Energy Efficiency of Fixed-Rate Transmissions with Markov Arrivals under Queueing Constraints

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***Abstract*—Energy efficiency of wireless transmissions is analyzed in the presence of Markov sources and queueing constraints. Not knowing the channel conditions, the transmitter is assumed to send the data at a fixed rate over a Rayleigh fading channel. This fixed-rate transmission is modeled as a two-state (ON/OFF) continuous-time Markov chain. Using the effective bandwidth of Markov sources and the effective capacity of the Markov fluid transmission model, a characterization of the maximum average arrival rate that can be supported in the Rayleigh fading channel under buffer constraints is given, and energy efficiency is analyzed by determining the minimum energy per bit and wideband slope expressions. The impact of queueing constraints, source and channel parameters, and fixed-rate transmissions on the energy efficiency is identified.**

***Index Terms*—Energy efficiency, fixed-rate transmissions, Markov arrivals, quality of service (QoS), Rayleigh fading.**

I. INTRODUCTION

DUE to rapid growth in mobile wireless applications and systems and rising energy costs and environmental concerns, energy efficiency in wireless communications has been attracting much interest recently (see e.g., [1] and [2]). Verdú in [3] identified the fundamental limits on the energy efficiency, following an information-theoretic approach. In this work, minimum energy per bit and wideband slope are studied as the performance metrics.

Recent years also witnessed an exponential growth in wireless transmission of multimedia content. Indeed, multimedia traffic is expected to be the dominant traffic in data communications. Such wireless multimedia traffic requires certain quality-of-service (QoS) guarantees. For instance, in voice over IP (VoIP) systems and multimedia streaming , constraints on delay, packet loss, or buffer overflow probabilities are imposed so that acceptable performance and quality levels can be met for the end-users. Finally, today's wireless networks carry heterogeneous traffic. Therefore, successful design of networks, efficient use of resources, and effective QoS provisioning for multimedia communications critically depend on incorporating source traffic models in the analysis.

In [4] and [5], effective bandwidth and effective capacity formulations are provided as performance metrics to determine the maximum throughput in the presence of statistical QoS constraints in the form of limitations on buffer violation probability. In [6] and [7], effective bandwidths of different source models are derived. In [8], effective bandwidth and effective capacity of Markov fluid models are studied for resource allocation and QoS evaluation in wireless systems. In [9], we studied the energy efficiency in a setting in which the data arrivals are modeled as a discrete-time Markov process and transmissions are performed with variable rates given by the instantaneous channel capacity.

In this paper, we consider channel and source models different from those in [9]. Primarily, we assume that the data source is modeled as a continuous-time Markov chain while we briefly address the discrete-time Markov model as well. Additionally, we assume that the time-varying channel conditions are not known at the transmitter and consequently, transmission rate is fixed. Fixed-rate transmission over the Rayleigh fading channel is modeled as an ON-OFF Markov fluid process. Under these modeling assumptions, our main contributions are the introduction of a general framework for performance analysis in the low-power regime, determination of closed-form expressions for the minimum energy per bit and wideband slope, and characterization of the impact of source and channel parameters and queueing constraints on the energy efficiency.

II. CHANNEL MODEL AND FIXED-RATE TRANSMISSIONS

We consider a flat-fading channel between the transmitter and receiver. The channel input-output relation can be expressed as

$$y(t) = h(t)x(t) + n(t) \tag{1}$$

where $x(t)$ and $y(t)$ are the complex-valued (i.e., low-pass equivalent) input and output signals, respectively, and $n(t)$ denotes the zero-mean, circularly-symmetric, complex Gaussian noise. The signal-to-noise ratio is defined as $\text{SNR} = \frac{P}{N_0 B}$, where P denotes the power of the input signal, $N_0/2$ is the power spectral density of the noise and B is the channel bandwidth. Above in (1), $h(t)$ denotes the multiplicative fading component representing the attenuation and phase shift experienced in the channel. We consider a Rayleigh fading channel and assume that $h(t)$ is a zero-mean complex Gaussian process. Therefore, $z(t) = |h(t)|^2$ has an exponential distribution.

