived from the full eigenvalue analysis of the system under various PV penetration levels.

Similar to the critical mode being studied, the fourth mode presented in Table IV, i.e., the mode with a frequency of 1.72 Hz, has also shown to be detrimentally impacted by the increased PV penetration levels. A comparison of the eigenvalues corresponding to this mode for different PV levels is presented in Table VIII. These results illustrate that care should be taken in selecting the displaced generators since the loss of their inertia could result in unsatisfactory damping ratios of certain low frequency oscillatory modes.

## V. TRANSIENT ANALYSIS

The results of the modal analysis presented in Section IV point out the fact that reduced system inertia, which is a side

TABLE VIII  
CRITICAL MODE DETRIMENTALLY AFFECTED BY HIGH PV GENERATION

PV Penetration (%)	Real Part (1/s)	Imaginary Part (rad/s)	Frequency (Hz)	Damping Ratio (%)
0%	-0.92	10.83	1.72	8.43
20%	-0.9329	10.8293	1.7235	8.58
30%	-0.8414	10.6211	1.6904	7.9
40%	-0.8378	10.5162	1.6737	7.9
50%	-0.7944	10.4007	1.6553	7.62

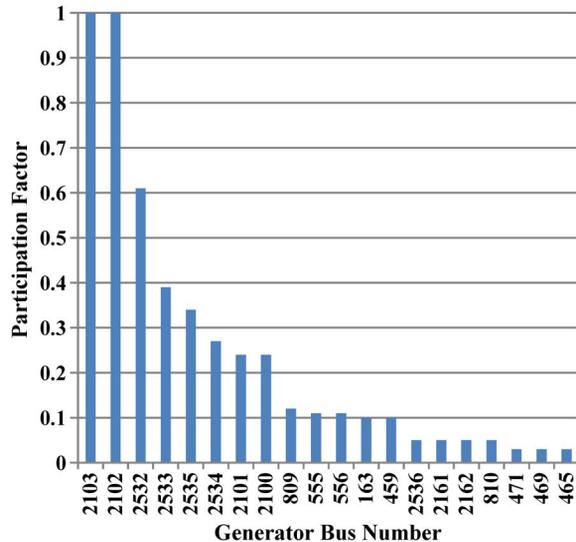


Fig. 4. Participation factor of the machines participating in the critical mode detrimentally affected by high PV penetration.

effect of increasing distributed rooftop PVs, could result in decreased system damping. Through extensive studies of systems with rooftop PVs, it has been found to be advisable to examine the transient response of the large scale transmission system to identify problematic damping. This is illustrated here through a continuation of the case study.

To illustrate the adverse effects of increased PV generation on small signal stability of the systems, the detrimentally impacted critical modes are scrutinized in the time domain. A disturbance that excites the most detrimentally impacted critical mode with respect to the PV penetration level is simulated. The purpose of this case study is to show how the system is impacted with decreased system inertia under various PV generation levels. Therefore, the cases studied are solely those that excite the modes analyzed by eigenvalue analysis. Additional transient stability studies with high PV penetration can be found in [23].

In order to identify the transient case that would excite a detrimentally impacted critical mode, the dominant machine with the highest participation factor in that mode needs to be identified. The participation factors of various machines participating in the first studied mode, i.e.,  $-1.09 \pm j11.21$ , are presented in Fig. 4. As seen from these results, the generators located at bus 2102 and 2103 have the highest participation factor in this mode. In view of the fact that this mode is related to the speed of the

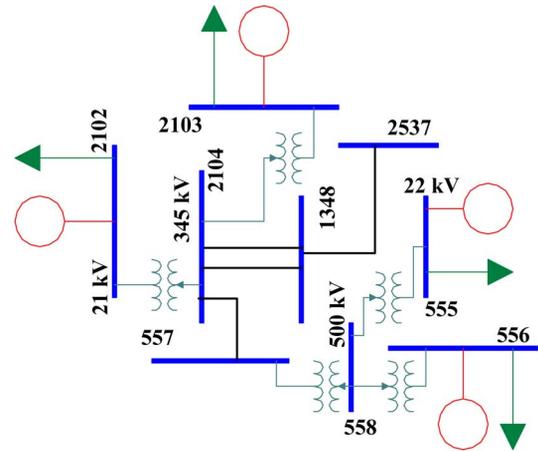


Fig. 5. Single line diagram of the system near the generator at bus number 2102.

