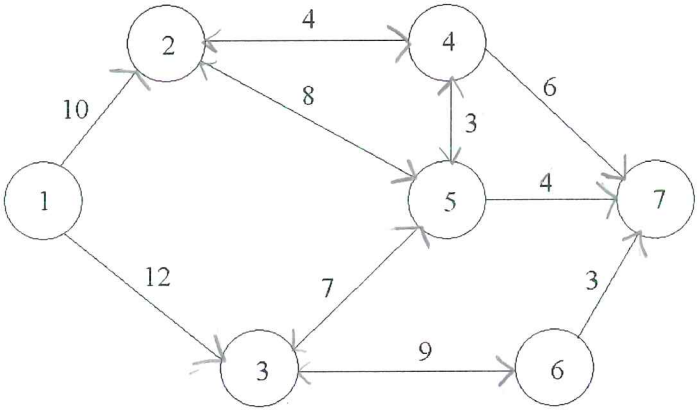
Driscoll, Dunning, Hopkins, Croneiser, Zarek

MFE411 – Intro to Management Science

Take-Home Assignment

Group No. 2 Date: 4/8/14

I. Military Intelligence has rated the road segments between Kabul (1) and Herat (7) according to their risk of attack from the Taliban, as indicated in the Graph below:



Find the "shortest path" from 1 to 7, i.e. *the one with the smallest total risk for a military convoy,* to make that move. Include all equations, the Solver program, its complete solution, and its interpretation.

Min: $10x_{12} + 12x_{13} + 4x_{24} + 8x_{25} + 7x_{35} + 9x_{36} + 4x_{42} + 3x_{45} + 6x_{47} + 8x_{52} + 7x_{53} + 3x_{54} + 4x_{57} + 9x_{63} + 3x_{67}$

$$x_{12} + x_{13} = 1$$
$$-x_{12} + x_{24} + x_{25} - x_{42} + x_{52} = 0$$
$$-x_{13} + x_{35} + x_{36} - x_{53} - x_{63} = 0$$
$$-x_{24} + x_{42} + x_{45} + x_{47} - x_{54} = 0$$
$$-x_{25} - x_{35} - x_{45} + x_{52} + x_{53} + x_{54} + x_{57} = 0$$
$$-x_{36} + x_{63} + x_{67} = 0$$
$$x_{47} + x_{57} + x_{67} = 1$$

Group #2 Prob. #1

X	x12	x13	x24	x25	x35	x36	x42	x45	x47	x52	x53	x54	x57	x63	x67	
	1	1														1
	-1		1	1			-1			-1						0
		-1			1	1					-1			-1		0
			-1				1	1	1			-1				0
				-1	-1			-1		1	1	1	1			0
						-1								1	1	0
									1				1		1	1
Cost	10	12	4	8	7	9	4	3	6	8	7	3	4	9	3	
Solutions	1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	

O.F. 20

S.T.			
	1	=	1
	0	=	0
	0	=	0
	0	=	0
	0	=	0
	0	=	0
	1	=	1

Microsoft Excel 15.0 Limits Report
Worksheet: [Take-Home 4-7-14.xlsx]Sheet1
Report Created: 4/8/2014 1:36:58 PM

Cell	Objective Name	Value
$B$16	O.F.	20

Cell	Variable Name	Value	Lower Limit	Objective Result	Upper Limit	Objective Result
$C$14	Solutions x12	1	1	20	1	20
$D$14	Solutions x13	0	0	20	0	20
$E$14	Solutions x24	1	1	20	1	20
$F$14	Solutions x25	0	0	20	0	20
$G$14	Solutions x35	0	0	20	0	20
$H$14	Solutions x36	0	0	20	0	20
$I$14	Solutions x42	0	0	20	0	20
$J$14	Solutions x45	0	0	20	0	20
$K$14	Solutions x47	1	1	20	1	20
$L$14	Solutions x52	0	0	20	0	20
$M$14	Solutions x53	0	0	20	0	20
$N$14	Solutions x54	0	0	20	0	20
$O$14	Solutions x57	0	0	20	0	20
$P$14	Solutions x63	0	0	20	0	20
$Q$14	Solutions x67	0	0	20	0	20