Not knowing the channel conditions, the transmitter sends the data at the fixed rate of R bits/s/Hz. If the wireless channel changes slowly and hence $h(t)$ stays almost a constant over a coding block, the instantaneous channel capacity of the fading Gaussian channel can be formulated in bits/sec/Hz as

$$C(t) = \log_2\left(1 + \frac{P}{N_0 B} z(t)\right) = \log_2\left(1 + \text{SNR} z(t)\right). \tag{2}$$

Then, we assume that if $R < C(t)$, reliable communication is attained and hence the transmitted message is decoded correctly. If, on the other hand, $R \geq C(t)$, outage occurs and retransmission is needed. Under these assumptions, we, following the approach in [8], model the wireless link as a two-state continuous-time Markov chain. The channel is assumed to be in the ON state if $R < C(t)$ or equivalently $z(t) > \zeta$, where $\zeta = \frac{2^R - 1}{\text{SNR}}$. The channel is in the OFF state when

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$z(t) \leq \zeta$. We denote transition rates from ON to OFF state as λ and from OFF to ON state as μ. Now, the transition rate matrix can be expressed as $\mathbf{Q} = \begin{bmatrix} -\lambda & \lambda \\ \mu & -\mu \end{bmatrix}$.

These transition rates need to be consistent with the properties of the channel. The stationary probabilities are easily obtained as $\frac{\lambda}{\lambda+\mu}$ for the ON state and as $\frac{\mu}{\lambda+\mu}$ for the OFF state. Without loss of generality, we assume that $z(t)$ has unit variance. Then, we can write

$$\Pr\{z(t) > \zeta\} = \int_{\zeta}^{\infty} e^{-z}\,\mathrm{d}z = e^{-\zeta} = \frac{\lambda}{\lambda+\mu}, \tag{3}$$

$$\Pr\{z(t) \leq \zeta\} = \int_{0}^{\zeta} e^{-z}\,\mathrm{d}z = 1 - e^{-\zeta} = \frac{\mu}{\lambda+\mu}. \tag{4}$$

Hence, we have $\lambda = \kappa e^{-\zeta}$ and $\mu = \kappa(1-e^{-\zeta})$ where $\kappa = \lambda+\mu$ can be seen as the exponential decay rate of the memory of the underlying Rayleigh channel as discussed in [8] and can be determined from the channel statistics.

III. Markov Sources and Queueing Constraints

A. Effective Bandwidth of Markov Fluid Sources

In this paper, we assume that the data to be sent is generated by Markov fluid sources and is initially stored in a buffer before transmission. Statistical constraints are imposed on the buffer length. In particular, we assume that the buffer violation probability satisfies

$$\lim_{q\to\infty} \frac{\log \Pr(Q \geq q)}{q} = -\theta \tag{5}$$

where Q denotes the stationary queue length, and θ is the decay rate of the tail distribution of the queue length. The above limiting formula implies that for large $q_{\max}$, we have $\Pr(Q \geq q_{\max}) \approx e^{-\theta q_{max}}$. Hence, for a sufficiently large threshold, the buffer violation probability should decay exponentially with rate controlled by the QoS exponent θ. Note that as θ increases, stricter queueing or QoS constraints are imposed.

Next, we briefly describe the effective bandwidth, which characterizes the minimum constant transmission (or service) rate needed to support the given time-varying data arrivals while the buffer violation probability satisfies (5). Assume that the data arrivals are modeled as a continuous-time Markov process with n states and generating matrix $\mathbf{G}$. The arrival rate is ν_i in state i and $\mathbf{\Lambda} = diag\{\nu_1, \nu_2, ..., \nu_n\}$ is the diagonal matrix of the arrival rates. The effective bandwidth of this Markov fluid source is given by [7]

$$a(\theta) = \mu\left(\mathbf{\Lambda} + \frac{1}{\theta}\mathbf{G}\right) \tag{6}$$

where $\mu(\cdot)$ denotes the maximum real eigenvalue of its argument matrix. Also, the stationary distribution π_i for each state can be found from the equations $\pi\mathbf{1} = 1$ and $\pi\mathbf{G} = \mathbf{0}^T$ where $\pi = [\pi_1, \pi_2, \ldots \pi_n]$, $\mathbf{0} = [0, \ldots, 0]^T$ and $\mathbf{1} = [1, \ldots, 1]^T$. The generating matrix for the two-state case is in the form of $\mathbf{G} = \begin{bmatrix} -\alpha & \alpha \\ \beta & -\beta \end{bmatrix}$ where α and β are the transition rates from one state to another. When the arrival rates for the two-state model are r and 0 and hence we basically have ON and OFF states, the effective bandwidth expression specializes to

$$a(\theta, r) = \frac{1}{2\theta}\left[\theta r - (\alpha+\beta) + \sqrt{(\theta r - (\alpha+\beta))^2 + 4\alpha\theta r}\right]. \tag{7}$$