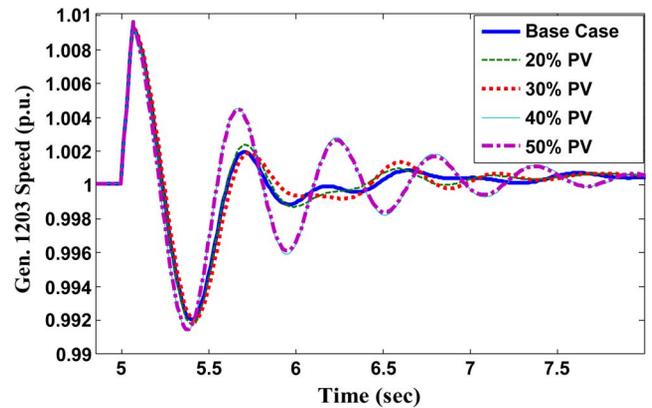


Fig. 6. Generator 2103 speed (p.u.) following a three phase fault at bus 2104.

forementioned generators, the speed of either of these generators is monitored during the transients. The one line diagram of the system in vicinity of the two participating generators is presented in Fig. 5.

The two participating generators are located in the same area and are connected to a 345 kV level bus with a step-up transformer. To identify the location of the disturbance that excites the detrimentally affected critical mode, various faults are simulated in the vicinity of the generator located at bus number 2103. The types of the faults considered for this study are mainly three phase faults on the transmission system and single phase faults are not considered for this study. Among the simulated fault scenarios, a three phase fault at the 345 kV bus of 2104, which is cleared after four cycles, has shown to excite the mode observed in full eigenvalue analysis results. Later in this paper, the validity of the studied transient case is verified with the aid of *Prony analysis* [24].

Fig. 6 illustrates the speed of the generator located at bus 2103 following the aforementioned three phase fault. As observed from Fig. 6, at higher PV penetration levels, more oscillations are observed in speed of this generator. In other words, the decreased damping observed in modal analysis is verified by the transient analysis results.

At a PV penetration level of 40% and above, the generator speeds attain higher peaks and deeper dips. Higher oscillations

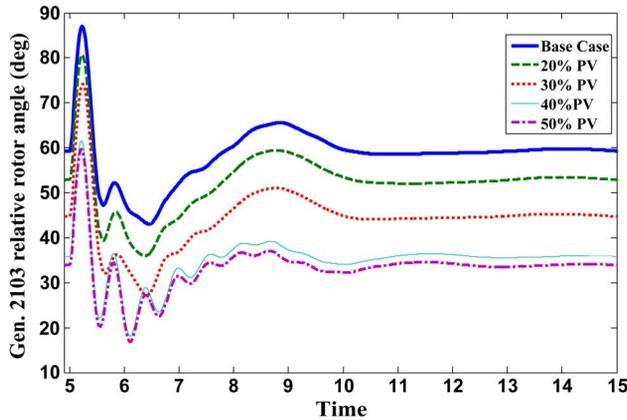


Fig. 7. Generator 2103 relative rotor angle (deg) following a three phase fault at bus 2104.

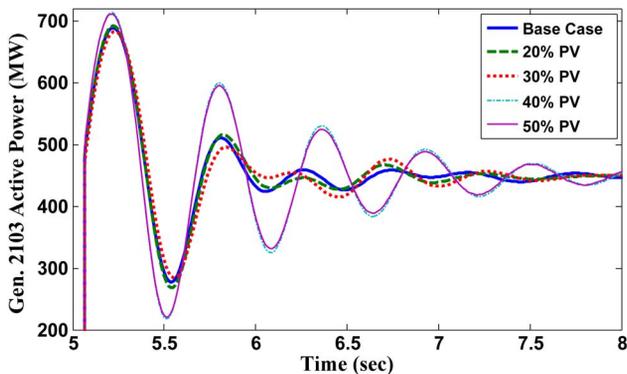


Fig. 8. Active power output of the generator located at bus 2103 following a three phase fault at bus 2104.

are also observed in the relative rotor angle of the generator located at bus 2103, under various PV penetration levels. Variations of the relative rotor angles are presented in Fig. 7. Detrimental impact of the PV penetration levels, which result in increased generator speeds, is also evident in the active power output of the generator located at bus 2103. Variations of the generator active power output are shown in Fig. 8.

