# Cognitive Radio Transmissions Exploiting Multi-User Diversity under Channel and Sensing Uncertainty

Gozde Ozcan and M. Cenk Gursoy

***Abstract*—In this paper, a cognitive multiple access channel (MAC) in which the secondary transmitters opportunistically communicate with a secondary receiver (i.e., a secondary base station) is considered. It is assumed that the base station first determines the channel availability through channel sensing and then broadcasts a pilot symbol with energy depending on the channel sensing decision in order to facilitate the linear minimum mean-square estimation (LMMSE) of the channel fading coefficient between itself and each secondary user. The user with the largest estimated channel gain is selected opportunistically for data transmission during each frame. In this setting, achievable rates and energy efficiency are studied by exploiting the multiuser diversity in the presence of channel sensing and channel estimation errors. The interactions between number of users, channel sensing parameters and performance (e.g., detection and false-alarm probabilities), achievable rates, and energy efficiency are analyzed. Performance gains due to collaborative sensing and multi-user diversity are investigated.**

***Index Terms*—Achievable rate, channel sensing, energy efficiency, linear minimum mean-square error channel estimation, multiple access channel, multiuser diversity, probability of detection, and probability of false alarm.**

## I. Introduction

MULTI-USER diversity has been recently exploited in cognitive radio systems with multiple independent communication links by selecting a single user with instantaneously best channel condition to transmit data in a given frame. In particular, the authors in [1] studied the achievable normalized capacity of secondary users in spectrum sharing systems with multiple primary receivers by choosing the user with the highest received signal-to-noise (SNR). The more recent work in [2] mainly focused on the performance of uplink multi-user cognitive radio systems by opportunistically selecting users with the largest capacity based on data and sensing channel quality. Moreover, multiuser diversity gain was analyzed through approximate average capacity in a spectrum sharing system by considering the interference constraint of primary users in [3]. In addition, the authors in [4] investigated the error rate performance of underlay cognitive radio systems with opportunistic user selection under peak transmit power and interference constraints. In [5], the multiuser diversity gain in both multiple-access channel (MAC) and broadcast channel (BC) spectrum sharing cognitive radio networks was determined. Also, the authors in [6] analyzed the multiuser interference diversity for cognitive MAC, BC and parallel access channel in a spectrum sharing environment. The recent work in [7] investigated the average throughput of a cognitive MAC with multiuser scheduling and transmit power control at the secondary transmitters.

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The authors are with the Department of Electrical Engineering and Computer Science, Syracuse University, Syracuse, NY 13244 (e-mail: {gozcan, mcgursoy}@syr.edu).

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Opportunistic selection and transmission are highly dependent on the channel conditions, which can only be learned imperfectly in dynamic environments. However, aforementioned works have not taken into account imperfect estimation of the data link between secondary users together with channel sensing errors. With this motivation, in this paper, we consider a practical scenario in which multiple secondary transmitters opportunistically access to a secondary base station with imperfect channel sensing and channel estimation results. We first assume that the base station combines the measurements of secondary transmitters to decide whether the primary user is active or not, and then feeds back the sensing decision. Following channel sensing, the base station broadcasts a pilot symbol with energy depending on the sensing decision. The channel between each secondary transmitter and the base station is estimated via LMMSE at each secondary transmitter based on the received pilot symbol by using the recent results in [8]. It is assumed that the channels of different secondary transmitters experience independent and identical fading and the user with the largest estimated channel gain is selected to transmit in each frame. Under these assumptions, we derive an achievable rate expression for opportunistic user selection and data transmission when both channel sensing and estimation are performed with possible errors. Then, we investigate the energy efficiency defined as rates normalized by the average energy consumption.

## II. System Model

We consider a cognitive radio model where secondary users can coexist with the primary user in the channel as long as their interference level is below a threshold that the primary user can tolerate. Secondary users are assumed to employ frames of $M$ symbols in which the initial duration of $N$ symbols is allocated for channel sensing.