B. Effective Capacity for Markov Fluid Transmission Model

Effective capacity, as a dual concept of effective bandwidth, provides the maximum constant arrival rate that a given time-varying service process can support while satisfying the buffer constraint in (5) [5]. For the ON-OFF Markov fluid model of fixed-rate wireless transmissions described in Section II, the effective capacity is given by

$$C_E(\text{SNR}, \theta, R) = \frac{1}{2\theta}\left[\theta R + (\lambda+\mu) - \sqrt{(\theta R + (\lambda+\mu))^2 - 4\lambda\theta R}\right]$$

where R is the fixed transmission rate, and λ and μ are the transition rates from transition rate matrix $\mathbf{Q}$. As can be seen from (3) and (4) in which ζ is a function of the SNR, the dependence of effective capacity on SNR is through λ and μ.

IV. Energy Efficiency Metrics

In this section, we formulate the energy efficiency metrics for wireless transmissions in the presence of random source arrivals and queueing constraints. Specifically, we consider two-state Markov arrival models in which the arrival rates are r and 0 in the ON and OFF states, respectively. Stationary distribution is denoted as π_1 for the OFF state and π_2 for the ON state. Therefore, the average arrival rate is simply

$$r_{\text{avg}} = \pi_2 r = P_{\text{ON}} r \tag{8}$$

which is equal to the average departure rate when the queue is in steady state [10]. Then, we seek to determine the maximum average arrival rate r^*_{avg} that can be supported by the fading channel described in Section II while satisfying the statistical QoS limitations given in the form in (5). As shown in [10, Theorem 2.1], if the effective bandwidth of the arrival process is equal to the effective capacity of the service process, i.e.,

$$a(\theta, r) = C_E(\text{SNR}, \theta, R), \tag{9}$$

then, (5) is satisfied, i.e., buffer violation probability decays exponentially fast with rate controlled by the QoS exponent θ. Hence, we can determine from (9) the arrival rate $r(\text{SNR}, \theta, R)$ that can be supported in the wireless channel for given SNR, fixed-rate R, QoS exponent θ. We can further optimize this arrival rate over all possible choices of the transmission rates R and determine the maximum arrival rate $r^*(\text{SNR}, \theta)$. Then, the maximum average arrival rate is

$$r^*_{\text{avg}}(\text{SNR}, \theta) = r^*(\text{SNR}, \theta) P_{\text{ON}}. \tag{10}$$

In this paper, we employ energy per bit as the performance metric of energy efficiency. In our setup, we define energy per bit as $\frac{E_b}{N_0} = \frac{\text{SNR}}{r^*_{\text{avg}}(\text{SNR},\theta)}$. In particular, as the ultimate limit on energy efficiency, we are interested in the minimum energy per bit required for reliable communications. The minimum energy per bit $\frac{E_b}{N_0}_{\text{min}}$ under QoS constraints can be obtained from [3]

$$\frac{E_b}{N_0}_{\text{min}} = \lim_{\text{SNR}\to 0} \frac{\text{SNR}}{r^*_{\text{avg}}(\text{SNR}, \theta)} = \frac{1}{\dot{r}^*_{\text{avg}}(0)} \tag{11}$$

where $\dot{r}^*_{\text{avg}}(0)$ is the first derivative of r^*_{avg} with respect to SNR at zero SNR. Due to minimum bit energy being an asymptotic performance measure in the limit as $\text{SNR} \to 0$, we also investigate the wideband slope, which is defined as the slope of the spectral efficiency curve at zero spectral efficiency. More specifically, at $\frac{E_b}{N_0}_{\text{min}}$, the wideband slope $\mathcal{S}_0$ of the spectral efficiency versus E_b/N_0 (in dB) curve can be computed from [3]