Shortest path = 1-2-4-7

total units traveled = 20

Microsoft Excel 15.0 Sensitivity Report
Worksheet: [Take-Home 4-7-14.xlsx]Sheet1
Report Created: 4/8/2014 1:36:58 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$14	Solutions x12	1	0	10	3	11
$D$14	Solutions x13	0	0	12	11	3
$E$14	Solutions x24	1	0	4	2	8
$F$14	Solutions x25	0	2	8	1E+30	2
$G$14	Solutions x35	0	3	7	1E+30	3
$H$14	Solutions x36	0	4	9	1E+30	4
$I$14	Solutions x42	0	8	4	1E+30	8
$J$14	Solutions x45	0	1	3	1E+30	1
$K$14	Solutions x47	1	0	6	1	5
$L$14	Solutions x52	0	14	8	1E+30	14
$M$14	Solutions x53	0	11	7	1E+30	11
$N$14	Solutions x54	0	5	3	1E+30	5
$O$14	Solutions x57	0	0	4	5	1
$P$14	Solutions x63	0	14	9	1E+30	14
$Q$14	Solutions x67	0	0	3	14	4

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$19	S.T.	1	17	1	0	0
$B$20		0	7	0	0	0
$B$21		0	5	0	0	0
$B$22		0	3	0	0	0
$B$23		0	1	0	0	0
$B$24		0	0	0	0	1E+30
$B$25		1	3	1	0	0

to change the final values the distance between points would have to increase or decrease by the allowable increase or decrease

Microsoft Excel 15.0 Answer Report
Worksheet: [Take-Home 4-7-14.xlsx]Sheet1
Report Created: 4/8/2014 1:36:58 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine

Engine: Simplex LP
Solution Time: 0.016 Seconds.
Iterations: 7 Subproblems: 0

Solver Options

Max Time Unlimited, Iterations Unlimited, Precision 0.000001
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$B$16	O.F.	0	20

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$C$14	Solutions x12	0	1	Contin
$D$14	Solutions x13	0	0	Contin
$E$14	Solutions x24	0	1	Contin
$F$14	Solutions x25	0	0	Contin
$G$14	Solutions x35	0	0	Contin
$H$14	Solutions x36	0	0	Contin
$I$14	Solutions x42	0	0	Contin
$J$14	Solutions x45	0	0	Contin
$K$14	Solutions x47	0	1	Contin
$L$14	Solutions x52	0	0	Contin
$M$14	Solutions x53	0	0	Contin
$N$14	Solutions x54	0	0	Contin
$O$14	Solutions x57	0	0	Contin
$P$14	Solutions x63	0	0	Contin
$Q$14	Solutions x67	0	0	Contin

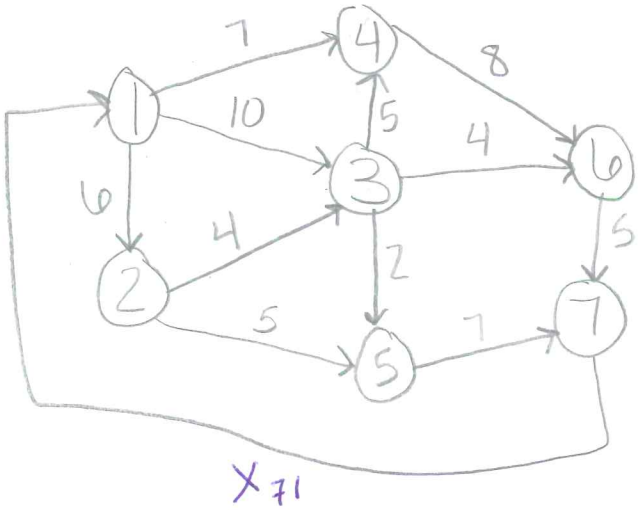
Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$19	S.T.	1	$B$19=$D$19	Binding	0
$B$20		0	$B$20=$D$20	Binding	0
$B$21		0	$B$21=$D$21	Binding	0
$B$22		0	$B$22=$D$22	Binding	0
$B$23		0	$B$23=$D$23	Binding	0
$B$24		0	$B$24=$D$24	Binding	0
$B$25		1	$B$25=$D$25	Binding	0