### A. Channel Sensing

We consider spatially distributed $K$ secondary users performing channel sensing collaboratively. Each secondary user $k \in \{1, 2, \dots, K\}$ collects $N$ samples. With these assumptions, the received signal at the $k^{\text{th}}$ secondary user in the channel sensing phase can be expressed as

$$\begin{aligned} &z_k(n) = w_k(n) && \text{in the absence of a primary user} \\ &z_k(n) = g_k(n)s(n) + w_k(n) && \text{in the presence of a primary user} \end{aligned} \tag{1}$$

where $n = 1, 2, \dots, N$. Above, $w_k$ denotes background Gaussian noise samples with zero-mean and variance $\mathbb{E}\{|w_k|^2\} = \sigma_w^2$. In addition, $s$ represents the primary user's signal with power $\rho$. Also, $g_k$ is zero-mean, circularly-symmetric complex fading coefficient between the primary transmitter and $k$-th secondary user with variance $\mathbb{E}\{|g_k|^2\} = \sigma_g^2$.

Each user measures the total energy gathered in a duration of $N$ symbols and transmit their measurements to the secondary base station. Subsequently, the base station employs an energy detector which combines the measurements of each secondary transmitter with equal gain. Under the above-mentioned statistical assumptions, the test statistic is obtained as [9]

$$T(z) = \frac{1}{NK}\sum_{n=1}^{N}\sum_{k=1}^{K}|z_k(n)|^2 \gtrless_{\hat{\mathcal{H}}_0}^{\hat{\mathcal{H}}_1} \lambda \tag{2}$$

where $\hat{\mathcal{H}}_1$ and $\hat{\mathcal{H}}_0$ denote busy and idle sensing decisions, respectively and $\lambda$ is the detection threshold. If the number of samples $N, K$ is large enough, $T(z)$ can be approximated as a Gaussian random variable by applying the Central Limit Theorem. With this characterization, the false alarm and detection probabilities are given by

$$\begin{aligned} P_f &= \Pr\{T(z) > \lambda|\mathcal{H}_0\} = \Pr(\hat{\mathcal{H}}_1|\mathcal{H}_0) = Q\left(\frac{\lambda - \sigma_w^2}{\sqrt{\frac{\sigma_w^4}{NK}}}\right), \\ P_d &= \Pr\{T(z) > \lambda|\mathcal{H}_1\} = \Pr(\hat{\mathcal{H}}_1|\mathcal{H}_1) = Q\left(\frac{\lambda - (\rho\sigma_g^2 + \sigma_w^2)}{\sqrt{\frac{(\rho\sigma_g^2+\sigma_w^2)^2}{NK}}}\right). \end{aligned} \tag{3}$$

Above, $Q(\cdot)$ is the Gaussian $Q$-function and, $\mathcal{H}_0$ and $\mathcal{H}_1$ denote the true hypotheses corresponding to the absence and the presence, respectively, of the primary user. After the base station determines the channel availability, the sensing result is fed back to the secondary transmitters.

Note that the remainder of the analysis depends on the sensing strategy only through the false alarm and detection probabilities. Hence, any other collaborative sensing scheme (e.g., in which secondary users send their decisions rather than the accumulated energy) can be considered as well.

### *B. Channel Estimation and Opportunistic User Selection*

The base station initiates the channel training phase following the channel sensing. Depending on whether the sensing decision is idle or busy, the base station sends the training symbol with energy $\mathcal{E}_{t0}$ or $\mathcal{E}_{t1}$, respectively, in order to facilitate at each secondary user the estimation of the corresponding channel fading coefficient. In general, we have $\mathcal{E}_{t1} \leq \mathcal{E}_{t0}$ in order to limit the interference inflicted on the primary user.

First, we express the pilot symbol received by the $k$-th secondary user under four possible cases, arising as a result of the true channel states and corresponding channel sensing decisions as follows:

$$y_k = \begin{cases} h_k\sqrt{\mathcal{E}_{t1}} + w_k + g_k s & \text{if } (\mathcal{H}_1, \hat{\mathcal{H}}_1) \\ h_k\sqrt{\mathcal{E}_{t0}} + w_k + g_k s & \text{if } (\mathcal{H}_1, \hat{\mathcal{H}}_0) \\ h_k\sqrt{\mathcal{E}_{t1}} + w_k & \text{if } (\mathcal{H}_0, \hat{\mathcal{H}}_1) \\ h_k\sqrt{\mathcal{E}_{t0}} + w_k & \text{if } (\mathcal{H}_0, \hat{\mathcal{H}}_0) \end{cases} \tag{4}$$

where $h_k$ is circularly-symmetric complex Gaussian distributed fading coefficient of the channel from the $k$-th secondary user to the base station with zero mean and variance $\sigma_h^2$.