$$\mathcal{S}_0 = -\frac{2\left(\dot{r}^*_{\text{avg}}(0)\right)^2}{\ddot{r}^*_{\text{avg}}(0)} \log_e 2 \tag{12}$$

where $\ddot{r}^*_{\text{avg}}(0)$ is the second derivative of r^*_{avg} with respect to SNR at zero SNR. $\frac{E_b}{N_0}_{\text{min}}$ and $\mathcal{S}_0$ provide a linear approximation of the spectral efficiency curve at low spectral efficiencies and hence characterize the energy efficiency at low but nonzero SNR levels.

V. Energy Efficiency of Fixed-Rate Transmission of ON-OFF Markov Sources

A. Markov Fluid Sources

For the two-state Markov fluid source, the average arrival rate is

$$r_{\text{avg}} = P_{\text{ON}}\, r = \frac{\alpha}{\alpha+\beta} r \tag{13}$$

where $P_{\text{ON}} = \alpha/(\alpha+\beta)$ is obtained from the steady state equations of the generating matrix $\mathbf{G}$. In the Markov fluid model, maximum arrival rate that can be supported by fixed-rate transmissions in the presence of buffer constraints can be obtained by solving (9) and the maximum *average* arrival rate $r^*_{\text{avg}}(\text{SNR}, \theta)$ can be determined from (13). In the following result, we characterize this maximum average arrival rate in the low-SNR regime and find the minimum energy per bit requirement and the wideband slope.

Theorem 1: Assume that the source arrivals and fixed-rate transmissions over the Rayleigh-fading channel are both modeled as ON-OFF continuous-time Markov processes. The decay rate of the memory of the Rayleigh channel is denoted by κ. Then, the minimum energy per bit and wideband slope expressions as a function of the channel and source parameters and the QoS exponent θ are given, respectively, by

$$\frac{E_b}{N_0}_{\text{min}} = e \log_e 2 = 2.7512 \text{ dB}, \text{ and} \tag{14}$$

$$\mathcal{S}_0 = \frac{1}{\frac{\theta}{\log_e 2}\left[\frac{e-1}{\kappa} + \frac{\beta}{\alpha(\alpha+\beta)}\right] + \frac{e}{2}}. \tag{15}$$

Proof: We first consider the condition in (9) and express it for Markov fluid transmission and source models as

$$\frac{\theta r-(\alpha+\beta)+\sqrt{(\theta r-(\alpha+\beta))^2+4\alpha\theta r}}{2\theta} = \frac{\theta R+(\lambda+\mu)-\sqrt{(\theta R+(\lambda+\mu))^2-4\lambda\theta R}}{2\theta}. \tag{16}$$

Note that the equality in (16) enables us to determine the maximum arrival rate, r^*, in the Markov fluid source model and the corresponding optimal fixed transmission rate R^* for the given channel parameters (e.g., the transition rates λ and μ) and the imposed queueing constraints specified by the QoS exponent θ. Note further that as seen in (11) and (12), we have to determine the first and second derivatives of r^* with respect to SNR at $\text{SNR} = 0$ in order to identify the minimum energy per bit and wideband slope. In (16), we have dependence on SNR through λ and μ. It is important to also note that optimum arrival and transmission rates r^* and R^* in general depend on SNR as well.

Initially, we consider an arbitrary fixed-rate transmission strategy $R(\text{SNR})$ for any given SNR. After multiplying both sides of (16) with 2θ and taking the derivative with respect to SNR with evaluating them at $\text{SNR} = 0$, we can obtain

$$\dot{r}_{\text{avg}}(0) = \dot{r}(0)\frac{\alpha}{\alpha+\beta} = \dot{R}(0)2^{-\dot{R}(0)}. \tag{17}$$