Group #2 4/8/14 Driscoll, Dunning, Hopkins, Croneiser, Zarek

II There is a need to transport the largest number of troops possible from Kabul (1) to Kandahar (7). The difficulties of the terrain, the conditions of the roads, and the attack risks are used to estimate the number of troop transports (i.e. trucks) per hour that can traverse these segments (given below).:

Segment	Trucks/Hr
1 to 2	6
1 to 3	10
1 to 4	7
2 to 3	4
2 to 5	5
3 to 4	5
3 to 5	2
3 to 6	4
4 to 6	8
5 to 7	7
6 to 7	5



Based on the above data, draw the network graph, develop the algebraic equations for this Max Flow problem, and use Solver to obtain the optimal troop flow from Kabul to Kandahar.

Max: X_{71}

① $X_{12} + X_{13} + X_{14} - X_{71} = 0$

② $-X_{12} + X_{23} + X_{25} = 0$

③ $-X_{13} - X_{23} + X_{34} + X_{36} + X_{35} = 0$

④ $-X_{14} - X_{34} + X_{46}$

⑤ $-X_{25} - X_{35} + X_{57}$

⑥ $-X_{46} - X_{36} + X_{67}$

⑦ $-X_{57} - X_{67} + X_{71}$

$X_{12} \leq 6$

$X_{13} \leq 10$

$X_{14} \leq 7$

$X_{23} \leq 4$

$X_{25} \leq 5$

$X_{34} \leq 5$

$X_{35} \leq 2$

$X_{36} \leq 4$

$X_{46} \leq 8$

$X_{57} \leq 7$

$X_{67} \leq 5$

	x12	x13	x14	x23	x25	x34	x35	x36	x46	x57	x67	x71
1	1	1	1									-1
2	-1			1	1							
3		-1		-1		1	1	1				
4			-1			-1			1			
5					-1		-1			1		
6								-1	-1		1	
7										-1	-1	1
	1											
		1										
			1									
				1								
					1							
						1						
							1					
								1				
									1			
										1		
											1	

Solution	5	6	1	0	5	0	2	4	1	7	5	12

O.F	12

Contraints

0	=	0
0	=	0
0	=	0
0	=	0
0	=	0
0	=	0
0	=	0
5	<=	6
6	<=	10
1	<=	7
0	<=	4
5	<=	5
0	<=	5
2	<=	2
4	<=	4
1	<=	8
7	<=	7
5	<=	5

Microsoft Excel 15.0 Answer Report
Worksheet: [Book1]Sheet1
Report Created: 4/8/2014 12:19:41 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine

Engine: Simplex LP

Solution Time: 0 Seconds.