Then, the secondary users perform linear MMSE (LMMSE) estimation by employing the received pilot symbol $y_k$. Since we assume imperfect channel sensing, the channel fading coefficient is estimated with possible channel sensing errors. Hence, the LMMSE estimator of $h_k$ under the sensing decision $\hat{\mathcal{H}}_i$ for $i \in \{0, 1\}$ is given by

$$\begin{aligned} \hat{h}_{k,i} &= \frac{\mathbb{E}\{h_k y_k^*|\hat{\mathcal{H}}_i\}}{\mathbb{E}\{|y_k|^2|\hat{\mathcal{H}}_i\}} y_k \\ &= \frac{\sqrt{\mathcal{E}_{t_{k,i}}}\sigma_h^2}{\Pr(\mathcal{H}_0|\hat{\mathcal{H}}_i)(\mathcal{E}_{t_{k,i}}\sigma_h^2 + \sigma_w^2) + \Pr(\mathcal{H}_1|\hat{\mathcal{H}}_i)(\mathcal{E}_{t_{k,i}}\sigma_h^2 + \sigma_w^2 + \sigma_g^2\rho)} y_k \\ &= a_{k,i} y_k. \end{aligned} \tag{5}$$

Above, $a_{k,i}$ represents the LMMSE coefficient for the $k$-th secondary user under the channel sensing decision $\hat{\mathcal{H}}_i$. Moreover, the conditional probabilities in (5) are given by

$$\Pr\{\mathcal{H}_j|\hat{\mathcal{H}}_i\} = \frac{\Pr\{\hat{\mathcal{H}}_i|\mathcal{H}_j\}\Pr\{\mathcal{H}_j\}}{\Pr\{\hat{\mathcal{H}}_i|\mathcal{H}_0\}\Pr\{\mathcal{H}_0\} + \Pr\{\hat{\mathcal{H}}_i|\mathcal{H}_1\}\Pr\{\mathcal{H}_1\}}$$

where $i, j \in \{0, 1\}$, and $\Pr\{\mathcal{H}_1\}$ and $\Pr\{\mathcal{H}_0\}$ are the prior probabilities of channel being busy and idle, respectively. Also, the conditional variance of the LMMSE estimate of the channel fading coefficient between the base station and $k$-th secondary user is obtained as

$$\sigma^2_{\hat{h}_{k|\mathcal{H}_j,\hat{\mathcal{H}}_i}} = \begin{cases} a_{k,i}^2(\mathcal{E}_{t_{k,i}}\sigma_h^2 + \sigma_w^2) & \text{if } j = 0 \\ a_{k,i}^2(\mathcal{E}_{t_{k,i}}\sigma_h^2 + \sigma_w^2 + \sigma_g^2\rho) & \text{if } j = 1 \end{cases}. \tag{6}$$

Additionally, the estimation error is defined as $\tilde{h}_k = h_k - \hat{h}_k$. Thus, given the true state of the channel and the corresponding sensing decision, the conditional error variance for each secondary transmitter $k$ can be written as

$$\sigma^2_{\tilde{h}_{k|\mathcal{H}_j,\hat{\mathcal{H}}_i}} = \sigma^2_{\hat{h}_{k|\mathcal{H}_j,\hat{\mathcal{H}}_i}} + \left(1 - 2a_{k,i}\sqrt{\mathcal{E}_{t_{k,i}}}\right)\sigma_h^2. \tag{7}$$

It should be noted that $\hat{h}_k$ has a zero-mean circularly symmetric complex Gaussian distribution with variance $\sigma^2_{\hat{h}_{k,i}}$. Hence, the square of the envelope of the channel estimate $|\hat{h}_k|^2$ under the sensing decision $\hat{\mathcal{H}}_i$ is exponentially distributed with $\sigma^2_{\hat{h}_{k,i}}$.