Assume that $R(\text{SNR})$ has the following second-order expansion at $\text{SNR} = 0$:

$$R(\text{SNR}) = a\text{SNR} + b\text{SNR}^2 + o(\text{SNR}) \tag{18}$$

for some constants a and b. Then, plugging the result in (17) into the formula of minimum energy per bit, we immediately obtain

$$\frac{E_b}{N_0}_{\text{min}} = \frac{2^a}{a}, \tag{19}$$

which characterizes the minimum energy per bit for a given transmission rate with $\dot{R}(0) = a$. The smallest value of $\frac{E_b}{N_0}_{\text{min}}$ can be obtained by optimizing over the choice of a. It can be easily seen that $a^* = 1/\log_e 2$ is the optimized value which we use in (19) in order to obtain the minimum energy per bit expression given in (14). As another equivalent approach, note that this optimal $a^* = 1/\log_e 2$ indeed maximizes $\dot{R}(0)2^{-\dot{R}(0)} = a2^{-a} = \frac{a}{2^a}$. Since maximizing $r_{\text{avg}}(\text{SNR}, \theta, R) = \dot{r_{\text{avg}}}(0)\text{SNR} + o(\text{SNR})$ in the low-SNR regime up to first order is equivalent to maximizing $\dot{r_{\text{avg}}}(0)$, we readily conclude from (17) that $\dot{R}^*(0) = a^* = 1/\log_e 2$. Hence, we have $\dot{r}^*_{\text{avg}}(0) = a^*\, 2^{-a^*}$. Plugging this $\dot{r}^*_{\text{avg}}(0)$ into (11), we again obtain the desired result in (14).

In order to determine the wideband slope, we first take the second derivative of both sides of (16) with respect to SNR and evaluate them at $\text{SNR} = 0$. With further simplifications, we can easily derive the second derivative of the maximum average arrival rate with respect to SNR at $\text{SNR} = 0$ as in (20). When we use the optimal $\dot{R}^*(0) = a^* = 1/\log_e 2$ value, we notice that $\ddot{r}^*_{avg}(0)$ does not depend on $b = \ddot{R}(0)/2$.

Finally, inserting (17) and (20) into (12) and using $a^* = 1/\log_e 2$, we obtain the wideband slope expression in (15). ∎

Remark 1: The minimum energy per bit in (14) does not depend on the QoS exponent θ and hence does not get affected by the presence of the buffer constraints. However, when compared with the ultimate limit of $\frac{E_b}{N_0}_{\text{min}} = \log_e 2 = -1.59$ dB achieved when the transmission rate is given by the ergodic Shannon capacity, we notice that fixed-rate transmissions incur a certain cost and the minimum energy per bit has significantly increased to 2.7512 dB.

Remark 2: The wideband slope expression in (15) depends on the QoS exponent θ, channel memory κ, and the Markov source characteristics through the transition rates α and β. In particular, we see that as θ increases (i.e., more strict buffer constraints are imposed), or κ decreases meaning that channel memory decays more slowly, we have smaller wideband slopes, resulting in smaller average arrival rates at the same energy per bit level or equivalently higher energy per bit to support the same arrival rate. Hence, stricter queueing constraints and/or more correlated channel adversely affect the energy efficiency in the low-SNR regime. Furthermore, increasing source burstiness also lowers the wideband slope and degrades the energy efficiency. Note that smaller α with $\alpha+\beta$ constant means that the stationary distribution of the

$$\ddot{r}_{avg}(0) = \theta(\dot{R}(0))^2 \left[\frac{2}{\kappa}\left(2^{-2\dot{R}(0)} - 2^{-\dot{R}(0)}\right) - \frac{2\beta}{\alpha(\alpha+\beta)}2^{-2\dot{R}(0)}\right] + 2^{-\dot{R}(0)}\left[2\ddot{R}(0)\left(1-\dot{R}(0)\log_e 2\right) - (\dot{R}(0))^3(\log_e 2)^2\right]. \quad (20)$$

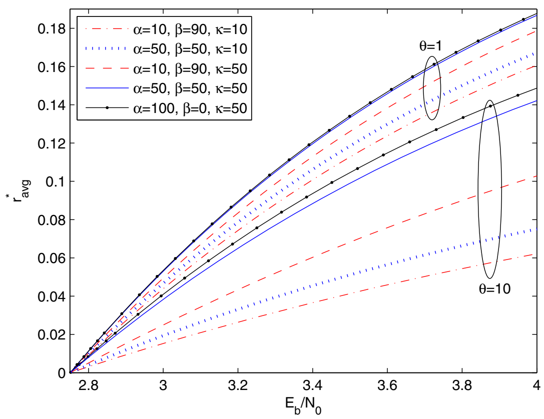

Fig. 1. Maximum average arrival rate r^*_{avg} vs. energy per bit $\frac{E_b}{N_0}$ when $\theta = 1, 10$.