Transient simulation results presented so far corroborate the results achieved by the small signal stability analysis. However, to reassert the premise of these simulations, a measure is required to ascertain the simulated disturbance in fact excites the detrimentally impacted mode. Prony analysis is carried out to ensure the presence of the studied mode in the transient response of the generator 2103 speed. This method is an extension of Fourier analysis that directly estimates the frequencies, damping, amplitude, and relative phase of the modes presents in a given signal [24].

The result of Prony analysis following a three phase fault at bus 2104 is presented in Table IX. As presented in this table, the same mode is observed in the generator speed variable as was observed in the eigenvalue analysis. The difference between the results presented in Tables V and IX is related to the inaccuracy of the discrete models used in the Prony analysis. However, as expected, with an increase in PV penetration levels, the damping of the studied mode is reduced. The transient stability results coupled with Prony analysis reinforce the results obtained by

TABLE IX  
PRONY ANALYSIS RESULTS OF THE GENERATOR 2103 SPEED FOR VARIOUS PV PENETRATION LEVELS

PV Level (%)	Magnitude	Frequency (Hz)	Damping (%)
Base Case	0.06	1.76	6.6
20%	0.06	1.76	6.4
30%	0.126	1.75	6.9
40%	0.42	1.75	4.8
50%	0.45	1.75	4.8

small signal stability analysis and illustrate the need to carefully select the conventional generators that are displaced when increased PV penetration occurs.

## VI. CONCLUSION

In this paper, the impact of high PV penetration on power system small signal stability is investigated. Two different types of PV generation units, i.e., rooftop PVs as well as utility scale PV units, are added to a large system to simulate a case with high PV penetration. Various levels of PV penetration are examined for their effect on system small signal stability. Modal analysis is performed on the system with no PVs as well as the cases after different PV levels were introduced. The approach illustrated via a case study is offered as an approach for the identification of the impact of large-scale PV penetration on power systems.

Eigenvalue analysis performed on the studied system identified the locations of the critical modes of the system. These modes are examined in the frequency range of 0.01–2 Hz with a damping ratio of less than 10%. Reduced system inertia as a result of displacing conventional generators with distributed rooftop PVs may result in a reduction in the damping torques of the system modes. In order to illustrate this effect, results of the modal analysis were compared to cases with PV penetration levels of up to 50%. The results of these comparisons identified the detrimental impacts of high PV penetration on decreasing the damping of the critical modes of the system.

To highlight the adverse effects of reduced inertia on electro-mechanical modes of oscillation, eigenvalue sensitivities to inertia of the displaced generators are calculated. The sensitivity analyses pinpoint the fact that the eigenvalues are detrimentally impacted as the inertia of conventional generators is reduced. The negative sensitivities of the real part of the eigenvalue to inertia show that the eigenvalues move closer to the right half plane with a reduction in generator inertia and, therefore, result in degraded damping performance during system transients.

It is recommended that in cases of high PV penetration, time domain analysis be performed as well: time domain analysis is performed to examine the detrimental impacts of high PV penetration during system transients. Using the concept of participation factors and by utilizing Prony analysis, a transient case that excites a detrimentally impacted critical mode was identified in an example test bed. Transient studies carried out for various PV penetration levels substantiate the results of small signal analysis together with eigenvalue sensitivity analysis. Based on the simulation results presented in this work, PV generation is found to have a detrimental impact on small signal stability of the studied system with increased penetration of PV systems.

In view of the fact that this detrimental impact of PV penetration is caused by reduced system inertia, maintaining the critical generators in service can be helpful in mitigating the impact of poor damping resulted from high PV penetration. The critical generators can be identified by examining the sensitivity of the critical modes to the inertia of the generators. Although the conclusions are derived only from the studied system, due to large size of the system and the fact that the system is representative of most of North American power systems, the conclusions can be safely generalized without loss of generality to the systems of same size and characteristics. In addition, the procedure to perform the study is general and applies to all systems.

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