The secondary user with the largest estimated channel gain among $K$ secondary users, i.e., with

$$|\hat{h}_i|^2 = \max_{k\in\{1,\ldots,K\}} |\hat{h}_{k,i}|^2 \quad \text{for } i \in \{0, 1\}, \tag{8}$$

is selected to transmit in each frame. Above, $|\hat{h}_0|^2$ is the maximum estimated channel gain if the channel is detected as idle whereas $|\hat{h}_1|^2$ denotes the maximum estimated channel gain if the channel is detected as busy. Furthermore, the probability density function of the maximum channel gain is given at the top of the next page in (9) with $\sigma^2_{\hat{h}_{k|\mathcal{H}_j,\hat{\mathcal{H}}_i}}$ defined in (6).

## III. Achievable Rates and Energy Efficiency

In this section, we first determine an achievable rate expression and then investigate the energy efficiency. Following opportunistic user selection among $K$ secondary users, the selected secondary transmitter initiates the data transmission. In the case of channel being sensed as busy, the secondary transmitter sends the signal $x_1$ with energy $\mathcal{E}_1 = \mathbb{E}\{|x_1|^2\}$ whereas the signal $x_0$ with energy $\mathcal{E}_0 = \mathbb{E}\{|x_0|^2\}$ is transmitted in the case of channel being sensed as idle. This two-level energy scheme is a protection mechanism to avoid the primary user from harmful interference. In addition, we impose

$$P_d\mathcal{E}_1 + (1 - P_d)\mathcal{E}_0 \leq I_0 \tag{10}$$

$$f_{|\hat{h}_i|^2}(x) = \sum_{j=0}^{1} \frac{K \Pr\{\mathcal{H}_j|\hat{\mathcal{H}}_i\}}{\sigma^2_{\hat{h}_{k|\mathcal{H}_j,\hat{\mathcal{H}}_i}}} \exp\left(-\frac{x}{\sigma^2_{\hat{h}_{k|\mathcal{H}_j,\hat{\mathcal{H}}_i}}}\right)\left\{1-\exp\left(-\frac{x}{\sigma^2_{\hat{h}_{k|\mathcal{H}_j,\hat{\mathcal{H}}_i}}}\right)\right\}^{K-1}. \tag{9}$$

in order to limit the average interference inflicted on the primary users, together with peak constraints, $\mathcal{E}_0 \leq \mathcal{E}_{pk,0}$ and $\mathcal{E}_1 \leq \mathcal{E}_{pk,1}$. Above, $I_0$ is proportional to the maximum average interference level that the primary user can tolerate[1]. Now, given the true nature of the primary user's activity and channel sensing results, the signal of the selected secondary transmitter received at the base station can be expressed as

$$y_r = \begin{cases} \hat{h}_1 x_1 + \tilde{h}_1 x_1 + n & \text{if } (\mathcal{H}_0, \hat{\mathcal{H}}_1) \\ \hat{h}_0 x_0 + \tilde{h}_0 x_0 + n & \text{if } (\mathcal{H}_0, \hat{\mathcal{H}}_0) \\ \hat{h}_1 x_1 + \tilde{h}_1 x_1 + n + p & \text{if } (\mathcal{H}_1, \hat{\mathcal{H}}_1) \\ \hat{h}_0 x_0 + \tilde{h}_0 x_0 + n + p & \text{if } (\mathcal{H}_1, \hat{\mathcal{H}}_0) \end{cases} \tag{11}$$

where $\hat{h}_i$ and $\tilde{h}_i$ denote estimate of the fading coefficient and estimation error, respectively, of the selected secondary transmitter under sensing decision $\hat{\mathcal{H}}_i$. In addition, $p$ denotes the active primary user's signal received at the base station. We assume that $p$ is Gaussian distributed with mean zero and variance $E\{|p|^2\} = \sigma_p^2$. Finally, $n$ denotes the zero-mean background noise with variance $\sigma_n^2$.