ON state, P_{ON}, is smaller. Hence, data arrivals occur in less frequent bursts.

Remark 3: In the absence of buffer constraints, wideband slope expression becomes $\mathcal{S}_0 = 2/e = 0.7358$. Hence, we have no dependence on channel memory and source characteristics. We also notice that the wideband slope is smaller compared to $\mathcal{S}_0 = 1$ achieved in Rayleigh fading channels when ergodic Shannon capacity is considered [3] which shows the cost of fixed-rate transmissions in the wideband slope.

In Figure 1, we plot the maximum average arrival rate r^*_{avg} as a function of the energy per bit $\frac{E_b}{N_0}$ when $\theta = 1, 10$. For given θ, different curves are obtained for different values of κ, α, and β while $\alpha + \beta$ is fixed. Note that the special case in which $\beta = 0$ corresponds to constant arrival rate. Confirming our discussions above, we observe that, regardless of the buffer constraints and source characteristics, all curves approach the same minimum energy per bit level of 2.7512 dB. However, smaller κ and hence more slowly decaying channel memory, lower α and larger β and hence more bursty source, and larger θ and hence stricter buffer constraints, all lower the wideband slope and hence result in degradations in the energy efficiency.

B. Discrete-Time Markov Sources

Finally, we note that we provide above a general framework for energy-efficiency analysis in the low power regime with time-varying sources. While we primarily apply this framework to Markov fluid sources, other source models can be analyzed by following a similar approach. For instance, for a discrete-time Markov ON-OFF source for which the state transition probability matrix is $\mathbf{J} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$, the effective bandwidth is given by $a(\theta, r) = \frac{1}{\theta}\log_e\left(\frac{p_{11}+p_{22}e^{r\theta}+\sqrt{(p_{11}+p_{22}e^{r\theta})^2-4(p_{11}+p_{22}-1)e^{r\theta}}}{2}\right)$. Using the techniques of the proof of Theorem 1, we readily have the following characterization for the discrete-time Markov source model with the same transmission and channel assumptions.

Theorem 2: Assume now that the source arrival follows the discrete-time ON-OFF model described above. Then, the minimum energy per bit and wideband slope expressions as a function of the channel and source parameters and the QoS exponent θ are given, respectively, by

$$\frac{E_b}{N_0}_{\min} = e\log_e 2 = 2.7512 \text{ dB}, \text{ and}$$

$$\mathcal{S}_0 = \frac{1}{\frac{\theta}{\log_e 2}\left[\frac{e-1}{\kappa} + \frac{(\tilde{\eta}-1)}{2}\right] + \frac{e}{2}}$$

where we have defined $\tilde{\eta} = \eta\left(\frac{1}{P_{\text{ON}}}\right)^2$ and

$$\eta = \frac{p_{22}^2+2(1-p_{11})}{2(2-p_{11}-p_{22})} - \frac{[p_{22}(p_{11}+p_{22})-2(p_{11}+p_{22}-1)]^2}{2(2-p_{11}-p_{22})^3}.$$

Based on Theorem 2, similar conclusions as in the Markov fluid source model can immediately be drawn for the discrete-time Markov source as well.

VI. Conclusion

We have investigated the energy efficiency of fixed-rate transmissions of Markov sources over Rayleigh fading channels, and obtained closed-form expressions of the minimum energy per bit and wideband slope. Interestingly, minimum energy per bit is shown to be independent of the source and channel characteristics and buffer constraints. On the other hand, source and channel parameters such as transition rates and probabilities, source burstiness, and channel memory, have significant impact on the wideband slope and hence the energy efficiency in the low-SNR regime when transmitter is operating under QoS constraints, i.e., when QoS exponent $\theta > 0$. In particular, we have observed that increased source burstiness, higher channel fading correlation, and stricter QoS constraints all tend to increase the energy expenditure.

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