Iterations: 10 Subproblems: 0

Solver Options

Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling

Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$B$24	O.F x12	0	12

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$22	Solution x12	0	5	Contin
$C$22	Solution x13	0	6	Contin
$D$22	Solution x14	0	1	Contin
$E$22	Solution x23	0	0	Contin
$F$22	Solution x25	0	5	Contin
$G$22	Solution x34	0	0	Contin
$H$22	Solution x35	0	2	Contin
$I$22	Solution x36	0	4	Contin
$J$22	Solution x46	0	1	Contin
$K$22	Solution x57	0	7	Contin
$L$22	Solution x67	0	5	Contin
$M$22	Solution x71	0	12	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$27	x12	0	$B$27=$D$27	Binding	0
$B$28	x12	0	$B$28=$D$28	Binding	0
$B$29	x12	0	$B$29=$D$29	Binding	0
$B$30	x12	0	$B$30=$D$30	Binding	0
$B$31	x12	0	$B$31=$D$31	Binding	0
$B$32	x12	0	$B$32=$D$32	Binding	0
$B$33	x12	0	$B$33=$D$33	Binding	0
$B$34	x12	5	$B$34<=$D$34	Not Binding	1
$B$35	x12	6	$B$35<=$D$35	Not Binding	4
$B$36	x12	1	$B$36<=$D$36	Not Binding	6

$B$37	x12	0	$B$37<=$D$37	Not Binding	4
$B$38	x12	5	$B$38<=$D$38	Binding	0
$B$39	x12	0	$B$39<=$D$39	Not Binding	5
$B$40	x12	2	$B$40<=$D$40	Binding	0
$B$41	x12	4	$B$41<=$D$41	Binding	0
$B$42	x12	1	$B$42<=$D$42	Not Binding	7
$B$43	x12	7	$B$43<=$D$43	Binding	0
$B$44	x12	5	$B$44<=$D$44	Binding	0

Microsoft Excel 15.0 Sensitivity Report
Worksheet: [Book1]Sheet1
Report Created: 4/8/2014 12:19:41 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$22	Solution x12	5	0	0	0	1
$C$22	Solution x13	6	0	0	0	0
$D$22	Solution x14	1	0	0	0	0
$E$22	Solution x23	0	0	0	0	1E+30
$F$22	Solution x25	5	0	0	1E+30	1
$G$22	Solution x34	0	0	0	0	1E+30
$H$22	Solution x35	2	0	0	1E+30	1
$I$22	Solution x36	4	0	0	1E+30	0
$J$22	Solution x46	1	0	0	0	1
$K$22	Solution x57	7	0	0	1E+30	1
$L$22	Solution x67	5	0	0	1E+30	1
$M$22	Solution x71	12	0	1	1E+30	1

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$27	x12	0	-1	0	12	0
$B$28	x12	0	-1	0	5	0
$B$29	x12	0	-1	0	6	0
$B$30	x12	0	-1	0	1	0
$B$31	x12	0	0	0	0	0
$B$32	x12	0	-1	0	1	0
$B$33	x12	0	0	0	0	1E+30
$B$34	x12	5	0	6	1E+30	1
$B$35	x12	6	0	10	1E+30	4
$B$36	x12	1	0	7	1E+30	6
$B$37	x12	0	0	4	1E+30	4
$B$38	x12	5	1	5	0	5
$B$39	x12	0	0	5	1E+30	5
$B$40	x12	2	1	2	0	2
$B$41	x12	4	0	4	1	4
$B$42	x12	1	0	8	1E+30	7
$B$43	x12	7	0	7	1E+30	0
$B$44	x12	5	1	5	6	1

Microsoft Excel 15.0 Limits Report
Worksheet: [Book1]Sheet1
Report Created: 4/8/2014 12:19:41 PM

Cell	Objective Name	Value
$B$24	O.F x12	12

Cell	Variable Name	Value	Lower Limit	Objective Result	Upper Limit	Objective Result
$B$22	Solution x12	5	5	12	5	12
$C$22	Solution x13	6	6	12	6	12
$D$22	Solution x14	1	1	12	1	12
$E$22	Solution x23	0	0	12	0	12
$F$22	Solution x25	5	5	12	5	12
$G$22	Solution x34	0	0	12	0	12
$H$22	Solution x35	2	2	12	2	12
$I$22	Solution x36	4	4	12	4	12
$J$22	Solution x46	1	1	12	1	12
$K$22	Solution x57	7	7	12	7	12
$L$22	Solution x67	5	5	12	5	12
$M$22	Solution x71	12	12	12	12	12