*Proposition 1:* An achievable sum rate expression for cognitive MAC channel with opportunistic user selection in the presence of channel sensing and estimation errors is given by

$$R(\mathcal{E}) = \sum_{i,j=0}^{1}\sum_{k=0}^{K-1} \Pr\{\mathcal{H}_j|\hat{\mathcal{H}}_i\}\Pr\{\hat{\mathcal{H}}_i\} \frac{(M-N-1)K}{M\log_e(2)} \frac{(-1)^{(k+1)}}{k+1} \times \binom{K-1}{k} e^{\frac{b_{j,i}}{d_i}} E_i\left(-\frac{b_{j,i}}{d_i}\right). \tag{12}$$

In the above expression, $\binom{K-1}{k}$ is the binomial coefficient, $E_i(.)$ is the exponential integral function [10, eq. 8.211.1] and the scaling term $\frac{M-N-1}{M}$ reflects the rate reduction due to allocating a duration of $N$ symbols for channel sensing and a duration of 1 symbol for pilot transmission in each frame of $M$ symbols. Also, $\Pr\{\hat{\mathcal{H}}_1\}$ and $\Pr\{\hat{\mathcal{H}}_0\}$ denote the probabilities of channel being sensed as busy and idle, respectively, whose expressions are given by

$$\begin{aligned} \Pr\{\hat{\mathcal{H}}_1\} &= \Pr\{\mathcal{H}_0\}P_f + \Pr\{\mathcal{H}_1\}P_d \\ \Pr\{\hat{\mathcal{H}}_0\} &= \Pr\{\mathcal{H}_0\}(1-P_f) + \Pr\{\mathcal{H}_1\}(1-P_d). \end{aligned} \tag{13}$$

Moreover, $b_{j,i}$ and $d_i$ in (12) are expressed as

$$b_{j,i} = \frac{k+1}{\sigma^2_{\hat{h}_{k|\mathcal{H}_j,\hat{\mathcal{H}}_i}}}, \quad d_i = \frac{\mathcal{E}_i}{\mathcal{E}_i\sigma^2_{\tilde{h}_i} + \sigma_n^2 + \Pr\{\mathcal{H}_1|\hat{\mathcal{H}}_i\}\sigma_p^2} \quad i,j \in \{0,1\}. \tag{14}$$

*Proof*: The achievable rate is determined through the mutual information between transmitted and received signals conditioned on the sensing decision and the estimate of the fading coefficient as follows:

$$I(x, y_r|\hat{h}, \hat{\mathcal{H}}) = \sum_{i=0}^{1} \Pr\{\hat{\mathcal{H}}_i\} \mathbb{E}_{\hat{h}_i, x_i, y_r}\left\{\log\left(\frac{f(y_r|x_i, \hat{h}_i, \hat{\mathcal{H}}_i)}{f(y_r|\hat{h}_i, \hat{\mathcal{H}}_i)}\right)\right\}. \tag{15}$$

[1]More specifically, maximum tolerable average interference is $I_0\mathbb{E}\{|g|^2\}$ where $g$ is the fading coefficient between the secondary transmitter and primary receiver.

In the case of Gaussian inputs, we have

$$\begin{aligned} f(y_r|x_i, \hat{h}_i, \hat{\mathcal{H}}_i) =& \frac{\Pr\{\mathcal{H}_0|\hat{\mathcal{H}}_i\}}{\pi(\sigma^2_{\tilde{h}_i}\mathcal{E}_i + \sigma_n^2)} e^{-\frac{|y_r - \hat{h}_i x_i|^2}{(\sigma^2_{\tilde{h}_i}\mathcal{E}_i + \sigma_n^2)}} \\ &+ \frac{\Pr\{\mathcal{H}_1|\hat{\mathcal{H}}_i\}}{\pi(\sigma^2_{\tilde{h}_i}\mathcal{E}_i + \sigma_n^2 + \sigma_p^2)} e^{-\frac{|y_r - \hat{h}_i x_i|^2}{(\sigma^2_{\tilde{h}_i}\mathcal{E}_i + \sigma_n^2 + \sigma_p^2)}}, \end{aligned} \tag{16}$$

$$\begin{aligned} f(y_r|\hat{h}_i, \hat{\mathcal{H}}_i) =& \frac{\Pr\{\mathcal{H}_0|\hat{\mathcal{H}}_i\}}{\pi(|\hat{h}_i|^2\mathcal{E}_i + \sigma^2_{\tilde{h}_i}\mathcal{E}_i + \sigma_n^2)} e^{-\frac{|y_r|^2}{(|\hat{h}_i|^2\mathcal{E}_i + \sigma^2_{\tilde{h}_i}\mathcal{E}_i + \sigma_n^2)}} \\ &+ \frac{\Pr\{\mathcal{H}_1|\hat{\mathcal{H}}_i\}}{\pi(|\hat{h}_i|^2\mathcal{E}_i + \sigma^2_{\tilde{h}_i}\mathcal{E}_i + \sigma_n^2 + \sigma_p^2)} e^{-\frac{|y_r|^2}{(|\hat{h}_i|^2\mathcal{E}_i + \sigma^2_{\tilde{h}_i}\mathcal{E}_i + \sigma_n^2 + \sigma_p^2)}}. \end{aligned} \tag{17}$$

It should be noted that the above conditional probabilities are actually mixture of Gaussian distributions due to channel sensing errors. Therefore, a closed-form expression for mutual information in (15) is not available. On the other hand, we can still find a closed-form expression by replacing the conditional distributions in (16) and (17) with pure Gaussian densities having the same corresponding variances. Since Gaussian noise is the worst-case noise, we obtain the following lower bound:

$$\mathbb{E}\left\{\log\left(\frac{f(y_r|x_i, \hat{h}_i, \hat{\mathcal{H}}_i)}{f(y_r|\hat{h}_i, \hat{\mathcal{H}}_i)}\right)\right\} \geq \mathbb{E}\left\{\log\left(1 + \frac{\mathcal{E}_i|\hat{h}_i|^2}{\sigma^2_{\tilde{h}_i}\mathcal{E}_i + \sigma_n^2 + \Pr\{\mathcal{H}_1|\hat{\mathcal{H}}_i\}\sigma_p^2}\right)\right\}.$$

Applying the above lower bounds to the mutual information in (15), and using the identity [10, eq. 4.337.2] to evaluate the expectation with respect to the distribution of the channel gain of the selected secondary transmitter given in (9), we obtain the expression in (12). □

Since we assume that power consumption in channel sensing is negligible, the average energy consumption in a block is expressed as $\mathcal{E}_{avg} = \Pr\{\hat{\mathcal{H}}_0\}\mathcal{E}_0 + \Pr\{\hat{\mathcal{H}}_1\}\mathcal{E}_1$. Hence, the achievable rate in (12) normalized by average energy consumption represents the energy efficiency (in bits per joule) as follows

$$\frac{R(\mathcal{E})}{\mathcal{E}_{avg}} = \frac{R(\mathcal{E})}{\Pr\{\hat{\mathcal{H}}_0\}\mathcal{E}_0 + \Pr\{\hat{\mathcal{H}}_1\}\mathcal{E}_1}. \tag{18}$$

## IV. Numerical Results

In this section, numerical results are presented. In the computations, it is assumed that $M = 1000, N = 100$, noise power $\sigma_w^2 = \sigma_n^2 = 0.2$, the variances $\sigma_g^2 = 0.1$ and $\sigma_h^2 = 0.1$, the primary user's received signal power is 0 dB. The energy of the training symbol is set to $-10$ dB or 0 dB in the cases of channel being sensed to be busy or idle, respectively. It is assumed that $\Pr\{\mathcal{H}_0\} = 0.6$, $\Pr\{\mathcal{H}_1\} = 0.4$.

In Fig. 1, we plot achievable rate $R(\mathcal{E})$, the probabilities of detection $P_d$ and false-alarm $P_f$ vs. number of secondary transmitters $K$. The achievable rate is maximized over transmission energies $\mathcal{E}_0$ and $\mathcal{E}_1$ subject to average interference constraint of $I_0 = 5$ dB together with peak constraints, $\mathcal{E}_{pk,0} = \mathcal{E}_{pk,1} = 10$ dB. In cases 1, 2 and 3, detection and false alarm probabilities are fixed to some constants.

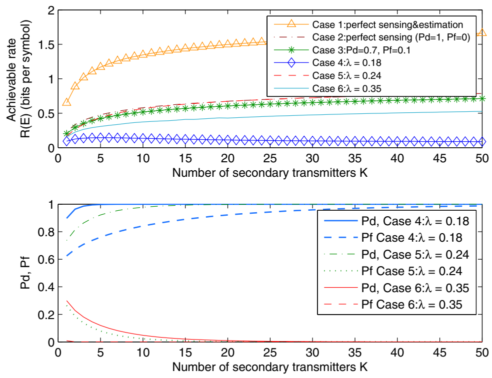


Fig. 1. The achievable rate $R(\mathcal{E})$, the probabilities of detection and false-alarm, $P_d$, $P_f$, vs. number of secondary transmitters $K$.

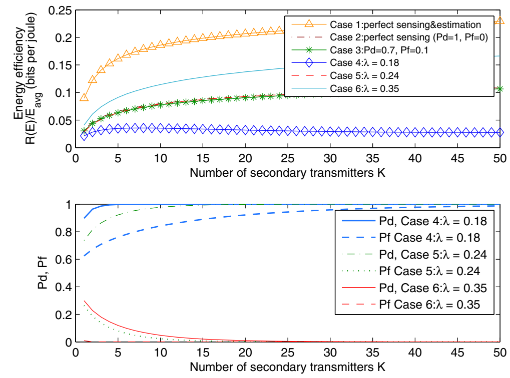


Fig. 2. The energy efficiency $R(\mathcal{E})/\mathcal{E}_{avg}$, the probabilities of detection and false-alarm, $P_d$, $P_f$, vs. number of secondary transmitters $K$.

On the other hand for cases 4, 5 and 6, the secondary transmitters perform cooperative sensing with different choices of detection thresholds $\lambda$ and corresponding detection and false alarm probabilities are displayed in the lower subfigure. We observe that achievable rates tend to increase with the number of secondary transmitters in all cases except case 4 (in which detection and false-alarm probabilities increase to 1 as $K$ increases). This improvement is because the more secondary transmitters are present, the more likely the selected user has better channel condition in each frame, achieving a higher rate. When the channel sensing and estimation are performed without any errors, i.e., $P_d = 1$, $P_f = 0$ and $\sigma^2_{\tilde{h}_k} = 0$ corresponding to case 1, we have the highest achievable rate. In case 2, we assume perfect sensing with possible channel estimation errors whereas imperfect sensing result with $P_d = 0.7$ and $P_f = 0.1$ is considered in case 3. Since the probabilities $\Pr\{\mathcal{H}_1|\hat{\mathcal{H}}_0\}$ and $\Pr\{\mathcal{H}_0|\hat{\mathcal{H}}_1\}$ are zero in perfect sensing, diminishing the effect of additive disturbance of primary user's signal, we have higher rate in case 2 compared to case 3. In case 4, the probabilities of detection and false-alarm increase to 1 with increasing $K$. Thus, the base station detects the channel as busy more often and sends pilot symbol with lower energy (i.e., $-10$ dB), which affects the channel estimation performance adversely. Subsequently, based on the channel sensing decision of the base station, the best secondary transmitter performs data transmission with lower energy, which leads to lower rate among other cases. In case 5, we have reliable sensing performance due to the fact that $P_d$ and $P_f$ approach 1 and 0, respectively, as $K$ increases. In this case, we basically benefit from collaborative sensing. Achievable rate increases as well due to improved sensing results and a similar performance is obtained as in case 2 in which we assume perfect sensing. In case 6 (i.e., $\lambda = 0.35$), as $K$ increases, the probabilities of detection and false-alarm start to diminish and approach 0. Therefore, the base station detects the channel as idle more frequently, hence the transmission energy of the selected secondary transmitter is limited by the interference constraint to protect the primary user's transmission, which results in lower achievable rates.

In Fig. 2, we display the energy efficiency $R(\mathcal{E})/\mathcal{E}_{avg}$, the probabilities of detection $P_d$ and false-alarm $P_f$ as a function of the number of secondary transmitters $K$. Note that transmission energies $\mathcal{E}_0$ and $\mathcal{E}_1$, which maximize the achievable rate, are used to compute the energy efficiency. We observe that energy efficiency generally increases with the number of secondary users (except again in case 4). Additionally, the highest energy efficiency is achieved with perfect spectrum sensing and channel estimation corresponding to case 1. On the other hand, we have the lowest energy efficiency in case 4 due to the lower achievable rates experienced because of increasing false-alarm probability. Hence, we conclude that channel sensing performance plays an important role on the energy efficiency as well.

## V. Conclusion

We have studied the performance of cognitive transmissions in an opportunistic multiple-access setting under the practical assumptions of imperfect channel sensing and channel estimation. We have determined an achievable sum-rate expression and demonstrated that channel sensing and estimation performance, as well as multi-user diversity, have significant impact on both the rates and energy efficiency.

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